Analysis of Optimal Restart Policies for Software Systems

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Abstract: The restart is one of the typical environmental diversity techniques in dependable computing, and is quite effective to rejuvenate software systems at low cost. In this paper we generalize the seminal results on restart mechanisms by van Moorsel and Wolter (2004, 2006) and analyze optimal restart policies under more general conditions. We further develop a statistical algorithm to estimate the optimal restart policies from the empirical data of task processing time.

Key words: restart policy, fault tolerance, software systems, statistical estimation, total time on test

1 INTRODUCTION

Fault-tolerant computing techniques can be classified into two categories; design diversity technique and environmental diversity technique [2]. The former denotes expensive redundancy techniques including recovery block and N version programming, and the latter corresponds to preventive and proactive solutions by properly changing the operational circumstance of the software system. Checkpointing is a typical example of the environmental diversity technique, because it dynamically changes the data-backup circumstance using checkpoints [3, 7, 12, 16]. Software rejuvenation is another environmental diversity technique, and receives considerable attention, for preventing transient failures caused by software aging [1, 8–10, 14, 19, 24]. For instance, the software aging is caused by memory leaks of an operating system or application software, it is known that periodic process restarting may be effective to release allocated memories. The typical examples of software rejuvenation are garbage collection, memory compaction, defragmentation and hardware reboot. In many cases, since software faults cannot be fixed in the operational phase, such proactive fault management techniques are rather efficient to achieve the high availability requirement for critical software systems.

In this paper, we consider the restart mechanism that is the simplest environmental diversity technique with low cost. It is frequently used to determine the timing to response transmission in the Internet services such as web applications. Dimitrov et al. [5] analyzed the total processing time with a retry/restart mechanism in software systems from checkpointed states. Maurer and Huberman [15] was the first work to introduce the basic concept of restart. Reinecke et al. [17, 18] and van Moorsel et al. [20–22] considered stochastic models for the restart mechanism, and derived the optimal restart time by minimizing the mean net completion time of one task and maximizing the task completion probability. Especially, a statistical inference algorithm to estimate the optimal restart time by minimizing the mean net completion time is discussed in Reinecke et al. [17]. In recent years, restart algorithms are applied to service-oriented architectures on cloud computing [25] and distributed database management system [13]. For a comprehensive survey on the restart mechanism for computer-based systems, see the study by Wolter [23].

The purpose of this paper is to generalize the results by van Moorsel and Wolter [20–22]. More specifically, we generalize the basic restart model from the viewpoints of the Laplace-Stieltjes transform (LST), the finite number of restarts and random restart time. Next, we derive a nonparametric inference algorithm to estimate the optimal restart time from complete task completion time samples which is regarded as non-censored data. The fundamental idea is to apply the basic concept of the total time on test (TTT) statistics. Based on the
TTT concept, we develop a graphical method to determine and estimate the optimal restart time. It is quite useful to estimate the optimal restart time even when the underlying probability law of task completion time is unknown, and can be categorized into a nonparametric inference approach. Our results in this paper complement the seminal results of van Moorsel and Wolter [20–22] and provide new insights to the optimal restart mechanism. Throughout simulation experiments, it is shown that the resulting nonparametric estimator asymptotically converges to the real (but unknown) optimal restart time as the volume of data increases. Finally, the paper is concluded with some remarks regarding future studies.

2 MODEL DESCRIPTION

2.1 Periodic Restart Model

Following van Moorsel and Wolter [20–22], let \( T \) denote the completion time of a task or transaction without restarts and the non-negative random variable having the cumulative probability distribution (c.d.f.) \( F(t) \), probability density function (p.d.f.) \( f(t) \) and finite mean \( E[T] = \lambda \ (\lambda > 0) \). Without any loss of generality, it is assumed that \( F(t) \) is absolutely continuous and non-decreasing in \( t \ (\geq 0) \). Suppose that a software system starts operating at time \( t = 0 \) and restarts at \( t = \tau \) periodically if the task is not completed by time \( \tau \), where \( \tau \) is called the restart time, or the restart policy in this paper. The overhead time associated with restarting is given by constant \( c \ (\geq 0) \), so that a restart takes place periodically every \( \tau \) time unit until the task is completed. Define the net completion time of a task with an unbounded number of restarts by \( T_{\tau} \), which is a non-negative random variable having c.d.f. \( F_{\tau}(t) \) and p.d.f. \( f_{\tau}(t) \). Figure 1 depicts the schematic illustration of the restart mechanism under consideration.

To complete our discussion, let \( T_k \ (k = 1, 2, \cdots) \) be independent and identically distributed (i.i.d.) random variables having the same c.d.f. as \( T \). Then, the net completion time of a task with an unbounded number of restarts, \( T_{\tau} \), is represented by

\[
T_{\tau} = \sum_{k=1}^{\nu_0-1} (\tau + c) + T_{\nu_0} = (\nu_0 - 1)(\tau + c) + T_{\nu_0},
\]

where \( \nu_0 = \min\{k \geq 1; T_k < \tau\} \). Since this is regarded as a Bernoulli trial with success probability \( F(\tau) \), \( \nu_0 \) reduces to a simple geometric distributed random variable, and the probabilistic law of random variable \( T_{\tau} \) is evident. For instance, it is straightforward to see that the mean completion time of a task with an unbounded number of restarts, \( E[T_{\tau}] \), is given by

\[
E[T_{\tau}] = \{E[\nu_0] - 1\}(\tau + c) + E[T_{\nu_0}]
\]

\[
= \frac{\tau}{1 - F(\tau)} + \int_0^\tau tf(t)du \frac{F(t)}{F(\tau)}
\]

\[
= \int_0^\tau F(t)dt + cT \frac{F(t)}{F(\tau)}
\]

from the elementary probabilistic argument, where \( F(\cdot) = 1 - F(\cdot) \).

van Moorsel and Wolter [20–22] derived the c.d.f. of the net completion time of a task with an unbounded number of restarts by

\[
F_{\tau}(t) = \begin{cases} 
1 - F(\tau)^k F(t - k(\tau + c)) & \text{if } k(\tau + c) \leq t < k(\tau + c) + \tau \\
1 - F(\tau)^{k+1} & \text{if } k(\tau + c) + \tau \leq t < (k + 1)(\tau + c)
\end{cases}
\]

for \( k = 0, 1, 2, \cdots \). For a more general probabilistic argument, see Dimitrov et al. [6]. Based on the result in Eq. (3), van Moorsel and Wolter [20–22] obtained a recursive formula to compute the \( n \)-th moment \( E[T_{\tau}^n] \) \((n = 1, 2, 3, \cdots)\) of the completion time with an unbounded number of restarts:

\[
E[T_{\tau}^n] = \frac{M_n(\tau)}{F(\tau)} + \frac{F(\tau)}{F(\tau)} \sum_{k=0}^{n-1} \binom{n}{k} \times (\tau + c)^{n-k} E[T_{\tau}^n],
\]

where

\[
M_n(\tau) = \int_0^\tau t^n F(t) \ dt
\]

is the partial moment of \( T \) and \( E[T_{\tau}^0] = 1 \). When
\( n = 1 \), Eq. (4) reduces to Eq. (2). As shown in the latter discussion, it is not so difficult to compute \( E[T^n] \) for an arbitrary \( n \) because the Laplace-Stieltjes transform (LST) of \( T \) can be obtained in the closed form. Nevertheless, the recursive formula in Eq. (4) may still be useful when the computation is executed in an algorithmic manner for relatively small \( n \).

### 2.2 Aperiodic Restart Model

Here, we introduce an extended model with an aperiodic restart time, where the restart time is given by \( \tau_0, \tau_1, \tau_2, \ldots \), before the completion of a task. Let \( \hat{T}_i \), denote the net completion time of the \( i \)-th task when the \((i - 1)\)-th task fails to complete, where \( \pi_i = \{\tau_i, \tau_{i+1}, \ldots\} \) is a set of the restart policy after the \( i \)-th restart decision. Define the optimal restart policy that minimizes the mean net completion time of a task by \( \pi^*_i = \{\tau^*_i, \tau^*_{i+1}, \ldots\} \). Then, it holds from the principle of optimality that the optimal restart policy has to satisfy the following dynamic programming equation:

\[
E[\hat{T}_{\pi^*_i}] = \min_{\tau_i} \left\{ \int_0^{\tau_i} F(t) dt + F(\tau_i) \right\}
\]  
\[\times \left( c + E[\hat{T}_{\pi^*_{i+1}}] \right) \text{ for } i = 1, 2, \ldots \]  
\[ \text{(6)} \]

**Theorem 1:** Suppose that the number of restarts for one task is not unbounded. Then, \( E[\hat{T}_{\pi^*_i}] = E[\hat{T}_{\pi^*_2}] = \cdots \) holds, and the periodic restart policy is superior or equivalent to any aperiodic restart policy.

The proof is omitted for simplicity. From Theorem 1, it would be enough to find the optimal restart time \( \tau^* \) minimizing Eq. (2) in terms of the minimization of the mean net completion time of a task.

### 3 PROBABILISTIC RESTART MECHANISM

We generalize the periodic restart model by introducing a random restart time. Suppose that the software system restarts at the i.i.d. random time \( X_k \) periodically if the task is not completed, where \( X_k \) denotes the \( k \)-th restart time \((k = 1, 2, \cdots)\) having the c.d.f. \( G(t) \), p.d.f. \( g(t) \) and finite mean \( 1/\mu \) \((> 0)\). This situation occurs when an opportunity to trigger the restart arrives randomly. Additionally, it is assumed that the overhead time associated with restart is given by \( Y \), which is a non-negative random variable with c.d.f. \( H(t) \) and finite mean \( E[Y] = c \) \((> 0)\) as well. Suppose

\[
G(t) = U(t - \tau) = \begin{cases} 1 & \text{if } t \geq \tau, \\ 0 & \text{otherwise,} \end{cases} \quad \text{(7)}
\]

where \( U(\cdot) \) is the unit step function. Then, the probabilistic restart policy based on c.d.f. \( G(t) \) is reduced to the periodic restart policy with a constant restart time interval. In other words, our probabilistic restart policy involves the periodic policy as a special case. On the other hand, generalization on the random recovery overhead may be regarded as a minor modification at first glance. However, since the c.d.f. of the net completion time of a task with an unbounded number of restarts cannot be given in a simple form as Eq. (3), by putting \( E[Y] = c \), it is not a trivial case.

The net completion time of a task with an unbounded number of restarts, \( T_G \), is then given by

\[
T_G = \sum_{k=1}^{\nu-1} (X_k + Y) + T_v, \quad \text{(8)}
\]

where \( \nu = \min \{k \geq 1; T_k < X_k\} \). This is regarded as the total service time under a service discipline called the preemptive-repeat-different, so that when task execution is interrupted by a server failure, it will be repeated anew upon recovery of the server. Khalil and Dimitrov [11] obtained the following theorem, but unfortunately the proof was not presented in their study. We give the mathematical proof to complete our discussion.

**Theorem 2:** The LST of the net completion time \( T_G \) under the probabilistic restart policy is given by

\[
E[e^{-sT_G}] = \frac{\int_0^{\infty} e^{-st} G(t) dF(t)}{1 - H^*(s)} \int_0^{\infty} e^{-st} F(t) dG(t),
\]

where

\[
H^*(s) = \int_0^{\infty} e^{-st} dH(t), \quad \text{(10)}
\]

denotes the LST of \( Y \).
Proof: Since the renewal points for the underlying stochastic model are the task completion time point and restart point, we obtain the following renewal-type equation on the net completion time of a task:

\[
E[e^{-sT_G}] = \int_0^\infty \int_0^{\tau} e^{-st} dF(t) dG(\tau) + \int_0^\tau e^{-st} E[e^{-s(Y+\tau_T G)}] F(\tau) dG(\tau).
\]

Solving the above equation with respect to \(E[e^{-sT_G}]\) yields Eq. (9).

Since the LST of \(T_G\) involves the information on arbitrary higher moments, deriving the recursive formula on the \(n\)-th moment in Eq. (4) is almost equivalent to deriving the LST of the net completion time \(T_G\), when

\[
H(t) = U(t - c) = \begin{cases} 
1 & \text{if } t \geq c, \\
0 & \text{otherwise}. 
\end{cases}
\]

The following corollaries are the immediate results from Theorem 2.

**Corollary 1:** The LST of net completion time \(T_r\) under the periodic restart policy is given by

\[
E[e^{-sT_r}] = \frac{\int_0^{\tau} \exp(-st) dF(t)}{1 - H^*(s) F(\tau) \exp(-s\tau)).}
\]

**Corollary 2:** The mean net completion time under the probabilistic restart policy is given by

\[
E[T_G] = \frac{\int_0^\infty G(t) F(\tau) dt + c \int_0^\infty G(t) F(\tau) dt}{\int_0^\infty G(t) F(\tau) dt}.
\]

**Corollary 3:** Under the exponentially distributed restart policy, the LST of the net completion time of a task and its mean value are given by

\[
E[e^{-sT_G}] = \frac{F^*(s + \mu)}{1 - \mu H^*(s) F^*(s + \mu)/(s + \mu)}; \quad \text{E}[T_G] = \frac{(1/\mu + c) F^*(\mu)}{F^*(\mu)},
\]

respectively, where

\[
F^*(s) = \int_0^\infty e^{-st} dF(t).
\]

Corollary 3 indicates the moments of net completion time of a task when the restart is triggered by a constant hazard rate under the probabilistic restart policy. The following is a comparison theorem for the periodic and probabilistic restart policies.

**Theorem 3:** If \(T\) is the decreasing hazard rate (DHR), i.e., \(f(t)/F(t)\) is decreasing in \(t\), then \(E[T_G] \geq E[T\tau]\) does not always hold. Therefore, the periodic restart policy is not always better than the probabilistic restart policy.

Apart from the feasibility in practice, this is a quite negative result for validating the periodic restart policy when \(T\) is DHR.

Before closing this section, we remark on the case where \(X_k (k = 1, 2, \cdots)\) are generally distributed random variables with c.d.f. \(F(t)\), but \(T\) is the exponentially distributed random variable. This case is quite interesting for characterizing exponential distribution. As a different service discipline from our restart mechanism, we consider the preemptive-resume service, i.e., a task is continued upon restarting the software system. In this case, the net completion time of a task is represented by

\[
S_G = T + \sum_{k=1}^{\nu'} Y_k,
\]

where \(\nu' = \min\{k \geq 1; T < X_1 + \cdots + X_k\}\). Dimitrov and Khalil [5] and Khalil and Dimitrov [11] proved that if \(T\) is exponentially distributed, then \(E[e^{-sT_G}] = E[e^{-sS_G}]\) and \(E[T_G] = E[S_G]\), i.e., net completion times of a task under the two different disciplines coincide in distribution as well as in expectation. In particular, under the periodic restart policy, the net completion time of a task without restart is exponentially distributed if and only if \(E[e^{-sT_T}] = E[e^{-sS_T}]\), where \(S_T\) is defined with Eqs. (7) and (12).

### 4 OPTIMAL RESTART POLICY

van Moorsel and Wolter [20,22] considered a minimization problem of the mean net completion time of a task with an unbounded number of restarts, and characterized the optimal restart time \(\tau^*\) minimizing \(E[T_T]\) in Eq. (2). Unfortunately, their results seem to be somewhat incomplete, because they just give a sufficient condition based on only the first-order condition of optimality. Here, we derive the complete characterization theorem on the optimal
restart policy. Define
\[ q(\tau) = F(\tau) - h(\tau) \left\{ c + \int_0^\tau F(t) dt \right\}, \tag{19} \]
where \( h(t) = f(t)/F(t) \) is the hazard rate of completion time \( T \).

**Theorem 4:** (1) Suppose that the completion time of a task, \( T \), is strictly DHR, i.e., the function \( h(t) \) is strictly decreasing in \( t \). If \((c + \lambda)^{-1} > h(\infty)\), then there exists a finite and unique optimal restart time \( \tau^* (0 < \tau^* < \infty) \) satisfying \( q(\tau^*) = 0 \), and its associated minimum mean net completion time of a task is given by
\[ E[T_{\tau^*}] = h(\tau^*)^{-1} - c, \tag{20} \]
otherwise, \((c + \lambda)^{-1} \leq h(\infty)\), it is optimal not to restart the system \((\tau^* \to \infty)\) and the corresponding minimum mean completion time of a task is given by \( E[T_{\tau^*}] = \lambda \).

(2) Suppose that the completion time of a task, \( T \), is the increasing hazard rate (IHR), i.e., the function \( h(t) \) is increasing in \( t \). Then, the optimal restart time is always given by \( \tau^* \to \infty \).

**Proof:** Let \( N(\tau) \) and \( D(\tau) \) be the numerator and denominator of Eq. (2), respectively. Differentiating Eq. (2) with respect to \( \tau \), we have
\[ \frac{d}{d\tau} E[T_{\tau}] = \frac{N'(\tau)D(\tau) - N(\tau)D'(\tau)}{D(\tau)^2}, \tag{21} \]
where \( ' \) denotes the derivative with respect to \( \tau \). Dividing the numerator in Eq. (21) by \( F(\tau) \), we get the following function, \( q(\tau) \), whose sign is equivalent to the sign of the first derivative of \( E[T_{\tau}] \):
\[ q(\tau) = \frac{1}{F(\tau)} \left( N'(\tau)D(\tau) - N(\tau)D'(\tau) \right) = (1 - ch(\tau)) D(\tau) - h(\tau) N(\tau). \tag{22} \]
It is evident that the sign of function \( q'(\tau) \) is equivalent to the sign of the following function:
\[ \tilde{q}(\tau) = -h'(\tau) \{ cD(\tau) + N(\tau) \}. \tag{23} \]
For any \( \tau \), if \( h'(\tau) \) is strictly negative, i.e., \( F(t) \) is strictly DHR, it holds that \( \tilde{q}(\tau) > 0 \) and that the function \( q(\tau) \) is monotonically increasing. Since \( q(0) = -ch(0) < 0 \), if \( q(\infty) > 0 \), or equivalently \((c + \lambda)^{-1} > h(\infty)\), then there exists a uniquely optimal restart time \( \tau^* (0 < \tau^* < \infty) \) satisfying \( q(\tau^*) = 0 \). On the other hand, if \( h'(\tau) \) is positive, namely, \( F(t) \) is IHR, then \( \tilde{q}(\tau) \leq 0 \) and \( q(\tau) \) is decreasing in \( \tau \). From \( q(0) = -ch(0) < 0 \), the mean net completion time of a task is decreasing in \( \tau \). The proof is completed.

It should be noted that the monotone case (IHR or DHR) in Theorem 4 is enough to characterize the optimal restart policy in various cases. For instance, van Moorsel and Wolter [20] introduced a mixed hyper/hypo-exponential distribution with a unimodal hazard rate to describe the completion time of different sizes of tasks. Supposing that the hazard rate \( h(\tau) \) is a unimodal function at \( \tau = \tau_0 \), the function \( q(\tau) \) decreases to \( \tau = \tau_0 \) and turns to increase. If \( q(\infty) > 0 \), then there exists a unique \( \tau^* (\tau_0 < \tau^* < \infty) \), otherwise \( q(\tau) < 0 \) and \( \tau^* \to \infty \).

For better understanding the restart model, we interpret the underlying algebraic minimization problem \( \min_{0 \leq \tau \leq \infty} E[T_{\tau}] \) as a geometrical one. Based on the same idea as Dohi et al. [8], we define
\[ \phi(p) = \frac{1}{\lambda} \int_0^{F^{-1}(p)} F(t) dt, \tag{24} \]
where
\[ F^{-1}(p) = \inf \{ \tau \geq 0; F(\tau) \geq p \}, \quad (0 \leq p \leq 1). \tag{25} \]
Function \( \phi(p) \) is called the scaled total time on test (TTT) transform and is sometimes called the equilibrium distribution of the random variable \( T \). It is well known that \( F(\tau) \) is IHR (DHR) if and only if \( \phi(p) \) is concave (convex) on \( p \in [0, 1] \) (see Dohi et al. [8]). From a few algebraic manipulations, we have the following result.

**Theorem 5:** On the two-dimensional plane \( (p, \phi(p)) \in [0, 1] \times [0, 1] \), the minimization problem, \( \min_{0 \leq \tau \leq \infty} E[T_{\tau}] \), is reduced to \( \min_{0 \leq p \leq 1} \{ c/\lambda + \phi(p) \} / p \), so that the optimal \( p^* = F(\tau^*) \) gives the minimum tangent slope from \( (p, \phi(p)) = (0, -c/\lambda) \) to the curve \( \phi(p) \).

The above result can be obtained using a similar TTT technique to Dohi et al. [8]. Figure 2 illustrates the graphical determination of \( \tau^* = F^{-1}(p^*) \). If the hazard rate \( h(t) \) is the inversely unimodal (bath-tub) function, then \( \phi(p) \) draws an S-shaped curve. From Fig. 2, it is seen that the optimal restart policy \( p^* = F(\tau^*) \) is uniquely determined so as to minimize the tangent slope from
\( (p, \phi(p)) = (0, -c/\lambda) \) to the curve \( \phi(p) \) on a two-dimensional graph. From this result it can be understood that once the c.d.f. \( F(t) \) is given, the determination of the optimal restart time is possible in spite of the monotone property.

5 STATISTICAL INFERENCE SCHEME

Our next concern is to develop a statistical algorithm to estimate the optimal restart policy. Suppose that \( n \) observations, i.e., \( 0 = x_0 \leq x_1 \leq x_2 \leq \cdots \leq x_n \), ordered samples from a completion time distribution of task \( T \), are available. If \( F(t) \) is known in advance, the problem is to estimate the model parameters involved in \( F(t) \) from the data. We consider here a nonparametric approach to estimate the task completion time distribution. We consider the empirical distribution function:

\[ F_n(t) = \left\{ \begin{array}{ll} j/n & \text{for } x_j \leq t < x_{j+1}, \\
1 & \text{for } x_n \leq t \end{array} \right. \]  

for \( 0 \leq t < \infty \). It is well known that the empirical distribution is strongly consistent, namely, \( F_n(t) \rightarrow F(t) \) almost surely as \( n \rightarrow \infty \) from the law of large numbers. Based on the empirical distribution, we define the scaled TTT statistics by \( \phi_{n,j} = \psi_j/\psi_n \), where

\[ \psi_j = \sum_{k=1}^{j} (n-k+1)(x_k-x_{k-1}), \quad j = 1, 2, \ldots, n \]  

and \( \psi_0 = 0 \). Note that \( \phi_{n,j} \) is a numerical counter part of \( \phi(p) \) in Eq. (24) and possesses a strongly consistent property, \( \phi_{n,j} \rightarrow \phi(t) \) almost surely as \( n \rightarrow \infty \). On the two-dimensional plane, plotting \( [j/n, \phi_{n,j}] (j = 1, 2, \ldots, n) \) and connecting by line segments yields the scaled TTT plot. The following result is a direct application of Theorem 5.

**Theorem 6:** Let \( 0 = x_0 \leq x_1 \leq x_2 \leq \cdots \leq x_n \) be \( n \) i.i.d. completion time samples for an identical task. Then, a nonparametric estimate of the optimal restart time minimizing the mean net completion time is given by \( \hat{\tau}^* = x_{j^*} \), where index \( j^* \) satisfies

\[
\min_{0 \leq j \leq n} \frac{c}{\sum_{j=1}^{n} x_j/n} + \phi_{n,j}
\]  

and tends to be \( \hat{\tau}^* \rightarrow \tau^* \) almost surely as \( n \rightarrow \infty \).

From Theorem 6, it is possible to estimate the optimal restart time directly from the ordered samples of the task completion time. It complements the result by Reinecke et al. [17], though they consider the grouped data of the task completion time and never refer to its asymptotic property.

6 NUMERICAL ILLUSTRATIONS

For the illustrative purpose to estimate the optimal restart time, we conduct Monte Carlo simulation. Suppose that the task completion time obeys the Weibull distribution:

\[ F(t) = 1 - e^{-(t/\eta)^m}, \quad t \geq 0, \]  

where \( m \) and \( \eta \) are the scale and shape parameters, respectively. Since task completion time should be strictly DHR to guarantee the existence of a finite optimal restart time, we consider two cases of \( m = 0.5 \) and \( m = 0.8 \), where the shape parameter \( \eta \) is adjusted as \( \lambda = 1 \) in respective cases. We assume three overhead costs for the restart as \( c = 1.0, 0.5 \) and \( 0.1 \). In the simulation experiments, we generate 10 data sets, where each data set consists of 500-1,000 task completion time data. In each data set, we calculate estimates of the optimal restart time and its associated minimum mean net completion time and take the average for 10 data sets. Figures 3 to 14 illustrate the asymptotic behavior of the estimated optimal restart time and the corresponding minimum mean net completion time for varying parameters, \( m = 0.5, 0.8 \) and \( c = 1.0, 0.1 \), where the horizontal line in each figure denotes the analytical value on the optimal restart time or the minimum mean net completion time.

In the case with \( m = 0.5 \) (Figs. 3-8), it can be seen that the nonparametric estimation algorithm can provide excellent goodness-of-fit performance even for small samples. On the other hand, in the case of \( m = 0.8 \) and \( c = 1.0 \) (Figs. 9 and 10), it is found that the resulting estimates tend to deviate from the analytical solutions. This is because probability \( \Pr\{T > \tau^*\} \) is extremely small for both the task completion time \( T \) and the optimal restart
time $\tau^* \approx 17.3$. In fact, since $\Pr\{T > \tau^*\} = 2.0\times10^{-5}$, more than 100,000 samples for task completion time will be needed to guarantee sufficiently accurate estimates, while we set the maximum number of sample data to 1,000. However, even though the estimates deviate from the theoretical values, it is seen from Fig. 10 that the minimum mean net completion time of a task takes the closed value to the theoretical result. Figures 11-14 show the asymptotic convergence behavior of these nonparametric estimators and reveal a similar tendency to Figs. 3 to 14 for the small restart overhead.
Fig. 7  Asymptotic behavior of nonparametric estimates on the optimal restart time ($m = 0.5$, $c = 0.1$).

Fig. 8  Asymptotic behavior of nonparametric estimates on the minimum mean net completion time ($m = 0.8$, $c = 0.1$).

Fig. 9  Asymptotic behavior of nonparametric estimates on the optimal restart time ($m = 0.8$, $c = 0.5$).

Fig. 10  Asymptotic behavior of nonparametric estimates on the minimum mean net completion time ($m = 0.8$, $c = 1.0$).

Fig. 11  Asymptotic behavior of nonparametric estimates on the optimal restart time ($m = 0.8$, $c = 0.5$).

Fig. 12  Asymptotic behavior of nonparametric estimates on the minimum mean net completion time ($m = 0.8$, $c = 0.5$).
7 CONCLUSIONS

In this paper, we considered the restart mechanism for computer-based systems and generalized the seminal results on restart mechanisms by van Moorsel and Wolter [20–22]. We analyzed three types of restart policies, periodic restart, aperiodic restart and probabilistic restart, and examined these interrelations. We further developed a graphical method to determine/estimate the optimal restart time minimizing the mean net completion time of a task. In simulation experiments, we carried out the Monte Carlo simulation and investigated the asymptotic behavior of the resulting estimates. In the future, we will consider a more complex restart model with the bounded number of restarts. Although this has been studied by van Moorsel and Wolter [20–22], no analytical results on the optimal restart policies have been completed. Additionally, an adaptive estimation scheme to estimate the optimal online restart time should be considered, because it is difficult to collect the sufficient volume of data for task completion time in practical computer-aided systems.

REFERENCES


