Morphological Analysis for Multistage Logistic Network Optimization under Disruption Risk

Muhammad Rusman and Yoshiaki Shimizu

Abstract: This study is concerned with multistage logistic network optimization under disruption risk. In its development, we consider three types of logistic network models, i.e., the multi-multi allocation, multi-single allocation and single-single allocation model. These logistic models are formulated as respective mixed-integer programming models with the expected cost function. Commercial software is then applied to find the optimal solutions. Moreover, defining a metric to stand for a certain service level or simplicity of network, we show a procedure to derive a final decision through morphological analysis. Through a numerical experiment, we show that the proposed idea is promising for use to design a resilient logistic network available for business continuity plan/management.

Key words: disruption risk, mixed-integer programming, morphological analysis, resilient logistic network

1 INTRODUCTION

Supply chains are subject to a wide range of risks such as demand uncertainty, natural disasters, terrorist attacks and so on. A recent disruption caused by the Tohoku earthquake and tsunami in Japan in March, 2011 had a dramatic impact on the supply chains and logistic distribution of many companies, including those in the automotive, electronics and chemical industries. The resulting slowdown and cessation of operations seriously affected some companies. For instance, a certain company producing electronic components for car manufactures was disrupted by this disaster. Accordingly, many manufactures were faced to slow down production due to the shortage of materials from the company. Similar circumstances were repeated as a result of the flooding in Thailand in October, 2011.

We can anticipate disruptions by considering preventive action. If the supply chain takes into account preventive action against disruptions, such actions are amenable to mitigation planning. Under such mitigation plans, the supply chain must build a resilient system that will minimize the impact of disruption in the future [1,2]. One such mitigation mechanism would be to have backup facilities that makes it possible to continue supply if the primary facility is disrupted.

In this study, therefore, we focus on the issue related to facility disruption risk for multistage logistic networks and present three different types of allocation models each of which will present, in parallel, the respective backup design and operational aspects associated with the design. Consequently, the aim of this study is to evaluate the properties of these models including the interest in the morphological aspect, which is essential for resilient system development, but hard to capture explicitly in practice. This is a novel approach to cope with decision making in an adverse environment.

The rest of the paper is organized as follows. Section 2 outlines the associated studies that form the background and the prospects of this study. A brief review is given in Section 3. In Section 4, descriptions and formulations of the models are presented. Then, numerical experiments are provided in Section 5. Finally, we will conclude in Section 6.

2 PROSPECTS FOR DECISION MAKING UNDER DISRUPTION RISK

2.1 Business Continuity Management/Plan

According to the increase in various risks, American and European countries have started establishing
institutes as a countermeasure for unexpected disruptions. Business continuity management (BCM) is an idea inclusive of a business continuity plan (BCP) and defined as a holistic management process that identifies potential impacts that threaten an organization, providing a framework for building resilience with the capability for an effective response for recovery or continuity in the event of a disaster. Meanwhile, the BCP identifies the organization's exposure to internal and external threats and synthesizes hard and soft assets to provide effective prevention and recovery for the organization while maintaining competitive advantage and value system integrity.

In Japan, starting with some semiconductor enterprises in 2003, this activity is now being adopted gradually in many industries. Notably, after the recent significant disasters in eastern Japan and Thailand, it seems that the importance of such movement will be extremely high throughout the manufacturing industry. The basic idea of a BCP is illustrated as Fig.1 [3]. Ultimately, the aim of this plan is to reduce the recovery time objective (RTO) to a period as short as possible. In other words, it must involve a preventive and remediable plan against emergencies for management and/or decision making to maintain business continuity.

2.2 Risk Drivers in Supply Chains

In addition to preventive measures like BCM/BCP, it is essential to construct a resilient logistic network to capture the properties of risk imbedded thereat. Such risks are classified into three categories listed below.

1. Outside risk refers to an abnormal climate, natural disasters, changes/enactments of laws/regulations, riot, terrorism and exhaustion of resources, etc.
2. Inside risk is caused by:
   · Inbound logistics such as: crashes; problems associated with quality, safety, productivity, tardiness and delivery of raw materials and parts; strikes, scandals; violations of laws and regulations.
   · Outbound logistics such as: unexpected changes of demand; problems from order processing and solvency; frequent deviations of specification.
3. Risk caused within the company refers to risk:
   · Peculiar to operations incidents; malfunctioning of production, human errors, etc.
   · Caused by management and decision making; safety level of inventory; schedule of delivery; location/allocation of sites/resources, etc.

To understand previous studies in a well-organized manner and to construct definite future prospects, it is helpful to refer to Cao and Chu who classified the risk drivers in supply chain [4]. In particular, they claim the importance of organizational cooperation throughout the society. Looking at the recent worldwide affairs, it makes sense to prepare against various disruption risks and move toward adopting a suitable BCP/BCM. This is also a consequence guided by the "All over the processes" row in a table in the study cited above. As a summary of this section, the importance of a backup system for a supply chain is very understandable for this purpose.

3 BRIEF LITERATURE REVIEW

There are some previous studies on supply chains considering disruption risk. For instance, the research of Tomlin [5] investigates the impact of considering unreliable facilities for facility location problems. Snyder and Daskin [6] introduced several models based on traditional facility location problems, in which some facilities may fail with a given probability. They assumed that in normal circumstances, customers are assigned to primary facilities and other facilities will serve the customers if the primary facility fails. Lim et al. [7] studied the facility reliability problem (FRP) which is an extension of the uncapacitated facility location problem (UFLP). They studied the FRP from the aspect of how to design a reliable supply chain network in the presence of random facility dis-
ruption. Chopra and Sodhi [8] and Kleindorfer and Saad [9] studied supply chain disruption from a risk management perspective. These studies were concerned with two-echelon logistic problems, and solved only small problems.

Considering the risk associated with demand fluctuation, Shimizu et al. applied a flexibility analysis for a three-echelon logistic problem [10]. A scenario-based approach was taken to provide a solution procedure via the recourse model [11]. Moreover, Rusman and Shimizu [12] compared some properties among the network configurations characterized by delivery manners. A general review on the logistic networks design is discussed in the literature [13].

4 PROBLEM FORMULATION

Among the sources of risk within a supply chain, the risk associated with demand is shown to have greater impact and higher likelihood of occurrence [14]. In addition, three-echelon logistics are popular [15] and more flexible compared to two-echelon logistics. In this study, therefore, we focus attention on the demand aspect and consider three-echelon problems consisting of the distribution center (DC), relay station (RS) and customer (RE).

Having proper backup facilities is an essential element to reduce the RTO in a BCP and to create resiliency as mentioned in Section 2. This study attempts to present a resilient network from sound DCs to customers via RSs potentially affected by the incident. Generally speaking, since every RS is located nearer to the customer site than the DC, it is adequate to consider the decision problem at the RS level. We provide two kinds of RSs, i.e., reliable RSs (RRSs) and unreliable RSs (URSs). URS is no longer available for serving customers when an RS fails or a disruption occurs. This means alternative sources of supply become necessary to provide service to the customers. On the other hand, an RRS is stronger and has additional capacity and/or an external alternative sourcing strategy, for example. An RRS can continue business even after an incident but it costs more to manage such facilities.

Figure 2 illustrates the three-echelon logistic network where the RSs are potentially being disrupted. Two DCs distribute products to three RSs, which consist of two RRSs and one URS. If customer demand is satisfied by the RRS (RE i), then single assignment is sufficient. On the other hand, if the customer is assigned to a URS (RE j), backup assignment is required besides the primary assignment. This means that when a disruption occurs at a URS, items demanded by the customer will be distributed from the RRS assigned as the backup for the customer.

Noticing the correlation of the risk derivers hidden in wide societal and human activities, we know it is insufficient simply to consider the economical aspect. Hence, this study attempts to consider a morphological structure of the network to evaluate certain intangible factors behind the cost. For this purpose, we take three models, i.e., the multi-multi allocation (MMA), multi-single allocation (MSA) and single-single allocation (SSA) models into consideration. Figure 3 shows the differences of configuration among them. In the MMA model, each RS will receive products from multiple DCs and each customer also receives products from multiple RSs. In the MSA model, each RS can receive products from multiple DCs while customers can only receive products from a single RS. Finally, in the SSA model, only each RS receives products from a single DC and customers also only receive products from a single RS.

A major factor to consider in these three structures is to emphasize the importance of intangible factors that may be associated with the flexibility, service, resiliency or other elements. Though those are important aspects in practical decision making, it is difficult to give certain evaluation metrics for them. In fact, no studies have addressed such an idea in terms of multiple models. As a preliminary study to concisely account for such factors, we
consider the numbers of paths necessary to constitute the networks in both normal and abnormal situations. It is natural to suppose that an increase in this number will raise complexity and impose additional load on every member of the logistic network.

The following notations are used to describe the present mathematical models.

**Index sets**
- \( I \): Set of distribution centers (DCs)
- \( J \): Set of relay stations (RSs)
- \( K \): Set of customers

**Parameters**

\[
\begin{align*}
F^p_{ij} & : \text{Fixed cost for opening URS } j \\
F^b_{ij} & : \text{Fixed cost for opening RRS } j \\
C^p_i & : \text{Shipping cost at DC } i \text{ as primary assignment} \\
C^b_i & : \text{Shipping cost at DC } i \text{ as backup assignment} \\
H^p_j & : \text{Handling cost at RS } j \text{ as primary assignment} \\
H^b_j & : \text{Handling cost at RS } j \text{ as backup assignment} \\
T^P_{ij} & : \text{Transport cost from DC } i \text{ to RS } j \text{ as primary assignment} \\
T^B_{ij} & : \text{Transport cost from DC } i \text{ to RS } j \text{ as backup assignment} \\
T^P_{jk} & : \text{Transport cost from RS } j \text{ to customer } k \text{ as primary assignment} \\
T^B_{jk} & : \text{Transport cost from RS } j \text{ to customer } k \text{ as backup assignment} \\
U_j & : \text{Capacity of RS } j \\
P^u_i & : \text{Maximum supply ability of DC } i \\
P^l_i & : \text{Minimum supply ability of DC } i \\
d_k & : \text{Demand of customer } k \\
q_i & : \text{Probability of disruption at RS } j (0 < q_i < 1)
\end{align*}
\]

**Decision variables**

\[
\begin{align*}
d^p_{ij} & : \text{Shipped amount from DC } i \text{ to RS } j \text{ as primary assignment} \\
d^b_{ij} & : \text{Shipped amount from DC } i \text{ to RS } j \text{ as backup assignment} \\
b^p_{jk} & : \text{Shipped amount from RS } j \text{ to customer } k \text{ as primary assignment} \\
b^b_{jk} & : \text{Shipped amount from RS } j \text{ to customer } k \text{ as backup assignment} \\
x^U_{ij} & = \begin{cases} 1, & \text{if } RS \text{ is opened as an URS;} \\ 0, & \text{otherwise.} \end{cases} \\
x^R_{ij} & = \begin{cases} 1, & \text{if } RS \text{ is opened as a RRS;} \\ 0, & \text{otherwise.} \end{cases}
\end{align*}
\]

\[
\begin{align*}
y^p_{jk} & = \begin{cases} 1, & \text{if } RS \text{ is distributed to customer } k \text{ as primary assignment;} \\ 0, & \text{otherwise.} \end{cases} \\
y^b_{jk} & = \begin{cases} 1, & \text{if } RS \text{ is distributed to customer } k \text{ as backup assignment;} \\ 0, & \text{otherwise.} \end{cases} \\
z^p_{ij} & = \begin{cases} 1, & \text{if } DC \text{ is distributed to RS } j \text{ as primary assignment;} \\ 0, & \text{otherwise.} \end{cases} \\
z^b_{ij} & = \begin{cases} 1, & \text{if } DC \text{ is distributed to RS } j \text{ as backup assignment;} \\ 0, & \text{otherwise.} \end{cases}
\end{align*}
\]

In these models, we commonly assume the following conditions. Each customer \( k \in K \) has a demand \( d_k \). The product will be distributed from DC \( i \) to RS \( j \) and from RS \( j \) to customers \( k \), respectively. At each customer \( k \), we may locate either RS with opening cost \( F^p_j \) or URS with opening cost \( F^U_j \). The fixed cost for opening an RRS is higher than for a URS (\( F^p_j > F^u_j \)) due to disruption related reasons.

The transportation cost per unit demand from DC \( i \) to RS \( j \) is given by \( T^P_{ij} \) and \( T^B_{ij} \) for the primary assignment and backup assignment, respectively. We also assume that \( T^P_{ij} < T^B_{ij} \) due to the consequence of using the backup resources. Similarly, the transportation cost per unit demand from RS \( j \) to the customer \( k \) is given by \( T^P_{jk} \) and \( T^B_{jk} \) for the primary assignment and backup assignment, respectively. We also assume that \( T^P_{jk} < T^B_{jk} \) due to the same reason given above. Moreover, the shipping cost at DC \( i \) and the handling cost at RS \( j \) are denoted as \( C^p_i \) and \( C^b_i \), \( H^p_j \) and \( H^b_j \) for the primary and backup conditions, respectively. We also assume that \( C^p_i < C^b_i \) and \( H^p_j < H^b_j \).

In this model, we assume that each RS has a certain disruption probability denoted by \( q_i \). The primary event occurs with probability \( 1-q_i \) under the normal costs while the backup event occurs with probability \( q_i \) under the abnormal costs.

Finally, we formulate each model as the mixed-integer programming problem described below. The objective function is expected costs that consist of shipping cost at the DC, the transportation cost between each facility and the handling cost at each RS besides the fixed cost for opening each RS. Among the decision variables, the binary variables are closely related to the structure of the network (design), while the integer variables closely relate to the operations under the prescribed structure. In other words, the resulting different structure will return the corresponding different operations.

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4.1 MMA Model

The MMA model is described as follows:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{ij} F^{ij}_x x^{ij} + \sum_{ij} F^{ij}_r r^{ij} \\
& \quad + \sum_{ij} (1 - q_{ij}) \left( \sum_{i} (c^{ij}_p + T_{ij}^p) a^{ij}_p + \sum_{k \in K} (H^{ij}_p + T_{ij}^p) b^{ij}_{jk} \right) \\
& \quad + \sum_{ij} q_{ij} \left( \sum_{i} (c^{ij}_q + T_{ij}^q) a^{ij}_q + \sum_{k \in K} (H^{ij}_q + T_{ij}^q) b^{ij}_{jk} \right)
\end{align*}
\]

subject to
\[
\begin{align*}
x^{ij}_x + x^{ij}_r & \leq 1 \quad \forall j \in J \\
\sum_{i} x^{ij}_r & \geq 1 \quad \forall j \in J \\
a^{ij}_p & \leq U(x^{ij}_x + x^{ij}_r) \quad \forall j \in J \\
a^{ij}_p & \leq U_j x^{ij}_r \quad \forall j \in J \\
a^{ij}_r & \leq P_l \quad \forall i \in I \\
a^{ij}_r & \leq P_l \quad \forall i \in I \\
a^{ij}_p & \geq P_l \quad \forall i \in I \\
a^{ij}_p & \geq P_l \quad \forall i \in I \\
b^{ij}_{jk} & = d_k \quad \forall k \in K \\
\sum_{i} b^{ij}_{jk} & = d_k \quad \forall k \in K \\
x^{ij}_x & \in \{0,1\} \quad \forall j \in J \\
x^{ij}_r & \in \{0,1\} \quad \forall j \in J \\
a^{ij}_p & \geq 0 \quad \forall i \in I, \forall j \in J \\
a^{ij}_p & \geq 0 \quad \forall i \in I, \forall j \in J \\
b^{ij}_{jk} & \geq 0 \quad \forall j \in J, \forall k \in K \\
b^{ij}_{jk} & \geq 0 \quad \forall j \in J, \forall k \in K
\end{align*}
\]

Equation (1) states that either RRSSs or URRSSs can be open, but not both. Equation (2) requires at least one RRSS to be open. Equations (3) and (4) are capacity constraints for RRSSs as the primary and backup assignments, respectively. Equations (5) and (6) are the upper bounds for available supply as primary and backup assignments, respectively. Equations (7) and (8) are the lower bounds for available supply as the primary and backup assignments, respectively. Equations (9) and (10) are balances of product flow as the primary and backup assignments, respectively. Equations (11) and (12) mean that demand of every customer must be satisfied as the primary and backup assignments, respectively. Equations (13) and (14) are integer conditions on decision variables. Equations (15) - (18) are nonnegative conditions for the primary and backup amounts.

4.2 MSA Model

The MSA model is described as follows:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{ij} F^{ij}_x x^{ij} + \sum_{ij} F^{ij}_r r^{ij} \\
& \quad + \sum_{ij} (1 - q_{ij}) \left( \sum_{i} (c^{ij}_p + T_{ij}^p) a^{ij}_p + \sum_{k \in K} (H^{ij}_p + T_{ij}^p) d_k y^{ij}_{jk} \right) \\
& \quad + \sum_{ij} q_{ij} \left( \sum_{i} (c^{ij}_q + T_{ij}^q) a^{ij}_q + \sum_{k \in K} (H^{ij}_q + T_{ij}^q) d_k y^{ij}_{jk} \right)
\end{align*}
\]

subject to
\[
\begin{align*}
\sum_{i} y^{ij}_{jk} & = 1 \quad \forall k \in K \\
\sum_{i} y^{ij}_{jk} & = 1 \quad \forall k \in K \\
y^{ij}_{jk} & \leq x^{ij}_x + x^{ij}_r \quad \forall j \in J, \forall k \in K \\
y^{ij}_{jk} & \leq x^{ij}_r \quad \forall j \in J, \forall k \in K \\
\sum_{i} a^{ij}_p - \sum_{k \in K} d_k y^{ij}_{jk} & = 0 \quad \forall j \in J \\
\sum_{i} a^{ij}_p - \sum_{k \in K} d_k y^{ij}_{jk} & = 0 \quad \forall j \in J \\
y^{ij}_{jk} & \in \{0,1\} \quad \forall j \in J, \forall k \in K \\
y^{ij}_{jk} & \in \{0,1\} \quad \forall j \in J, \forall k \in K
\end{align*}
\]

In the MSA model, explanation of the objective function and the constraints are all equivalent to that of the MMA model except for Eqs. (19), (20), (25) and (26).
These equations express that each customer must be assigned to a single RS for both the primary and backup assignments, respectively. Equations (21) - (24) correspond to Eqs. (9) - (12).

4.3 SSA Model

The SSA model is described as follows:

Minimize \( \sum_{j \in J} F^U_j x^U_j + \sum_{j \in J} F^P_j x^P_j \)
\[ + \sum_{j \in J} (1 - q_j) \left( \sum_{i \in I} (C^p_i + T1^p_i)q^p_{ij} \right) \]
\[ + \sum_{k \in K} (H^P_k + T2^P_{jk})d_k y^P_{yk} \]
\[ + \sum_{j \in J} q_j \left( \sum_{i \in I} (C^p_i + T1^p_i)q^p_{ij} \right) \]
\[ + \sum_{k \in K} (H^P_k + T2^P_{jk})d_k y^P_{yk} \]

subject to

Eqs. (1), (2), (13) - (16), (19) - (22), (25), (26) and

\[ \sum_{j \in J} x^P_{ij} = 1 \quad \forall k \in K \] (27)
\[ x^P_{ij} = 1 \quad \forall k \in K \] (28)
\[ a^p_{ij} x^P_{ij} \leq U_j (x^U_j + x^P_j) \quad \forall j \in J \] (29)
\[ a^p_{ij} x^P_{ij} \leq U_j x^P_j \quad \forall j \in J \] (30)
\[ a^p_{ij} x^P_{ij} \leq PU_i \quad \forall i \in I \] (31)
\[ a^p_{ij} x^P_{ij} \leq PU \quad \forall i \in I \] (32)
\[ a^p_{ij} x^P_{ij} \geq PL_i \quad \forall i \in I \] (33)
\[ a^p_{ij} x^P_{ij} \geq PL_i \quad \forall i \in I \] (34)
\[ a^p_{ij} x^P_{ij} - \sum_{k \in K} d_k y^P_{yk} = 0 \quad \forall j \in J \] (35)
\[ a^p_{ij} x^P_{ij} - \sum_{k \in K} d_k y^P_{yk} = 0 \quad \forall j \in J \] (36)
\[ z^U_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in J \] (37)
\[ z^P_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in J \] (38)

Equations (27), (28), (37) and (38) are necessary for single allocation for DCs. Equations (29) - (36) correspond to Eqs. (3) - (10).

Since this model involves bi-linear terms like \( a^p_{ij} x^U_j \) and \( a^p_{ij} x^P_j \), we need to introduce new variables \( Z^U_j \) and \( Z^P_j \) (\( \geq 0 \)) and the additional constraints to linearize them as follows:

\[ Z^U_j = a^p_{ij} x^U_j \quad \forall i \in I, \forall j \in J \] (39)
\[ Z^P_j = a^p_{ij} x^P_j \quad \forall i \in I, \forall j \in J \] (40)
\[ a^p_{ij} - Z^U_j \leq 0 \quad \forall i \in I, \forall j \in J \] (41)
\[ a^p_{ij} - Z^P_j \leq 0 \quad \forall i \in I, \forall j \in J \] (42)
\[ a^p_{ij} - B \leq Z^U_j - Bz^P_j \quad \forall i \in I, \forall j \in J \] (43)
\[ a^p_{ij} - B \leq Z^P_j - Bz^P_j \quad \forall i \in I, \forall j \in J \] (44)

where \( B \) is a certain large value.

5 NUMERICAL EXPERIMENTS

In this section, we show the results of numerical experiments. We provide benchmark problems by randomly generating every system parameter within the prescribed extents. The probability of disruption \( q_j \) is assumed to be same for \( \forall j \in J \) (hereinafter, denoted simply as \( q \)). The fixed cost required for opening RSs \( F^P_j \) is double for that of URS \( F^U_j \). Every node denoting the members of the facilities is generated randomly. The distances between them are calculated based on the Euclidian norm. We then obtain the transportation cost by multiplying the unit factor 1.5 and 1.0 with the distance between DC to RS and RS to customer, respectively. Every backup cost is set to 1.5 times the normal values.

We then solved the formulated problems using commercial software known as CPLEX 12.2 on a computer with 2.66GHz core 2 duo processor and 2 GB of RAM.

5.1 Results for Small Size Models

Results for the MMA model for the problem involving two DCs, five candidate RSs and 50 customers (hereafter, such a feature will be denoted as (2-5-50)) is illustrated in Fig. 4. We show the results in Table 1 to compare the models. We know the DCs will distribute products to three open RSs (RS#2, RS#4 and RS#5) for such probabilities of disruption that \( q=\{0.01, 0.1, 0.2, 0.3 \) and 0.5\} in a multiple distribution manner. For \( q=0.4 \), two reliable RSs exist in the system.

Moreover, it is interesting to see that RS#4 is used as the unreliable RS when \( q \) is low (<0.3) while it is used as
the reliable one at the higher $q (\geq 0.5)$ and disappears during the middle range ($q=0.4$). This is because the opening cost of RS#4 is quite low among RSs. Hence, opening this RS as a URS can reduce transportation costs greatly when $q$ is low. In contrast, when $q$ becomes higher, it makes sense to open this RS as an RRS to cope with the disruption as well as realize transportation cost savings. The situation when $q=0.4$ appears as the transient status of these two cases. This fact can conditionally verify the adequateness of the proposed model.

In the case of the MSA model, customers are forced to receive products only from one RS. This requirement causes an increase in normal costs since customers have no choice to receive products from another RS. We obtained the same result as for the MMA model regarding the number of open RSs. The expected cost is slightly higher than for the MMA model due to the above-mentioned reason.

The SSA model is originally a non-linear model. After the linearization shown already, the SSA model is readily solvable by CPLEX. All open RSs are reliable ones over the disruption probabilities, and the configuration becomes simple.

In Table 1, we also compare the results for these three models in detail. These are all global optimal solutions (gap is almost 0.0). Regarding the expected cost, as expected a priori, we obtained results such that the MSA is best, the MSA follows it and the SSA worst for all disruption probabilities. This is because the MSA model is least tightly restricted and the SSA is most tightly restricted.

In Table 2, we compare the problem sizes for the MMA, MSA and SSA models. The MMS and MSA are mixed-integer programming problems, while the SSA is simply an integer programming problem. From the comparison of CPU time, we know that not only the number of binary variables and constraints but also the nature of the problem has a strong impact on the computational load.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Number of decision variables and constraints.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Real variable number</td>
</tr>
<tr>
<td>MMA</td>
<td>520</td>
</tr>
<tr>
<td>MSA</td>
<td>520</td>
</tr>
<tr>
<td>SSA*</td>
<td>540</td>
</tr>
</tbody>
</table>

*After linearization

5.2 Morphological Analysis
Since we can derive the different network configurations, it makes sense to evaluate such features depending on the models. They are representatively characterized by the numbers of paths in both the normal and the abnormal states. Then we try to use the number of multiple distributions from both DCs to RSs and from RSs to customers as

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Comparison of results for the three models for the (2-5-50) problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability ($q$)</td>
<td></td>
</tr>
<tr>
<td>Relay station</td>
<td>URS</td>
</tr>
<tr>
<td>Number of facilities (RS#)</td>
<td>MMA</td>
</tr>
<tr>
<td>Expected cost</td>
<td>MMA</td>
</tr>
<tr>
<td>CPU time [s]</td>
<td>MMA</td>
</tr>
<tr>
<td>Gap [%]</td>
<td>MMA</td>
</tr>
</tbody>
</table>

URS: Unreliable relay station  RRS: Reliable relay station
a surrogate for evaluating a certain factor associated with the morphological structure. Receiving products from different multiple suppliers will increase the tedious treatments and extra handling costs compared with receiving from a single supplier. Reserving the backup paths against the disruption also requires additional countermeasures that will add spare loads in the normal state.

Eventually, we propose to view such an intangible attribute as complexity of the network $C \in [0, 1]$ defined by Eq. (48). It is an inverse index of simplicity, and we prefer the simple configuration in general.

$$C = (1 - w)\left(\frac{m_{RS}}{2J} + \frac{m_{CS}}{2K}\right) + w \left(\frac{m_{RS}}{K}\right)^{\alpha}$$

(48)

where $m_{RS}$, $m_{CS}$ and $m_{Rk}$ are numbers of multi-served RSs, multi-served customers and backup paths, respectively. Moreover, $w$ denotes a weighting factor and $\alpha$ and $\beta$ are elasticity coefficients in the primary and backup states, respectively. Hence, each fraction represents a normalized value of each number. It makes sense to set these values in such that $w=q$, $\alpha<1$ and $\beta>1$ since the abnormal state corresponds to the disruption probability and certain scale merit can reduce the spare loads while urgent tasks will need them increasingly.

We then depict the relationship between the total cost and the complexity in Fig. 5 where we can observe the trade-off relationship between the cost and the complexity. From this, the optimal network obtained from the MSA model seems to be the most adequate under the present propositions since nearly the minimum total cost can be attained while improving the simplicity considerably. If we could transform this merit into the cost, it is possible to draw a more definite final decision. To our knowledge, however, this is the first attempt to include such an

---

![Graph showing relationship between total cost and complexity.](image)

**Fig. 5 Relationship between total cost and complexity.**

---

<table>
<thead>
<tr>
<th>$q$</th>
<th>Expected cost</th>
<th>CPU time [s]</th>
<th>Gap [%]</th>
<th>Expected cost</th>
<th>CPU time [s]</th>
<th>Gap [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>7,401,882</td>
<td>0.7</td>
<td>0.29</td>
<td>7,597,109</td>
<td>63.09</td>
<td>0.01</td>
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<tr>
<td>0.05</td>
<td>7,569,112</td>
<td>0.66</td>
<td>0.47</td>
<td>7,756,400</td>
<td>57.16</td>
<td>0.01</td>
</tr>
<tr>
<td>0.1</td>
<td>7,777,467</td>
<td>0.75</td>
<td>0.57</td>
<td>7,955,037</td>
<td>106.06</td>
<td>0.01</td>
</tr>
<tr>
<td>0.3</td>
<td>8,541,065</td>
<td>0.36</td>
<td>0.00</td>
<td>8,674,913</td>
<td>195.92</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>9,238,925</td>
<td>0.33</td>
<td>0.00</td>
<td>9,339,990</td>
<td>62.09</td>
<td>0.01</td>
</tr>
</tbody>
</table>

SSA model: Infeasible

---

![Graph showing profile of open RSs with disruption probability.](image)

**Fig. 6 Profile of open RSs with disruption probability.**

---

<table>
<thead>
<tr>
<th>$q$</th>
<th>Expected cost</th>
<th>CPU time [s]</th>
<th>Gap [%]</th>
<th>Expected cost</th>
<th>CPU time [s]</th>
<th>Gap [%]</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>0.11</td>
<td>19,074,122</td>
<td>447.89</td>
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<tr>
<td>0.3</td>
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<td>1.34</td>
<td>0.00</td>
<td>20,830,578</td>
<td>507.05</td>
<td>0.01</td>
</tr>
<tr>
<td>0.5</td>
<td>22,530,650</td>
<td>1.27</td>
<td>0.00</td>
<td>22,533,988</td>
<td>1147.42</td>
<td>0.01</td>
</tr>
</tbody>
</table>

SSA model: Infeasible

---

Table 3 Numerical result for (4-15-150) problem

Table 4 Numerical result for (6-25-250) problem
intangible attribute in logistic network design through morphological analysis using multiple models.

5.3 Results for Larger Size Models

For further inspections, we solved several larger size problems. Among them, in Tables 3 and 4, we summarize the results for the (4-15-150) and (6-25-25) problems, respectively. In both cases, however, the SSA models have no feasible solutions. Similarly to the foregoing smaller size problem, URSs will shift to RRSs along with the increase in $q$ as illustrated in Fig.6(a) and (b). Similarly, a more flexible structure (MMA) can achieve smaller total cost than the inflexible SSA structure while increasing the complexity. We also notice the CPU time becomes rapidly large with the size as a nature of the problems.

6 CONCLUSION

In this paper, we presented three allocation models in multistage logistic network design considering disruption risk. We formulate each model as mixed-integer programming problems and use commercial optimization software to solve the models. Through the numerical experiments, we have shown each model is promising for use to design the multi-stage logistic networks available for the mitigation planning. Moreover, defining a metric to stand for a certain quality of the structure or complexity, we have shown a procedure to derive a final decision through morphological analysis.

Future studies should be devoted to evaluating the solving ability of the commercial software, and apply the models to some real-world applications in addition to developing a more sophisticated morphological analysis.

REFERENCES