Vibration Reduction of Reciprocating Engines*

Kuninori ITO^{*1}, Nobuaki SATO^{*2} and Kenichi KOSUGE^{*3}

Abstract

This report treats the design procedures for the crank-angle arrangements and firing sequences for multi-cylinder reciprocating engines. The authors developed the most powerful design method of finding uneven crank-angle arrangements for four and two stroke cycle reciprocating engines of multi-cylinders. The study reveals how the various kinds of engine-exciting forces and/or moments are simply expressed and how the procedure contributes toward the vibration reduction of multi-cylinder engines.

In order to build the design procedure, a new type of optimization algorithm is introduced: a method in which the feasible domain due to the constraint conditions is clarified. In other words, the idea of "modified cyclotomic polynomial (MCP)" analysis is contrived. This report points out that the introduced concept is significant not only for the reciprocating engine design but also for general engineering and science. Actually new and effective crank-angle arrangements have been looked for. This paper also establishes new ways in solving non-linear optimization problems with non-linear constraint conditions.

Key Words: Engine Vibration, Uneven Crank-angle Arrangement, New Optimization Procedure, Modified Cyclotomic Polynomial, Vibration and Control, Complex Vector Space and Complex Manifold.

1. Introduction

Seeking after higher efficiency, higher power density and better environmental conformity is the perpetual task for reciprocating engines. In developing new engines, designers always bear marginal design in their mind so as to achieve minimum strength and rigidity for better cost effectiveness. At the same time, they have to pay attention to restraining the possible vibration level of new engines under development. Reliability and comfort of reciprocating engines and engine rooms must not be spoiled. On the other hand, large commercial engines are of multi-cylinder type: 6 ~ 10 cylinders in line type and/or 12 ~ 20 cylinders in Vee type. Therefore, the advance estimation of engine exciting forces and/or moments of each kind for multi-cylinder engines is essential.

In usual design practices, all sorts of engine exciting forces and/or moments are evaluated after the crank arrangements and firing sequences are adopted. This study, in opposition, shows how the crank-angle arrangement and/or firing sequence can be decided. Conventional crank arrangements are almost of even angle arrangement and the primary and/or secondary external couples are minimized by selecting pertinent firing sequences. If residual external couples remains, expensive balancing equipment is often introduced. This study gives a way to dispense with such balancers or to minimize them.

In order to demonstrate and verify the powerfulness of the solving algorithm of the study, some examples of new

solutions are presented. Especially by means of an example result of reducing X-type exciting moment, modal analysis simulation of an engine laid on an elastic mounting was carried out to show fairly-well expected effectiveness.

It had been shown by Ferrand [1] or Sugiura et al. [2] that good uneven crank arrangements can exist. However, the authors have not been able to find any paper which took such a step into the general theory of uneven crank-angle arrangement as this report did.

To build the general theory, the authors induced a nonlinear optimization problem in which independent variables are located within high-dimensional complex vector spaces or high-dimensional complex manifolds. The authors also invented special polynomial equations to solve, which the authors named as MCP (modified cyclotomic polynomial) equations. Thanks to "MCP concept", the feasible domain of the optimization problem becomes quite easy to grasp. This makes the method versatile. The authors also point out that the introduced method can be applied to many other difficult engineering problems. Fig.1 shows the illustrative expression of the complex manifold and "MCP".

2. Exciting Forces and Couples

2.1 Scope

This report treats of the inertia forces, external couples, inner-moments, exciting forces due to H-type vibration, exciting moments due to X-type vibration and exciting vector-sums of the torsional vibration for multi-cylinder reciprocating engines of both two stroke cycle and four stroke cycle. The introduced theory is applicable to both in-line engines and V-type engines. Because a V-type crank-throw assembly is usually modeled like an in-line one, the authors mainly focus on in-line engines.

2.2 Dimension and Scale

In this report the discussion is held chiefly by means of non-dimensional coefficients for various kinds of exciting forces and/or moments (or torques). Consequently, there appear few values with scale units unless needed.

*Received March 8, 2006

*^{1,2,3} Mitsubishi Heavy Industries, Ltd.

Yokohama Research & Development Center

1-8-1 Sachiura, Kanazawa-ku Yokohama, JAPAN

*¹A regular member, born 1948, graduated from Tokyo Univ., applied mechanics / *²Born 1968, graduated from Sophia Univ., structural design / *³Born 1972, graduated from Nihon Univ., mechanical engineering.

The domain to be sought:

$$(P_A \cup P_B) \cap \{\text{Small Circles} \mid z_i \bar{z}_i = 1\}$$

$$S_p: 1 + z_2 \bar{z}_2 + \dots + z_n \bar{z}_n = (\sqrt{n})^2$$

$$\text{Quadric Hypersurface: } 1 + z_2^2 + \dots + z_n^2 = 0$$

$$\text{Hyperplane: } 1 + z_2 + \dots + z_n = 0$$

$$\text{MCP (Modified Cyclotomic Polynomial):}$$

$$(t - z_2)(t - z_3) \dots (t - z_n) = 0$$

Fig.1 Illustrative expression of the feasible domain

Suppose that L is cylinder distance and F is the typical inertia force for one cylinder. Further suppose that

$$F = F_i \text{ or } F_{cent} \quad (1)$$

Where F_i ($i=1,2,3,4,5,6,\dots$) is the i -th order inertia force and F_{cent} is the centrifugal force. Then, the total exciting force of a multi-cylinder engine is made dimensionless by being divided by $|F|$. In the same way, the total exciting couple is made dimensionless by $|F \cdot L|$. Here, the authors have to note that, if the gas pressure forces and/or side thrust forces for four stroke engines are paid attention to instead of inertia forces, the order of positive integer multiplied by 0.5 is to be taken into account.

Table 1 Typical crank-arrangement (four stroke)

cyl. number	crank-arrangement	firing sequence	coefficient of couple	
			1st.	2nd.
5 (10)		1-2-4-5-3	0.4490	4.980
6 (12)		1-5-3-6-2-4	0	0
6 (12)		1-2-4-6-5-3	0	0
7 (14)		1-2-4-6-7-5-3	0.2673	1.006
8 (16)		1-2-4-6-8-7-5-3	0	0
8 (16)		1-3-7-5-8-6-2-4	0	0
8 (16)		1-7-4-6-8-2-5-3	0	0
9 (18)		1-2-4-6-8-9-7-5-3	0.1937	0.5477
9 (18)		1-5-9-4-7-8-2-3-6	0.1264	1.577
10 (20)		1-6-9-3-7-10-5-2-8-4	0	0

2.3 Conventional Approach

When ordinary even-angle-decompositions and firing-sequences are reminded of, they are summarized as shown in "Table 1 (for four stroke engines)" accompanying external-couple-coefficients.

In connection with "Table 1", the primary couple coefficient for five cylinder four stroke engines, for example, is calculated as follows.

$$| \langle n, \xi_{s1} \rangle | = \text{abs} \left\{ \left\langle \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ \xi_5^4 \\ \xi_5^2 \\ \xi_5^3 \\ \xi_5 \end{bmatrix} \right\rangle \right\}$$

$$= \text{abs} \left\{ \left\langle \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ \xi_5^2 \\ \xi_5^3 \\ \xi_5^4 \\ \xi_5 \end{bmatrix} \right\rangle \right\} \quad (2)$$

Where, ξ is one of the 5-th roots of 1 and a complex number. Namely, we may consider.

$$\xi = \exp(j \cdot 2 \pi / 5), [j = \sqrt{-1}] \quad (3)$$

In more general way, the primary couple for n -cylinder engines is described in the expression below.

$$| \langle n, \xi_{n1} \rangle | = \text{abs} \{ \langle n, \xi_{n1} \rangle \}$$

$$= \text{abs} \{ \langle p, \xi_{p1} \rangle \} \quad (4)$$

Where,

$$n = [1 \ 2 \ 3 \ \dots \ n]^t \quad (5)$$

$$p = [p_1 \ p_2 \ p_3 \ \dots \ p_n]^t \quad (6)$$

$$\xi_{p1} = [1 \ \xi \ \xi^2 \ \dots \ \xi^{n-1}]^t \quad (7)$$

$$\xi_{n1} = [\xi^{q_1-1} \ \xi^{q_2-1} \ \dots \ \xi^{q_n-1}]^t \quad (8)$$

Here, the series $q_1, q_2, \dots, q_n$ is obtained by calculating the reverse permutation as follows.

$$\begin{pmatrix} 1 & 2 & 3 & \dots & n \\ q_1 & q_2 & q_3 & \dots & q_n \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ p_1 & p_2 & p_3 & \dots & p_n \end{pmatrix}^{-1} \quad (9)$$

The authors here note that, if the center of the couple should be positioned at the middle center of the engine, n vector has to be replaced by another one. For sample, for five cylinder case, the expression of n should be not

$$n = [1 \ 2 \ 3 \ 4 \ 5]^t \quad (10)$$

but

$$n = [-2 \ -1 \ 0 \ 1 \ 2]^t \quad (11)$$

For further example of six cylinder case, the expression is not

$$n = [1 \ 2 \ 3 \ 4 \ 5 \ 6]^t \quad (12)$$

but

$$n = [-2.5 \ -1.5 \ -0.5 \ 0.5 \ 1.5 \ 2.5]^t \quad (13)$$

It is important that each kind of the coefficient of total exciting moments and torques is described in a form of inner-product of two vectors as shown in "Eq. (4)".

As to the coefficient of total force, the following n -dimensional vector 1 , instead of n , is substituted into "Eq. (4)"

$$1 = [1 \ 1 \ 1 \ \dots \ 1]^t \quad (14)$$

Consequently, the coefficient for the primary total exciting force is described as follows.

$$|\langle \mathbf{1}, \xi_{n1} \rangle| = 1 + \xi + \xi^2 + \dots + \xi^{n-1} \quad (15)$$

Where, the right side of the above equation is 0.

The coefficient for the m -th order total exciting force is similarly shown as below.

$$|\langle \mathbf{1}, \xi_{nm} \rangle| = 1 + \xi^m + \xi^{2m} + \dots + \xi^{(n-1)m} \quad (16)$$

When one calculates the m -th order coefficient $|\langle \mathbf{n}, \xi_{nm} \rangle|$ for the primary coefficient $|\langle \mathbf{n}, \xi_{n1} \rangle|$ of "Eq.(4)", ξ_{nm} is given as below.

$$\xi_{nm} = [1 \quad \xi^{(q_2-1)m} \quad \xi^{(q_3-1)m} \dots \xi^{(q_n-1)m}]^T \quad (17)$$

The coefficients $|\langle \mathbf{1}, \xi_{nm} \rangle|$ and $|\langle \mathbf{n}, \xi_{nm} \rangle|$ are applicable to H-type vibration and X-type vibration respectively.

2.4 Inner-moment

For brief explanation, the authors here would like to stick to only primary inner-moment. Inner-moment diagram is a kind of bending moment diagram as shown in "Fig.2". The inner-moment distribution $\{M_1, M_2, M_3, \dots, M_n\}$ is calculated in accordance with makers' usual practice.

Fig.2 Inner-moment distribution

Suppose

$$\mathbf{M} = [M_1 \ M_2 \ M_3 \ \dots \ M_n]^T \quad (18)$$

and

$$\mathbf{K} = [K_1 \ K_2 \ K_3 \ \dots \ K_n]^T \quad (19)$$

Then, the following expression is obtained.

$$\mathbf{M} = -\mathbf{L}[\mathbf{S}]\mathbf{K} \quad (20)$$

Where, the matrix $[\mathbf{S}]$ is described as

$$[\mathbf{S}] = \frac{1}{n-1} \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ (n-2) & (n-3) & (n-4) & \dots & 3 & 2 & 1 & 0 \\ 2(n-3) & 2(n-4) & \dots & 6 & 4 & 2 & 0 \\ & 3(n-4) & \dots & 9 & 6 & 3 & 0 \\ & & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & \text{sym.} & 3(n-4) & 2(n-4) & (n-4) & 0 \\ & & & & 2(n-3) & (n-3) & 0 \\ & & & & & (n-1) & 0 \\ & & & & & & 0 \end{bmatrix} \quad (21)$$

Thus we are able to consider the coefficient for the inner-moment as

$$\frac{\max\{|M_i|\}}{F \cdot L} \quad (i=2, 3, \dots, n-1) \quad (22)$$

Therefore, the coefficient concerned is described in the following form.

$$|\langle s_i, \xi_{n1} \rangle| \quad (23)$$

Where, s_i is a pertinent column vector in the matrix $[\mathbf{S}]$ of "Eq.(21)".

2.5 Torsional Vector-sum

Because the vector-sum in torsional vibration calculations is regarded as the non-dimensional coefficient for the total vibratory torque-drive for crankshafts, it is requested to express a vector-sum in a form of the inner-product of two vectors. To satisfy this requirement, one may only take a vector which elements consist of specific amplitude distribution series as shown in "Fig.3". That is, let us suppose

$$\mathbf{a} = [a_1 \ a_2 \ a_3 \ \dots \ a_n]^T$$

where, $a_1, a_2, \dots, a_n$ are the specific amplitudes on the normal elastic curve for the node to be considered. By adopting the vector $\mathbf{a}$ instead $\mathbf{n}$ in "Eq. (4)", the coefficient concerned is expressed as

$$|\langle \mathbf{a}, \xi_{n1} \rangle| \quad (24)$$

Fig.3 Typical normal elastic curve

Summing up the above discussion, the various kinds of non-dimensional coefficients are summarized in "Table 2". And, "Table 3" shows a score sheet concerning whether, for the usual multi-cylinder engines, zero balance is achieved or not. According to "Table 3", it can be mentioned that, in case of usual even-crank-angle engines, some pertinent firing sequences are adopted so as to minimize both of external couples or to minimize either of couples under the condition that total forces are balanced. This is one of the important points when one thinks of uneven crank-angle arrangement.

Table 2 Exciting Drive Coefficient (non-dimensional expression)

Type of Exciting Drive	Stroke Cycle	Order k	Expression
Reciprocating force	2, 4	1	$ \langle \mathbf{1}, \xi_{n1} \rangle $
	2, 4	2	$ \langle \mathbf{1}, \xi_{n2} \rangle $
Centrifugal force	2, 4	1	$ \langle \mathbf{1}, \xi_{n1} \rangle $
Couple due to reciprocating force	2, 4	1	$ \langle \mathbf{n}, \xi_{n1} \rangle $
	2, 4	2	$ \langle \mathbf{n}, \xi_{n2} \rangle $
Couple due to centrifugal force	2, 4	1	$ \langle \mathbf{n}, \xi_{n1} \rangle $
Inner-moment	2, 4	1	$ \langle s, \xi_{n1} \rangle $
Torsional Vector-sum	2	1,2,3,4,...	$ \langle \mathbf{a}, \xi_{nk} \rangle $
	4	0.5,1,1.5,2,...	$ \langle \mathbf{a}, \xi_{nk} \rangle $
H-type (force)	2	1,2,3,4,...	$ \langle \mathbf{1}, \xi_{nk} \rangle $
	4	0.5,1,1.5,2,...	$ \langle \mathbf{1}, \xi_{nk} \rangle $
X-type (moment)	2	1,2,3,4,...	$ \langle \mathbf{n}, \xi_{nk} \rangle $
	4	0.5,1,1.5,2,...	$ \langle \mathbf{n}, \xi_{nk} \rangle $

n : number of cylinders

k : order number to be considered

Table 3 Score sheet on zero balancing

	n (number of cylinders)	Force		Moment	
		Primary $\langle \mathbf{1}, \xi_{p1} \rangle$	Secondary $\langle \mathbf{1}, \xi_{p2} \rangle$	Primary $\langle \mathbf{1}, \xi_{m1} \rangle$	Secondary $\langle \mathbf{1}, \xi_{m2} \rangle$
4 stroke cycle	5(10)	○	○	×	×
	6(12)	○	○	○	○
	7(14)	○	○	×	×
	8(16)	○	○	○	○
	9(18)	○	○	×	×
	10(20)	○	○	○	○
2 stroke cycle	5	○	○	×	×
	6	○	○	○	○
	7	○	○	×	×
	8	○	○	×	×
	9	○	○	×	×
	10	○	○	○	×

2.6 Further Advanced View

Through the above discussion up to the previous section, the authors have already succeeded in bringing an ordering relation into the set of firing sequences. The set of firing sequences of n -cylinder engines is isomorphic to the symmetric group S_n . Therefore, the authors have, in other words, succeeded in introducing a lattice structure, which is different from alphabet sequences, into the group S_n .

If p_1 and p_2 are different firing sequences of n -cylinder engines and if they give a same exciting coefficient, for example of primary external couple, then the following equations hold in accordance with “Eq. (4)”.

$$|\langle p_1, \xi_{p1} \rangle| = |\langle p_2, \xi_{p2} \rangle| \quad (25)$$

$$\xi_{p1} = \xi_{p2} = \xi \quad (26)$$

$$\|p_1\| = \|p_2\| = (1 + 2^2 + \dots + n^2)^{0.5} \quad (27)$$

“Eq. (25)” introduces an equivalence relation into the set of firing sequences, and also makes it possible to classify the set. “Eq. (27)” means that the tip points of the two vectors p_1 and p_2 are on the same sphere surface. Further, we can understand that the absolute values of the latitude for the tip points are the same, paying attention to “Eq. (26)” and “Eq. (27)”. Here, the authors note that the latitude corresponds to the rotating axis ξ in “Eq. (26)”.

Actually, the authors contrived the image of the ordering relation for three cylinder and four cylinder cases by means of the permutation polytopes as shown in “Fig.4” and “Fig.5” respectively.

However, as we originally started treating vectors of the n -dimension, we had better come back to the complex-vector-space C^n . Anyhow, it is important that the concept of distance or norm is brought into the set of firing sequences.

3. Generalization

3.1 Uneven Crank-Angle Arrangement

In case of even crank-angle arrangement, the components of ξ_{n1} vector or ξ_{p1} vector in “Eq. (4)” consist of the n -th roots of 1. Here, the authors define Z_1 vector instead of ξ_{p1} .

$$Z_1 = [z_1 \ z_2 \ z_3 \ \dots \ z_n]^t \quad (28)$$

And, we also define Z_2 vector as follows.

$$Z_2 = [z_1^2 \ z_2^2 \ z_3^2 \ \dots \ z_n^2]^t \quad (29)$$

Here, $z_1, z_2, \dots, z_n$ are complex numbers on the unit circle in Gaussian plane. And, we may assume $z_1=1$. The present subject is to minimize the following objective function f_{1n} or f_{2n} under the constraint conditions.

$$f_{1n}(Z_1) = |\langle p, Z_1 \rangle| \rightarrow \min \quad (30)$$

or

$$f_{2n}(Z_1) = |\langle p, Z_2 \rangle| \rightarrow \min \quad (31)$$

The constraint conditions are as follows.

$$g_1 = |\langle \mathbf{1}, Z_1 \rangle| = 0 \quad (32)$$

$$g_2 = |\langle \mathbf{1}, Z_2 \rangle| = 0 \quad (33)$$

$$|z_i| = 1 \quad (i=1, 2, \dots, n) \quad (34)$$

For the above problem of non-linear optimization, there has been not any firmly-established procedure to solve, though it is true that some software may give convergent solutions by chance.

Fig. 4 Permutation polytope for S_3 Fig. 5 Permutation polytope for S_4

Fig. 6 Uneven decomposition

Here, by the aid of Lagrange's method of indeterminate coefficients, the subject can be transformed into a variational problem or an "extended eigenvalue problem". Anyway, however, it is too complicated to solve the problem numerically. Actually one has to be confronted with a problem of minimum (or maximum) values of a real function defined by complex multi-variables. Further the problem is accompanied with non-linear multi-constraint conditions.

One difficulty lies on the multi-valley (or multi-peak) characteristics of the objective function as illustrated in "Fig.7" which shows an example objective function when one thinks of the primary external couple of three-cylinder's case. The function is expressed by two real substantially-independent polar-coordinate variables.

Fig.7 Multi-valley characteristic of functions (three cylinder)

Another difficulty is attributable to the vagueness of the feasible domain which is constituted by the constraint conditions ("Eq. (32)" to "Eq. (34)"). Without removing the vagueness, one is not capable of estimating if there were solutions. In fact, considering the feasible domain, one could conclude that there are only even-angle solutions for each of 3 to 5 cylinder's cases.

If the latter difficulty is overcome, the subject problem is fundamentally settled, because one could calculate the values of the objective function over the domain. Thus the authors are going to grasp all the local-solutions.

3.2 Feasible Domain

Let us discuss the dimension of the feasible domain at first. Suppose

$$z_i = x_i + jy_i \quad (i = 1, 2, \dots, n) \quad (35)$$

$$x_1, y_1 \in \mathbf{R}, j = \sqrt{-1}$$

$$x_1 = 1 \quad (36)$$

$$y_1 = 0 \quad (37)$$

Then, "Eq. (32)" to "Eq. (34)" give the following equations.

$$1 + \sum_{i=2}^n x_i = 0 \quad (38)$$

$$\sum_{i=2}^n y_i = 0 \quad (39)$$

$$1 + \sum_{i=2}^n x_i^2 - \sum_{i=2}^n y_i^2 = 0 \quad (40)$$

$$2 \sum_{i=2}^n x_i y_i = 0 \quad (41)$$

$$x_i^2 + y_i^2 = 1 \quad (i = 1, 2, \dots, n) \quad (42)$$

Paying attention to the numbers of unknown variables and equations, we obtain the degree of freedom concerning genuine independent parameters as shown in "Table 4".

Table 4 Degree of freedom (Dimension of feasible domain)

Number of cylinders	n	3	4	5	6	7	8	9
Number of unknowns	$2(n-1)$	4	6	8	10	12	14	16
Number of constraints	$(n+3)$	6	7	8	9	10	11	12
Degree of freedom	$(n-5)$	-2	-1	0	1	2	3	4
Degree in case of self-symmetry		-1	-1	0	0	1	1	2

"Eq.(32)" to "Eq.(34)" also lead the following equations.

$$1 + z_2 + z_3 + \dots + z_n = 0 \quad (45)$$

$$1 + z_2^2 + z_3^2 + \dots + z_n^2 = 0 \quad (46)$$

$$z_i \bar{z}_i = 1 \quad (i = 2, 3, \dots, n) \quad (47)$$

"Eq.(45)" to "Eq.(47)" mean that $z_2, z_3, \dots, z_n$ are the roots of an algebraic equation of the $(n-1)$ -th degree as below.

$$(t - z_2)(t - z_3) \dots (t - z_n) = t^{n-1} + (-1) \sigma_1 t^{n-2} + \dots + (-1)^{k-1} \sigma_{k-1} t^{n-k} + \dots + (-1)^{n-2} \sigma_{n-2} t + (-1)^{n-1} \sigma_{n-1} = 0 \quad (48)$$

All the root of "Eq.(48)" are on the unit circle and if

$$(-1)^{k-1} \sigma_{k-1} = 1 \quad (k = 2, 3, \dots, n) \quad (49)$$

Then "Eq.(48)" becomes what they call "cyclotomic polynomials". Therefore, the authors named the right side of "Eq.(48)" as "MCP (Modified cyclotomic Polynomials)".

"Eq.(45)" to "Eq.(47)" bring the following relations among $\sigma_1, \sigma_2, \dots, \sigma_{n-1}$ in "Eq.(48)".

$$(-1)^{n-1} \sigma_{n-1} = e^{j\mu}, [\mu \in \mathbf{R}] \quad (49)$$

$$\sigma_1 = -1 \quad (50)$$

$$\sigma_2 = 1 \quad (51)$$

$$\bar{\sigma}_k = \sigma_{n-k-1} / \sigma_{n-1} \quad (52)$$

Consequently, the MCP equation is described with a few real parameters. Further, as all the roots of the MCP equation are on unit circle, These real parameters must exist in some bounded domain within the real linear-space of rather a few dimensions. This domain is very the one which the authors have sought for.

3.3 MCP Examples

Although the expression of MCP equations are not unique, let us write down concrete forms of them. The MCP equations for four to seven cylinders are respectively as follows.

$$4\text{cyl. } t^3 + t^2 + t - e^{j\mu} = 0 \quad (53)$$

$$5\text{cyl. } t^4 + t^3 + t^2 + e^{j\mu}(t+1) = 0 \quad (54)$$

$$6\text{cyl. } t^5 + t^4 + t^2 - e^{j\mu}(t^2 + t + 1) = \{t^3 - e^{j\mu}\}(t^2 + t + 1) = 0 \quad (55)$$

$$7\text{cyl. } t^6 + t^5 + t^4 - \frac{C_R \cdot e^{j\mu}}{1 + e^{j\mu}} t^3 + e^{j\mu}(t^2 + t + 1) = 0 \quad (56)$$

Also in case of more cylinders, MCP equations are described with the plural number of real parameters in accordance with the degree of freedom shown in “Table 4”.

“Eq.(53)” and “Eq.(54)” indicate that only even-angle-arrangements satisfy the constraint condition for four cylinder and five cylinder respectively.

“Eq.(55)” shows that, for any real number μ , there exists a solution which satisfy the constraint condition, as illustrated in “Fig.8”. The solution (a) and (b) in “Fig.8” are usual even-angle decomposition for two cycle and four cycle engines respectively. The solution (c) is unsymmetrical concerning real axis, but self-symmetrical concerning the broken dotted line-axis.

Fig.8 Six cylinder decomposition

“Eq.(56)” is an example which give two dimensional feasible-domain. The domain is illustrated in “Fig.9”.

Fig.9 Feasible domain (7 cylinder)

The authors here note that root-locus methods, the Sturm’s algorithm or similar ways are useful when one confines the domain. New theoretical approach is shown in Appendix A3.

3.4 Optimization Examples

3.4.1 Six Cylinder (2 stroke)

With the intersection angle Δ in “Fig.8” as a parameter, the primary and secondary external couples are taken more arbitrarily in contrast with usual practices. This situation is shown in “Fig.10”, where abscissa and ordinate correspond primary external couple and secondary external couple respectively. The 31 points with ■-mark are of usual practice and the points with ◆-mark are of uneven arrangement

However, we have to notice that, if we take less than 50 degree as Δ angle (see Fig.8), we have to take care of some main bearing (strength) such as the bearing between No.3 and No.4 cylinders in the example of “Fig.11”.

And, as a matter of course, all sorts of $|\langle n, Z_{nm} \rangle|$ must be checked and countermeasures must be incorporated when needed.

Δ : Intersection angle
 ■ : even decomposition
 ◆ : uneven decomposition

Fig.10 Primary and secondary external couples (6 cylinder 2 stroke)

Fig.11 An example solution (2-stroke 6-cylinder)

3.4.2 Four Stroke Examples

Two examples of good results (new solutions) are shown in “Table 5” and “Table 6”. The former one is for 7 cylinder and the latter for 9 cylinder. The authors note there are many other new-solutions besides the examples.

When the dimension size of the constraint subspace exceeds three, we may introduce another constraint condition from some geometrical view points of arrangement.

Table 5 An example solution (4-stroke 7-cylinder)

firing seq.: 1-2-3-5-7-6-4	decomposition	(a) even	(b) uneven
		1 cyl.	2 cyl.
	2 cyl.	+102.8571°	+100.26°
	3 cyl.	-154.2857°	-166.08°
	4 cyl.	-102.8571°	-112.14°
	5 cyl.	-51.4286°	-72.97°
	6 cyl.	+154.2857°	+132.90°
	7 cyl.	+51.4286°	23.98°
	<1, Z ₁ >	0	2.56 × 10 ⁻⁵
	<1, Z ₂ >	0	2.89 × 10 ⁻⁵
	<n, Z ₁ >	1.3279	0.00071
	<n, Z ₂ >	1.5389	0.9815

Table 6 An example solution (4-stroke 9-cylinder)

firing seq.: 1-2-4-6-8-9-7-5-3	decomposition	(a) even	(b) uneven
		1 cyl.	2 cyl.
	2 cyl.	+ 80°	+ 80.25°
	3 cyl.	- 80°	- 80.40°
	4 cyl.	+160°	+157.67°
	5 cyl.	-160°	-158.98°
	6 cyl.	-120°	-122.33°
	7 cyl.	+120°	+119.15°
	8 cyl.	- 40°	- 41.24°
	9 cyl.	+ 40°	+ 37.59°
	<1, Z ₁ >	0	9.50 × 10 ⁻⁵
	<1, Z ₂ >	0	2.10 × 10 ⁻⁵
	<n, Z ₁ >	0.1937	0.00037
	<n, Z ₂ >	0.5477	0.4581

4. Experiment by Simulation

In ship propulsion systems with in-line four-stroke engines on resilient mountings, X-type and/or H-type vibration problems may sometime arise. The authors had sought the firing sequence and angle arrangement of a 7-cylinder in-line engine so as to definitely reduce the X-type coefficient. The sequence and geometry is explained in “Fig.12”, which is a newly obtained solution within the feasible domain of “Fig.9”. Further, the authors carried out modal response analyses according to the usual FEM manner. In contrast with the conventional even angle arrangement or the uneven geometry shown in “Table 5”, remarkable overall-stillness (approximately 60% reduction as to the acceleration response especially around the front and tail sides of the engine) has been obtained (“Fig.13”).

Fig.12 An example solution (4-stroke 7-cylinder, 4N restrained)

OA value (all direction)	even (a)	uneven (b)	ratio (a) / (b)
acceleration [gal]	852.4	348.0	0.408
velocity [mm/s]	53.245	29.444	0.553
displacement [mm]	0.579	0.387	0.668

Fig.13 Response simulation results (4-stroke 7-cylinder)

The authors are expecting that this kind of technology is applied to the propulsion system for the next generation.

5. Conclusion

It is sometimes mentioned that the traditional engineering on engine vibration problems is mature and that few subjects except routine work are left around the field. This study gave different views to the contrary.

This study challenged the nonlinear optimization problems for vibration reduction and obtained significant results. The method of confining feasible domain in complex manifold by means of MCP is introduced. There are many locally-optimum solutions which are not known. This paper gave some of unknown solutions which conventional approaches could not find out. The method is so versatile that it is applicable to more general engineering and science of wide range.

6. Acknowledgement

The authors sincerely thank **Nippon Foundation** and **Japanese Marine Equipment Association** for the support to this fundamental & essential study.

The authors expect this study and results to be further standardized and pervaded.

7. References

- [1] M.Ferrand, US patent 4002087, Jan.11,1977 / EU (FR) patent 576560, May.12,1975.
- [2] K.Sugiura & M.Ohtsu, JP patent H7-91274 Apr.4,1995.

Appendices

A1. Inner-product

The authors adopted the following definition of the inner-product of two vectors in C^n .

$$\langle x, y \rangle \equiv \bar{x}_1 y_1 + \bar{x}_2 y_2 + \cdots + \bar{x}_n y_n \quad (A1)$$

Where,

$$x = [x_1 \ x_2 \ x_3 \ \cdots \ x_n]^t \in C^n \quad (A2)$$

$$y = [y_1 \ y_2 \ y_3 \ \cdots \ y_n]^t \in C^n \quad (A3)$$

A2. About Response Simulation

The authors would like to show an additional result about the response comparison between the even arrangement and the uneven arrangement in "Table 5". In this comparison, there is little difference as to the vibration response as shown in the following figure ("Fig.A1").

Fig.A1 Response simulation results (4-stroke 7-cylinder)

The main reason why there is little difference in this case is that there exists a characteristic frequency of the engine on the resilient mounting system in the neighborhood of not two times engine-revolution speed but 4 times speed. In this case, the uneven arrangement of "Fig.12" is superior to that of "Table 5".

A3. MCP for Seven Cylinder

Let C_R in "Eq.(56)" be $R\{\exp(j\mu/2) + \exp(-j\mu/2)\}/2$, then the MCP equation for seven cylinder can be expressed also as follows.

$$t^6 + t^5 + t^4 - Re^{j\frac{\mu}{2}} t^3 + e^{j\mu} (t^2 + t + 1) = 0 \quad (A2)$$

Where,

$$\mu \in [-\pi, \pi] \quad (A3)$$

$$R \in R \quad (A4)$$

All the roots of "Eq.(A2)" are on the unit circle. Therefore, with a pertinent bi-linear transformation, "Eq.(A1)" is converted into another polynomial equation, all the roots of which are real numbers and all the coefficient of which are also real numbers. For example, when $\mu = -3\pi/2$, the transformed equation of w is

$$R = \frac{-2\cos\frac{\mu}{2}(w^6 - 9w^4 + 19w^2 - 3) - 8\sin\frac{\mu}{2}w(w^2 - 3)(w^2 - 1)}{(w^2 + 1)^3} \quad (A5)$$

Where,

$$w = j \frac{1-t}{1+t} \quad (A6)$$

All the root of "Eq. (A5)" must be real. Therefore, we obtain

$$-0.03 < R < 1.41 \quad (A7)$$

Fig.A2 Determination of feasible domain

The above method is simpler and more convenient than the Sturm's algorithm, and gives a reliable approach to the subspace in the complex manifold of high-dimension.

A4. Permutation Polytopes

Concerning permutation polytopes the authors consulted only the book of C.Berge, "Principes de Combinatoire", for which a Japanese translation is published. In the book, a permutation polytope for S_4 introduced by G.Guibaud and P.Rosenstiehl is quoted.

The polytope [Fig. 5(b)] which the authors made is similar to the above one, but different from it.(EOF)