Study on the Coefficients of Utilization of the Luminous Ceiling

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光天井の照明率に関する研究

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論文要旨

光天井の照明率については、これまで数多くの研究がなされているが、まだ完成したとはいえない。

光天井の問題は、光天井空間の効率とパネルによる固有照明率の問題に分けられる。光天井空間の効率算定式としては Moon-Spencer の式、イギリス IES の式、アメリカ IES の式など幾つかの式があるが、いずれも問題点を含んでいる。

さらに最近では、フリズムパネルが使用されるようになるに至り、パネルの配光を従来のように完全拡散として取り扱うことは、固有照明率の算定に誤差を与える要因となっている。

本報では、理論的に従来の効率算定式よりも正確と考えられる新しい効率算定式を導き、その実験的に簡単なチェックを行ない妥当性を確かめるとともに、光天井パネルが、任意の配光特性をもつ場合の固有照明率の算定を可能にした。これにより、従来よりも精度よく光天井の照明率を算定し得るようになった。

1. Introduction

In many cases the data of the coefficients of utilization generally found for luminous ceiling are not applicable to the conditions encountered in practice. Although there are several formulae⁽¹⁻⁴⁾ for the light output ratio of a luminous ceiling, sufficient information may not always be derived from these formulae. With the advent of prismatic panels, it becomes necessary to know more accurate value of utilance for a imperfectly diffusing luminous panel.

The purpose of this paper is to provide more valid formula for the light output ratio than the commonly used ones and more flexible data for computing utilance of a luminous ceiling with a

controlling medium (prismatic panel).

2. Light output ratio of a luminous ceiling

Let us consider a luminous ceiling system as shown in **Fig. 1 (a)**.

where ρ_1 = reflection factor of the ceiling,

ρ_2 = reflection factor of the plenum wall,

Fig. 1. (a) Schematic figure of a ceiling cavity.
(b) Equivalent ceiling cavity in which cavity 1 is replaced by an imaginary plane of fixtures with an equivalent reflection factor. The ceiling cavity index K_{cc} is $L \cdot W / (H \cdot L + W)$.

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ρ =reflection factor of a panel,

τ =transmission factor of a panel,

η' =upward flux ratio of the fittings in the ceiling cavity,

η'' =downward flux ratio of the fittings in the ceiling cavity.

In this case, it is very difficult to calculate inter-reflections in this ceiling cavity because of obstructions.

For simplicity, we shall consider a equivalent reflection factor ρ_1' of an imaginary fitting plane, which is given by

$$\rho_1' = \rho_f \delta + \rho_{eff}(K_{cc}')(1-\delta)^2 \quad \dots\dots\dots(1)$$

where ρ_f =reflection factor of obstructions (mainly fittings) as seen from below,

$\rho_{eff}(K_{cc}')$ =effective reflection factor of the cavity 1,

δ =obstruction factor defined by the ratio of the area occupied by obstructions (mainly fitting) to that of the imaginary fitting plane.

Especially in case of the ceiling mounted fittings,

$$\rho_1' = \rho_f \delta + \rho_1(1-\delta) \doteq \rho_1 \quad \dots\dots\dots(2)$$

on the condition of that δ is sufficiently small or $\rho_f = \rho_1$.

Assuming that all surfaces are perfectly diffusing and each has a uniform reflection factor, the following matrix equation holds in the cavity without-panel in Fig. 1 (b).

$$\begin{bmatrix} 1 & -\rho_1'S_{12}' \\ -\rho_2S_{21}' & 1-\rho_2S_{22}' \end{bmatrix} \begin{bmatrix} L_1' \\ L_2' \end{bmatrix} = \begin{bmatrix} L_{01}' \\ L_{02}' \end{bmatrix} \quad \dots\dots\dots(3)$$

where S_{ij} =shape factor between the surface element i and j ,

L_i =average luminous emittance of the surface i ,

L_{0i} =average initial luminous emittance of the surface i .

The solution of eq. (3) is given by:

$$\left. \begin{aligned} L_1' &= \frac{(1-\rho_2S_{22}')L_{01}' + \rho_1'S_{12}'L_{02}'}{1-\rho_2S_{22}'-\rho_2\rho_1'S_{21}'S_{12}'} \\ L_2' &= \frac{L_{02}'+\rho_2S_{21}'L_{01}'}{1-\rho_2S_{22}'-\rho_2\rho_1'S_{21}'S_{12}'} \end{aligned} \right\} \quad \dots\dots\dots(4)$$

If we normalize the total lamp flux to unity, we have

$$\left. \begin{aligned} L_{01}' &= \eta' \rho_{eff}(K_{cc}')(1-\delta) \doteq \eta' \rho_1(1-\delta) \\ L_{02} &= \eta''(1-S)\rho_2 \frac{K_{cc}}{2} \end{aligned} \right\} \quad \dots\dots\dots(5)$$

Here S , the shape factor, is the ratio of the flux received by the panel to the total flux emitted from the imaginary fitting plane.

Since direct and indirect flux received by the panel are expressed as

$$\left. \begin{aligned} F_p(\text{direct}) &= \eta''S \\ F_p(\text{indirect}) &= S\rho_1L_1' + S\rho_2L_2' \end{aligned} \right\} \quad \dots\dots\dots(6)$$

Hence, from eqs. (4)~(6) total flux received by the panel is given as follows with the aid of the reciprocal theorem and the enclosure property of radiation system.

$$\begin{aligned} F_p &= F_p(\text{direct}) + F_p(\text{indirect}) \\ &= \eta''S + \frac{1}{M} \left\{ S(1-\rho_2\{1-(1-S)K_{cc}\}) \right. \\ &\quad \left. + \rho_2 \frac{(1-S)^2}{2} K_{cc} \right\} \eta' \rho_1(1-\delta) \\ &\quad + \{(1+\rho_1')(1-S)\} \eta''(1-S)\rho_2 \frac{K_{cc}}{2} \quad \dots\dots\dots(7) \end{aligned}$$

where

$$M = 1 - \rho_2 + 0.5\rho_2(1-S)K_{cc}\{2 - \rho_1'(1-S)\} \quad \dots\dots\dots(8)$$

Eq. (7) is too complicated for practical application.

So the concept of effective reflection factor is introduced for practical use, and the following equation is proposed in place of eq. (7):

$$F_p = \eta''S + \left\{ \eta' \frac{\rho_1}{\rho_1'}(1-\delta) + \eta''(1-S) \right\} \rho_{eff} \quad \dots\dots\dots(9)$$

in which ρ_{eff} is the effective reflection factor⁶⁾ of the ceiling cavity which is a function of ρ_1' , ρ_2 and K_{cc} (See appendix 2).

Table 1 shows the comparison between two calculated values obtained from eqs. (7) and (9).

In the condition 1 often encountered in practice, somewhat larger values were estimated from eq. (9). Eq. (9), however, is better than eq. (7) for practical use due to its simplicity.

The increase in effective panel transmission factor ρ_{eff} taking account of the actual panel transmission factor τ , reflection factor ρ and the effective ceiling cavity reflection factor ρ_{eff} is given by

$$\tau_{eff} = \sum_{n=0}^{\infty} \tau(\rho \cdot \rho_{eff})^n = \frac{\tau}{1 - \rho \cdot \rho_{eff}} \quad \dots\dots\dots(10)$$

as is generally known.

Finally, the new formula for the light output ratio η of a luminous ceiling is obtained as follows:

$$\eta = \frac{\tau}{1 - \rho \cdot \rho_{eff}} \left[\eta''S + \left\{ \eta' \frac{\rho_1}{\rho_1'}(1-\delta) + \eta''(1-S) \right\} \right]$$

Table 1. Comparison of the values of F_p calculated by an exact expression (7) and an approximate expression (9).

K_{cc}	ρ_1'/ρ_2	Cond. 1. ($\delta=0.1, \eta'=0.22, \eta''=0.68$)				Cond. 2. ($\delta=0, \eta'=\eta''=1/2$)			
		70/70	70/50	50/70	50/50	70/70	70/50	50/70	55/50
2	Exact.	0.667	0.615	0.619	0.576	0.693	0.638	0.596	0.554
	Approx.	0.683	0.641	0.622	0.589	0.685	0.624	0.596	0.548
3	Exact.	0.715	0.677	0.668	0.636	0.742	0.702	0.643	0.612
	Approx.	0.734	0.706	0.674	0.650	0.740	0.695	0.645	0.607
5	Exact.	0.755	0.730	0.710	0.683	0.784	0.758	0.683	0.664
	Approx.	0.773	0.756	0.716	0.703	0.782	0.753	0.683	0.659
10	Exact.	0.786	0.773	0.743	0.733	0.816	0.803	0.716	0.705
	Approx.	0.802	0.797	0.752	0.746	0.814	0.804	0.716	0.705
15	Exact.	0.800	0.788	0.755	0.747	0.830	0.819	0.729	0.720
	Approx.	0.811	0.809	0.765	0.759	0.825	0.820	0.729	0.719
20	Exact.	0.802	0.795	0.761	0.755	0.833	0.826	0.733	0.727
	Approx.	0.817	0.815	0.770	0.768	0.832	0.827	0.733	0.727

$$\times \rho_{eff} \Big] \dots\dots\dots (11)$$

In particular, eq. (11) reduces to the equation of I.E.S. (London)³⁾

$$\eta = \frac{\tau(\eta'' + \eta' \rho_{eff})}{1 - \rho \cdot \rho_{eff}} \dots\dots\dots (12)$$

as $\delta=0, S=1$, and to the equation of I.E.S. (U.S.A.)⁴⁾

$$\eta = \frac{\tau \{S/2 + (1-S/2)\rho_{eff}\}}{1 - \rho \cdot \rho_{eff}} \dots\dots\dots (13)$$

as $\delta=0, \eta'=\eta''=\frac{1}{2}$.

Besides these eqs., the Moon-Spencer's equation⁵⁾

$$\eta = \frac{\tau(1+\rho_1)}{2(1-\rho \cdot \rho_1)} \dots\dots\dots (14)$$

and the Robertson-Hutchcroft's equation²⁾

$$\eta = \frac{\tau(A_p + A_{12}\rho_1')}{A(1-\rho \cdot \rho_1')} \dots\dots\dots (15)$$

are known as the main formulae for the output ratio, in which

$$\rho_1' = \frac{A_1}{A_{12}}\rho_1 + \frac{A_2}{A_{12}}\rho_2$$

- where A_p =area of the translucent diffuser,
- A_1 =area of ceiling,
- A_2 =area of side walls,
- A_{12} =area of the cavity surfaces, including sides and all obstructions in the cavity,
- $A=A_p+A_{12}$ =total area of the cavity.

Eqs. (13), (14) and (15) are valid only when lamps in a ceiling cavity are not installed in fittings. Eq. (13) is a good approximation when we can neglect the area of the bare lamps. As lamps, however, are usually installed in fittings, eq. (12) is more effective in the actual conditions. Eq. (12), however, has assumptions that all the downward flux emitted from fittings is considered to be directed to a panel com-

pletely and that the effect of fitting is neglected. On the contrary, eq. (11) developed by the author do not have any limitations to the applications as the other eqs. and is applicable not only to fitting but also to bare lamps.

To make the validity of eq. (11) clear, a model ceiling cavity was designed in such a way that the lamp spacing and the mounting height of lamps could be altered as shown in **Fig. 2**. The test

Fig. 2. The model cavity used for checking the validity of eq. (11). The ceiling cavity index can be adjusted by changing the position of the imaginary fitting plane and the ceiling panel.

cavity has a rectangular cross section of 1 m×1.3 m and a height of 36 cm, with the interior painted white with a reflectance of 80 per cent. As the panel materials, a diffusing plastic panel ($\tau=0.50, \rho=0.40$) and a prismatic plastic panel ($\tau=0.72, \rho=0.18$) were used.

The light output ratio was measured by using a large-size automatic distribution photometer. In this case temperature dependency of the flux was taken into account.

The calculated values of the light output ratio from eq. (11) were compared with measured values in **Table 2**. Both values show good agreement within our experimental errors, so that it can be concluded that the newly developed formula for the light output ratio, eq. (11), has sufficient accuracy for our

Table 2. Measured and calculated light output ratio for a diffusing panel and a prismatic panel.

K	S	δ	Light output ratio			
			Diffusing		Prismatic panel	
			Meas.	Calc.	Meas.	Calc.
5.65	0.84	0.4	0.53 ₅	0.54 ₅	0.62 ₅	0.65
4.25	0.80	0.2	0.55 ₅	0.52	0.64 ₅	0.62
3.77	0.78	0.4	0.52	0.53	0.61	0.63
2.82	0.72	0.2	0.54	0.54	0.62 ₅	0.64

practical use.

3. Utulance for a luminous ceiling

In order to get the utulance for a luminous ceiling, it is necessary to know the value of the direct ratio which is defined as the percentage to the total downward flux of downward flux that strikes on the working plane directly.

Now, if luminous intensity per unit area of the ceiling panel at an angle θ from the normal as shown in Fig. 3 is given by :

Fig. 3. Definition of coordinates.

$$I(\theta) = I_0 \cos^n \theta, \dots\dots\dots(16)$$

the flux emitted from this area is given by

$$\begin{aligned} dF &= \int I(\theta) dw = 2\pi \int_0^{\pi/2} I_0 \cos^n \theta \cdot \sin \theta d\theta \\ &= \frac{2\pi I_0}{n+1} \dots\dots\dots(17) \end{aligned}$$

In order to simplify further treatment, sizes and the total flux F emitted from the panel are so normalized that

$$L=W=1 \text{ and } F=1. \dots\dots\dots(18)$$

Then eq. (16) becomes

$$I(\theta) = \frac{n+1}{2\pi} \cos^n \theta \dots\dots\dots(19)$$

On the other hand, since

$$\cos \theta = \frac{H_m}{\sqrt{(x-x')^2 + (y-y')^2 + H_m^2}} \dots\dots\dots(20)$$

the illumination at a point $P(x, y)$ produced by a surface element $dx'dy'$ at a point $Q(x, y)$ is expressed as

$$dE(x, y) = \frac{I(\theta) \cos \theta dx'dy'}{r^2}$$

$$= \frac{n+1}{2\pi} \frac{H_m^{n+1} dx'dy'}{\{(x-x')^2 + (y-y')^2 + H_m^2\}^{\frac{n+3}{2}}} \dots\dots\dots(21)$$

where r and H_m are the distances from the point P to Q and the panel to the working plane, respectively.

Therefore, the direct ratio is given by

$$\begin{aligned} d &= \frac{n+1}{2\pi} H_m^{n+1} \iiint \frac{dx'dy'dxdy}{\{(x-x')^2 + (y-y')^2 + H_m^2\}^{\frac{n+3}{2}}} \dots\dots\dots(22) \end{aligned}$$

where room index $k_r = 1/2H_m$.

As it is impossible to evaluate the integral of eq. (22) analytically, the numerical integration on the basis of Simpson's formula was carried out by using a digital computer (IBM-1130) in such a manner that

$$\begin{aligned} &\iiint f(x', y', x, y) dx'dy'dxdy \\ &= \frac{h_x' h_y' h_x h_y}{3^4} \sum_{l=0}^{2n} \sum_{m=0}^{2n} \sum_{s=0}^{2n} \sum_{t=0}^{2n} K(l) K(m) K(s) K(t) \\ &\quad f(x_l', y_m', x_s, y_t) \dots\dots\dots(23) \end{aligned}$$

where $K(2^n) = 1$, $K(2i) = 2$ and $K(2i+1) = 4$, and h_x', h_y', h_x, h_y are partition widths.

Table 3 shows the calculated values of d in several combinations of n and k .

Table 3. Direct ratio as a function of room index K_r and light distribution index n .

$K_r \backslash n$	Direct ratio $d (K_r, n)$					
	0.0	1.0	1.5	2.0	3.0	4.0
0.6	0.14	0.25	0.29	0.33	0.39	0.45
0.8	0.20	0.34	0.39	0.44	0.50 ₅	0.56
1.0	0.26	0.41 ₅	0.47	0.51 ₅	0.58	0.63
1.25	0.32	0.49	0.54 ₅	0.59	0.65	0.69 ₅
1.5	0.37	0.55	0.60	0.64	0.70	0.74
2.0	0.45	0.63	0.68	0.72	0.77	0.80
2.5	0.51	0.69	0.74	0.77	0.81	0.84
3.0	0.55	0.73	0.78	0.81	0.84	0.87
4.0	0.62	0.79	0.83	0.85	0.88	0.90
5.0	0.67	0.83	0.86	0.88	0.90 ₅	0.92

The utulance of a luminous ceiling system is given by⁶⁾

$$\begin{aligned} U_0 &= U_0(d, K_r, \rho_c, \rho_w, \rho_f) \\ &= \frac{(1-d)}{2} K_r \frac{\rho_w}{\rho_f} G_{fw} + d \cdot G_{ff} \dots\dots\dots(24) \end{aligned}$$

where suffixes c, w and f denote the surface of the ceiling, wall and floor respectively, and G_{fw} and G_{ff} are flux transfer functions, the values of which may be found in some papers⁷⁾.

Now the effective ceiling reflection factor should be used in place of the reflection factor ρ of the panel. This is given by

Table 4. Utilance for a luminous ceiling system

	K_r	ρ_f	30			10			K_r	ρ_f	30			10		
		ρ_c/ρ_w	70/50	70/30	50/50	70/50	70/30	50/50		ρ_c/ρ_w	70/50	70/30	50/50	70/50	70/30	50/50
$n=0$ (BZ 8)	0.6	.32	.23	.31	.32	.23	.30	$n=2$ (BZ 3)	0.6	.48	.39	.46	.47	.40	.46	
	0.8	.41	.30	.39	.40	.30	.38		0.8	.60	.52	.58	.59	.52	.57	
	1.0	.49	.38	.47	.46	.37	.45		1.0	.70	.62	.68	.66	.59	.64	
	1.25	.57	.46	.53	.53	.43	.50		1.25	.79	.71	.76	.73	.67	.71	
	1.5	.63	.52	.59	.58	.48	.55		1.5	.84	.77	.81	.78	.73	.76	
	2.0	.72	.62	.68	.66	.57	.63		2.0	.94	.81	.89	.85	.80	.83	
	2.5	.79	.69	.74	.71	.63	.68		2.5	.99	.93	.94	.89	.85	.87	
	3.0	.83	.74	.78	.75	.67	.72		3.0	1.03	.98	.98	.92	.89	.90	
	4.0	.95	.87	.88	.80	.74	.77		4.0	1.11	1.07	1.05	.96	.93	.94	
5.0	.96	.89	.89	.84	.78	.82	5.0	1.12	1.08	1.05	.98	.96	.96			
$n=1.0$ (BZ 5)	0.6	.51	.32	.40	.41	.33	.40	$n=3$ (BZ 2)	0.6	.53	.45	.51	.52	.46	.51	
	0.8	.52	.43	.50	.51	.43	.49		0.8	.65	.57	.63	.64	.57	.62	
	1.0	.62	.53	.60	.58	.50	.57		1.0	.76	.68	.73	.71	.65	.70	
	1.25	.71	.62	.67	.66	.58	.64		1.25	.83	.78	.80	.78	.72	.76	
	1.5	.77	.68	.74	.71	.64	.69		1.5	.89	.82	.86	.82	.77	.80	
	2.0	.87	.78	.82	.78	.72	.76		2.0	.98	.91	.93	.88	.84	.86	
	2.5	.93	.86	.88	.84	.78	.81		2.5	1.02	.97	.97	.92	.88	.90	
	3.0	.97	.90	.92	.87	.82	.85		3.0	1.06	1.00	1.00	.94	.91	.92	
	4.0	1.04	.98	.97	.92	.88	.89		4.0	1.13	1.09	1.06	.98	.96	.96	
5.0	1.08	1.03	1.01	.95	.92	.93	5.0	1.14	1.10	1.07	1.00	.98	.98			
$n=1.5$ (BZ 4)	0.6	.45	.36	.43	.44	.37	.43	$n=4$ (BZ 1)	0.6	.57	.50	.56	.57	.51	.56	
	0.8	.56	.47	.54	.55	.47	.53		0.8	.69	.62	.67	.68	.62	.67	
	1.0	.67	.58	.64	.62	.55	.61		1.0	.80	.73	.78	.75	.69	.73	
	1.25	.75	.67	.72	.70	.63	.68		1.25	.87	.81	.84	.81	.78	.79	
	1.5	.81	.73	.78	.75	.68	.73		1.5	.93	.86	.89	.85	.81	.83	
	2.0	.91	.83	.86	.82	.76	.80		2.0	1.00	.94	.95	.90	.86	.88	
	2.5	.97	.90	.92	.87	.82	.85		2.5	1.05	.99	.99	.94	.91	.92	
	3.0	1.05	.95	.96	.90	.86	.88		3.0	1.08	1.03	1.03	.96	.94	.95	
	4.0	1.07	1.02	1.01	.94	.91	.92		4.0	1.14	1.10	1.07	.99	.97	.97	
5.0	1.10	1.06	1.04	.97	.94	.95	5.0	1.15	1.12	1.08	1.01	.99	.99			

$$\rho_c = \rho + \frac{\tau^2 \cdot \rho_{eff}}{1 - \rho \cdot \rho_{eff}} \quad (23)$$

as easily found in the later statement.

Table 4 gives the practical data of the utilance for the luminous ceiling.

Strictly speaking, luminous intensity of a panel is expressed as

$$I(\theta) = \sum_n I_{on} \cos^n \theta \quad (24)$$

where I_{on} is the constant determined by the light distribution data.

If we denote the utilance by U_0 when the light distribution index is n , we have the utilance as

$$U_0 = \sum_n I_{on} U_{on} \quad (27)$$

where,

$$\sum_n \frac{2\pi}{n+1} I_{on} = 1 \text{ (normalized)} \quad (28)$$

Finally, the coefficient of utilization for a luminous ceiling can be obtained as follows.

$$U = \eta \cdot U_0 \quad (29)$$

4. Discussion

The lamp spacing to mounting height ratio in a ceiling cavity influences the uniformity of the panel luminance, but it does not influence output ratio so directly. In case of the evaluation of light output ratio, the ceiling cavity index plays an important role. The lamp spacing has not been taken into account in the formulae for the light output ratio

already reported by several predecessors, but it has a weak effect on the light output ratio because it is related to the obstruction factor of the formula proposed by the author. In many cases mentioned in eq. (1) is approximately 0.1.

Moreover, it is the transmission factor and the reflection factor of a panel that comes into the question when we predict the light output ratio for a luminous ceiling. When a prismatic panel was used, it should be noted that the transmission factor and the reflection factor vary considerably according to the distance between the panel and the fitting plane, while in case of a diffusing panel the variation is comparatively small. Fig. 4 shows

Fig. 4. Relation between the transmission factor of a prismatic panel and the distance from the diffusing panel to the fitting plane.

an example of the dependency of transmission factor on the distance L .

A new formula for the light output ratio derived by the author may be understood to have an accuracy for the practical use as far as the reflection factor and the transmission factor are assumed correctly.

On the other hand, to calculate the utilisation we have to know the direct ratio of luminous panel, and for this purpose the data prepared in this report and the transmission factor are useful.

If the area of the rim of a luminous panel has a nonnegligible part of the ceiling area as shown in Fig. 5, it is worth while that we introduce the

Fig. 5. Room Division for calculation of correction factor for a ceiling with non-negligible rim 2.

following correction factor $c.f.$

$$c.f. = \Phi(1, 1'2') / \Phi(12, 1'2') \\ = \frac{1}{\Phi(12, 1'2')} \left\{ \Phi(ihlm, i'h'l'm') - 2\Phi(ih, i'h') + \Phi(ii') \right\} \dots\dots\dots(30)$$

where $\Phi(i, j) = A_i S_{ij}$

S_{ij} = shape factor,

A_i = area of a surface element i .

Then, the effective direct ratio is given by

$$d_{eff} \cong c.f. \cdot d(K_r, n) \dots\dots\dots(31)$$

The equality in eq. (31) holds only when a luminous panel is a perfect diffuser, but we can also apply this equation to a prismatic panel without making any serious errors.

5. Conclusion

The subjects discussed in this paper are summarized as follows.

(a) A new formula for the light output ratio of a luminous ceiling was developed and its validity was checked by experiment.

(b) The direct ratio for a luminous ceiling was calculated for several typical light distributions of panels. On the basis of these data, utilisation tables

were prepared for the lighting design of luminous ceiling.

In conclusion, the calculation method for the coefficients of utilization of a luminous ceiling described in this paper can be applied to practical purposes with sufficient accuracy.

6. Acknowledgement

The author wishes to express his sincere thanks to Dr. H. Satone and Mr. M. Morita for their valuable suggestions through this work. He is also indebted to Mr. T. Matsuba for help with experiment.

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Appendix 1. Shape factor S .

The shape factor of surface i in relation to j is defined as

$$S_{ij} = \frac{1}{\pi A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cdot \cos \theta_j}{r_{ij}^2} dA_i \cdot dA_j \dots\dots\dots(32)$$

where A_i, A_j are the areas of the respective surfaces which are assumed to be perfect diffusers, r_{ij} is the distance between infinitesimal areas dA_i and dA_j and θ_i, θ_j are the angles between the lines connecting dA_i and dA_j and the normals to the respective surfaces.

The shape factor S_{ij} represents the ratio of the radiant flux leaving the surface i and directing to another surface j to the total flux leaving the

surface i . This factor has been called variously shape factor, shape modulus, form factor, configuration factor and angle factor by other authors.

In the case of the rectangular parallelepiped, as shown in Fig. 6, the shape factor between two

Fig. 6. Surfaces i and j for calculation of the shape factor S_{ij} .

parallel planes S_{ij} is given by

$$S_{ij} = \frac{2}{\pi xy} \left\{ \log_e \left[\frac{(1+x^2)(1+y^2)}{1+x^2+y^2} \right]^{\frac{1}{2}} \right. \\ + y \sqrt{1+x^2} \tan^{-1} \left(\frac{y}{\sqrt{1+x^2}} \right) \\ + x \sqrt{1+y^2} \tan^{-1} \left(\frac{x}{\sqrt{1+y^2}} \right) \\ \left. - y \tan^{-1} y - x \tan^{-1} x \right\} ,$$

where $x=l/h$,

and $y=w/h$.

Appendix 2. Derivation of the eq. (9)

The effective reflection factor of the ceiling cavity

ρ_{eff} is given by⁶⁾

$$\rho_{eff} = \frac{1}{M} \left\{ S(1-\rho_2\{1-(1-S)K_{cc}\}) + \rho_2 \frac{(1-S)^2}{2} K_{cc} \right\} \\ \times \rho_1 S + (1-S)(1+\rho_1 S) \frac{(1-S)}{2} \rho_2 K_{cc} \dots (34)$$

where

$$M = 1 - \rho_2 + 0.5\rho_2(1-S)K_{cc}\{2-\rho_1'(1-S)\} \dots (8)$$

Whence the eq. (7) may be written in the form that

$$F_p(\text{exact}) = \eta''S + \frac{1}{M} \left\{ \eta' \frac{\rho_1}{\rho_1'} (1-\delta) + \eta''(1-S) \right\} \\ \times \rho_{eff} + \Delta F_p \dots (35)$$

where

$$\Delta F_p = \frac{1}{M} \left\{ S(1-\rho_2\{1-(1-S)K_{cc}\}) + \rho_2 \frac{(1-S)^2}{2} K_{cc} \right\} \\ \times [\eta' \rho_1 (1-\delta) - \rho_1' \eta'' S] (1-S) \\ + \frac{1}{M} (1+\rho_1 S) \left[\eta'' S - \eta' \frac{\rho_1'}{\rho_1'} (1-\delta) \right] \\ \times \frac{(1-S)^2}{2} \rho_2 K_{cc}$$

When the ceiling cavity index K_{cc} is large enough, ΔF_p can be neglected since the shape factor $S(K_{cc})$ nearly equals to unity.

To examine the influence of ΔF_p on F_p , we consider the following two cases.

(1) The case 1

For $\eta' = \eta'' = \frac{1}{2}$, $\delta = 0$ and $(\rho_1', \rho_2) = (0.7, 0.7)$, $(0.7, 0.5)$, $(0.5, 0.7)$ and $(0.5, 0.5)$, $\Delta F_p / F_p$ is approximately given by:

$$\frac{\Delta F_p}{F_p(\text{exact})} = 0 \sim 0.005.$$

(2) The case 2

For $\eta' = 0.22$, $\eta'' = 0.68$, $\delta = 0.1$ and $(\rho_1', \rho_2) = (0.7, 0.7)$, $(0.7, 0.5)$, $(0.5, 0.7)$ and $(0.5, 0.5)$, $\Delta F_p / F_p$ is:

$$\frac{\Delta F_p}{F_p(\text{exact})} = 0.01 \sim 0.03.$$

As seen above, $\Delta F_p / F_p(\text{exact})$ is fairly small. If the term ΔF_p is neglected from the practical aspect of view, eq. (7) reduces to eq. (9).

(受付1968年9月5日, 再受付1969年2月3日)