A Study about the Dynamic Interaction between Submersed Hanging Tube and Internal Flow

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Summary

Nowadays, risers and pipelines are widely used by the Offshore Industry. Such pipes are deployed mainly as a connection element for the stream of hydrocarbon or other service fluid during the offshore well drilling or the hydrocarbon production. The pipes that connect the platform at the sea surface down to the wellhead is named riser by the Industry. Several researches have been carried out about the risers’ mechanical behavior focusing mainly on the interaction among the pipe’s structure, floating platform’s motion, external hydrodynamic forces, and soil contact. However up till now, most of such investigations have neglected the effect of the internal flow on the pipe’s structure. Thus this work is focused on this gap: the interaction between the pipe’s structure and the forces and other effects caused by its internal flow. An Experiment was carried out at the Deep Sea Basin of the National Maritime Research Institute (Japan) using as a model, a silicon made tube of 10 m length. During the experiment, fresh water was pumped into the model in order to verify the effects of the internal flow. In addition, comments about the numerical simulation of the pipe’s mechanical behavior in the time domain are included.

1. Introduction

The Exploration & Production (E&P) of offshore petroleum and natural gas fields have grown tremendously in the last 30 years. During this period, the pipe and tubular elements have been widely deployed by the Offshore Industry.

One of the most important offshore application for pipe is as a connection element between two points wherein the hydrocarbon or another service fluid flow. The Offshore Industry names as riser, the pipe that connects the wellhead or another equipment on the sea floor up to the platform at the sea surface. The riser can be classified into production riser and drilling riser.

The production risers can also be classified into rigid riser and flexible riser. The rigid riser is a common pipe usually made of steel, and deployed mainly in a standing position; in this configuration it is usually called Top Tensioned Riser (TTR). The use of rigid riser is limited to platforms with none or low vertical motion, such as the Tension Leg Platform (Fig. 1a).

The flexible riser is made of several layers of polymeric materials, which are responsible for the riser sealing and fluid integrity, and metal armors, which gives strength to the riser1). The flexible risers are deployed mainly with conventionally moored floating platforms, such as semi-submersible and Floating Production Storage Offloading (FPSO) platforms. Figure 1b shows a semi-submersible platform with a flexible riser in a Lazy-Wave configuration.

The next riser’s application is during the offshore well drilling. The marine drilling riser is a rigid and top ten-
The well drilling is, actually, carried out in several phases; during the first and second phases, the well is drilled without riser, only the drillstring connects the platform to the well (Fig. 2a). There is no return of drilling mud or rock cuttings up to the platform. After the installation of the second casing, the Blow-Out Preventer (BOP) is lowered down attached to the drilling riser and installed on the wellhead, as shown by Fig. 2b. Then the rotary drilling operation is resumed.

In both production and drilling cases, the riser conveys a fluid. In the production riser case, either single phase or multiphase stream can flow for both directions upward and downward. The most common flow within the production riser is upward when a multiphase fluid (a mixture of water, oil, and gas) flows from the subsea petroleum well up to the platform’s facilities at the surface. But production riser also can have a downward flow as, for example, to inject water into the reservoir, to pump the gas used in gas-lift, or even to export the hydrocarbon production to the shore.

During the rotational drilling, a drilling mud is inject into the top of the drillstring; then the mud returns to the platform through the annular space between the drillstring and the drilling riser carrying the rock cuts up to the platform. In other words, during the drilling, such riser has an upward internal flow with solids in suspension. If a water-base mud is used, this internal fluid is heavier than the fresh water with a typical density equal to 11 lb/gal ($\sim 1320 \text{ kg/m}^3$)

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1 Drillstring is the rotary drilling column composed by the drill bit followed by the Bottom Hole Assembly (BHA) at the lower part and drill pipes at the upper part.
2 Casing is a pipe, which diameter is almost the same of the bottom of the well, lowered down into the openhole and cemented in place for the borehole stabilization.
3 BOP is a piece of safety equipment responsible for the well control in the event of a Kick (the undesirable influx, usually of gas, from the drilled formation into the well).

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Up till now, most of the riser’s investigations have been carried concerning about the riser’s mechanical behavior, the hydrodynamics loads and the boundary conditions: interaction with the soil and the effects of the platform’s motions.

For example, in the numerical analysis, Sparks features the effects of several parameters on the riser’s static behavior. Patel & Witz have, in its Chapter 11, an explanation about the use of Finite Element Method on the numerical analysis of riser’s mechanical behavior. About experimental analysis, Pereira et al. presented some experimental results using a reduced scale model of a Self-Standing Hybrid Riser. Morooka et al. presented an experiment of a Steel Catenary Riser carried out in a towing tank.

In addition, several researches have been carried out to understand better this hydrodynamic loads and to improve its mathematical models for its use by the Offshore Industry.

However, only a few of these works have been focused on the effects of internal flow on the riser’s mechanical behavior.

Among the theoretical studies that include the internal flow in their analysis, Wu & Lou features the dynamic equation that governs a TTR including the effect of an inviscid steady flow, their model assumes that the rigidity parameter, which the effects is limited to the portions near to both ends, can be neglected for deep water. Their results show that the effect of the internal flow is reduced, especially on the risers with high top tension ratios. The authors conclude that the riser’s “natural frequency increases with top tension and decreases with internal velocity”.

Olunloyo et al. investigated the effect of an inviscid on the dynamics of a buried pipeline. Their mathematical model includes, beside the internal flow, the effects of an elastic foundation, rigid and deformable porous bed. In this study, the internal flow velocity has a significant effect on the pipeline’s dynamics.

This divergent conclusion about the effect of the internal flow on the pipe behavior concern mainly due to the axial tension’s effect, which is presented in the TTR and neglected in the pipeline.

Among experimental investigations that includes internal flow, Guo & Lou carried out an experiment to verify the effect of the internal flow on the Vortex Induced Vibration (VIV). This experiment was carried out in a towing tank using a vertical model with 1.2 m of effective length. The results featured that the increase of the internal flow velocity reduces the model’s natural frequency. Further, in a constant current, the higher internal flow velocity also increased the pipe’s strains in both in-line and transverse directions.

Another contribution is the experiment carried out by
Bordalo et al.\textsuperscript{13}. They investigated the effect of a multiphase flow (gas-fluid) on the mechanical behavior of a \textit{catenary shaped riser}. This experiment was ran “dry” to discriminate the internal flow effect from hydrodynamics effects. The authors observed that the variation of the internal flow momentum can impose a whipping displacement on the pipe. Such variation of the internal flow may have 2 causes: the mass and velocity of the internal multiphase flow vary with time in each section; and the internal flow’s velocity vector change the direction due to the pipe’s shape.

This present work addresses the effects of a single phase internal flow on the dynamics of a hanging pipe. A program for the numerical simulation of riser dynamics have been developed trying to include the effect of the internal flow on the pipe's dynamic behavior. The numerical results are compared with experimental results. The experiment using, as a model, a silicon made tube of 10 m length was carried out at the Deep Sea Basin of the National Maritime Research Institute.

2. Experiment

The experiment was carried out at the Deep Sea Basin of the National Maritime Research Institute, located in the city of Mitaka, Tokyo Metropolis in Japan. The Deep Sea Basin has a circular basin with 16 m of diameter and 5 m of depth with 128 paddle type wave generators all around this circular basin. In addition, there is a deep pit of 5 m of diameter and 30 m of depth. Thus the total depth achieves 35 m; this basin can be assumed as one of the deepest marine test basin in the World.

2.1 Experimental Apparatus

The experimental apparatus was assembled on the deep pit. The circular basin and its wave generators had not been used during this experiment. Figure 3 shows a schematic of the experimental apparatus. It is composed by a positive displacement pump that inject fresh water from an auxiliary tank into the model. A flow meter measures the internal flow rate. The model’s top end is fixed to a pivoted tube and an oscillator generates the harmonic top motion.

During this experiment, glass balls were added to internal flow. In order to gather such glass balls, a basket was placed at bottom. Then the balls could be recovered using a winch.

2.2 Model

A silicon made tube of 10 m of length was deployed as model in this experiment. Table 1 shows the model’s main properties.

2.2.1 Model Instrumentation

Two different data acquisition system were used during the experiment. The first one is the Visual Measurement System. This system is composed by 10 pairs of camera distributed along the basin depth and a cluster of computers. Each pair of camera films a portion of the basin and the images are sent to the cluster. The cluster processes those images tracking and calculating the tridimensional position of each measurement station in \textit{real-time} with a sampling rate of 10 Hz. The model has 33 measurements stations along its length, and these stations are placed 30 cm of distance of each other, as shown by Fig. 4.

The second system is a conventional Analogical Data Acquisition System. During this experimental 11 channels have been acquired (Table 2). Further, the model itself has attached 4 accelerometers with 2 axis that measures the model’s axial and radial accelerations at 4 different positions as shown by Figure 5. Those accelerometers are a spare measurement system for the pipe’s dynamic response.

In addition, the internal flow is measured before to be injected into the model; as well the oscillator’s position and the vertical acceleration just above the model (Fig. 3). The reason of the vertical accelerometer is to measure the vibration induced by the rotating pump.

2.3 Parametric Analysis

During the experiment, the model’s top end was clamped to a pivoted tube as shown by Fig. 3. Actually, the oscillation of the top end has a prescribed trajectory in the shape of an arc as shown by Fig. 6. Such trajectory is kept the same during the whole experiment.

Despite the constant trajectory, the frequency of this oscillatory motion is a parametric variable; about fifteen different frequencies were used raging from 0.08 Hz to 0.5 Hz. In addition, the internal flow rate was the sec-

![Figure 3](image-url)
Fig. 4 The Model used during the experiment.

Fig. 5 Positions of the accelerometers.

Table 2 Analogical acquisition channels.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 2</td>
<td>Accelerometer #1</td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td>Accelerometer #2</td>
</tr>
<tr>
<td>5 &amp; 6</td>
<td>Accelerometer #3</td>
</tr>
<tr>
<td>7 &amp; 8</td>
<td>Accelerometer #4</td>
</tr>
<tr>
<td>9</td>
<td>Flow Rate</td>
</tr>
<tr>
<td>10</td>
<td>Oscillator’s Position</td>
</tr>
<tr>
<td>11</td>
<td>Vertical Accelerometer</td>
</tr>
</tbody>
</table>

Fig. 6 The prescribed trajectory of the pipe’s top end.

Table 3 Cases analyzed in this work.

<table>
<thead>
<tr>
<th>Case</th>
<th>Flow Rate [L/s]</th>
<th>Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.215</td>
</tr>
<tr>
<td>2</td>
<td>0.169</td>
<td>0.215</td>
</tr>
<tr>
<td>3</td>
<td>0.675</td>
<td>0.215</td>
</tr>
<tr>
<td>4</td>
<td>1.013</td>
<td>0.215</td>
</tr>
</tbody>
</table>

ond parametric variable. Five different internal flow rate were assumed, viz., 0.0 L/s (the pipe is filled with water, but there is NO internal flow); 0.169 L/s; 0.338 L/s; 0.675 L/s; and 1.013 L/s.

For lack of space, only the results for cases included in Table 3 will be discussed in this work.

3. Numerical Simulation

During this work, a computational program have been developed in order to simulate, in the time domain, the dynamic behavior of a riser including the effects of the internal flow. This program is coded in Fortran 90 and uses the Finite Element Method (FEM). Following, the analytical and numerical equations for the riser’s mechanical behavior are featured.

3.1 Analytical Model

Assuming the analytical model for a tensioned beam (Eq. 1), as following:

$$\frac{d^2}{dz^2} \left( EI \frac{d^2x_R}{dz^2} \right) - T \cdot \frac{d^2x_R}{dz^2} = q(z)$$ (1)

where $x_R$ is the riser deflection; $z$ is the coordinate along the unbent beam; $EI$ is the beam stiffness; $T$ is the axial tension; $q(z)$ is a transversal load per unit of length along the beam.

The first term of Eq. (1) corresponds to the beam’s bend stiffness. And the second term corresponds to the effect of the Tension $T$. We can see that such second term is related with the bending moment $d^2x_R/dz^2$. It means that the increase of tension reduces the bending moment and consequently the beam deflections.

Let’s take Equation (1) as basis, the analytical equation for the riser dynamics can obtained including the inertial
term and transverses loads due to effects of the hydrostatic and hydrodynamics pressures variation\(^7\), viz.,

\[ \begin{align*}
    m \frac{\partial^2 x_R}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 x_R}{\partial z^2} \right) + 2 \rho_l U \frac{\partial^2 x_R}{\partial t \partial z} & \quad - (T + A_0 \cdot P_0 - A_1 \cdot P_1 - \rho_l \cdot A_1 \cdot U^2) \frac{\partial^2 x_R}{\partial z^2} \\
    & \quad - \left[ \gamma_R \cdot (A_0 - A_1) - F_{ZR} - A_0 \cdot \gamma_0 + A_1 \cdot \gamma_1 \right] \cdot \frac{\partial x_R}{\partial z} \\
    & = \frac{1}{2} \rho_0 \cdot C_D \cdot D \cdot |u + U_C - \dot{x}_R| \cdot (u + U_C - \dot{x}_R) \quad (2)
\end{align*} \]

where \( EI \) is the beam stiffness; \( x_R \) is the riser deflection; \( z \) is the coordinate along the unbent beam; \( \rho_0 \) (\( \rho_l \)) is the density of the external (internal) fluid; \( T \) is the internal fluid velocity; \( U \) is the axial tension in \([N]\); \( A_0 \) (\( A_1 \)) is the external (internal) cross sectional area of the pipe; \( P_0 \) (\( P_1 \)) is the external (internal) pressure acting on the riser pipe wall; \( \gamma_R \) is the specific weight of the riser material; \( F_{ZR} \) is the resulting forces per unit length in the \( z \)-direction; \( \gamma_0 \) (\( \gamma_1 \)) specific weight of the external (internal) fluid; \( m \) is the mass per unit length (including the internal fluid mass and hydrodynamic added mass); \( C_D \) is the drag coefficient; \( D \) is the riser external diameter; \( u \) is the velocity of the external fluid particle due to surface waves; and \( U_C \) is the velocity of the external fluid particle around the pipe due to current.

Equation 2 includes the effects of an inviscid flow represented by the terms containing the internal flow velocity \( U \). According to Wu & Lou\(^1\), the internal flow introduces an additional coriolis force represented by \( 2 \cdot \rho_l \cdot U \cdot \frac{\partial^2 x_R}{\partial t \partial z} \). Despite this term has a time derivative, it does not dissipate energy from the system. Thus such term will be neglected in the numerical analysis.

### 3.2 Numerical Model

A computational program has been developed for the numerical simulation of the riser’s dynamic behavior in the time domain.

Instead of solve the dynamic equation (Eq. 2) with continuous distributed properties and infinite number of degrees of freedom, the riser is discretized in a finite number of elements; it is assumed that each element has 2 nodes (one node at each element’s end) with 3 degrees of freedom (2 translational DOF’s and 1 rotational DOF), summing 6 DOF’s per element. Then the riser’s dynamic properties are lumped into such nodes. Thus a discrete approximate dynamic equation is obtained, namely,

\[ \mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{B} \cdot \dot{\mathbf{x}} + \mathbf{K} \cdot \mathbf{x} = \mathbf{f}_e \quad (3) \]

where \( \mathbf{x}_e \) is the vector containing the displacement of each node in time domain, and the dot above represents its time derivative; \( \mathbf{M} \) is the global consistent mass matrix; \( \mathbf{B} \) is the global damping matrix; \( \mathbf{K} \) is the global stiffness matrix; \( \mathbf{f}_e \) is the vector of external loads that are concentrated on the nodes.

### The Inertial Properties

The inertia properties are lumped together into the nodes using an approximate method called Consistent Mass Method\(^1\). The inertial force (excluding the hydrodynamic added mass) for one single element is defined as,

\[ \mathbf{F}_{\text{Mass}} = \mathbf{M}_i \cdot \begin{bmatrix} \ddot{\mathbf{x}}_i \\ \dot{\mathbf{x}}_{i+1} \\ \mathbf{f}_{i+1} \end{bmatrix} \quad (4) \]

where \( \mathbf{M}_i \) is the consistent mass matrix for a single element defined by Eq. 5; \( \varpi_i, \tau_i, \) and \( \theta_i \) are the \( i \)-th nodal displacement in the local coordinate as shown by Fig. 7, and the two dots represents the second time derivative.

\[ \mathbf{M}_i = \frac{\pi L}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 156 & 22L & 0 & 54 & -13L \\ 4L^2 & 0 & 13L & -3L^2 \\ 140 & 0 & 0 \\ SYM & 156 & -22L \\ 4L^2 \end{bmatrix} \quad (5) \]

In the above equation, \( \mathbf{M}_i \) is the consistent mass matrix for a single element in the local coordinates as shown by Fig. 7; \( \varpi \) is the distributed mass per unit length in \([kg/m]\); \( L \) is the element’s length in \( [m] \); \( \tau_i, \) and \( \theta_i \) are the \( n \)-th nodal displacement in the local coordinate as shown by Fig. 7, and the two dots represents the second time derivative.

In order to achieve the global mass matrix \( \mathbf{M} \) (Eq. 3), it is necessary to compute the proper coordinate transformation for each element’s mass matrix and to carry out the matrix assembly operation\(^4\).

### The Elastic Properties

The elastic system force for a single element of riser is composed by two different terms as shown by Eq. 6. The first term \( \mathbf{K}_{G_i} \) is named elastic stiffness and corresponds to a bending beam including the axial deformation. The next term \( \mathbf{K}_{C_i} \), which is called geometric stiffness, corresponds to the effect of the axial...
tension on the bending properties. Both stiffness matrices can be obtained using Finite Element Method\textsuperscript{14).

\begin{equation}
\mathbf{F}_{\text{Elia}} = (\mathbf{K}_\text{Elia} + \mathbf{K}_\text{Gia}) \cdot \begin{pmatrix}
\pi_i \\
\theta_i \\
\pi_{i+1} \\
\theta_{i+1}
\end{pmatrix}
\end{equation}

The above equation represents the system stiffness matrix for a single element in the local coordinates. The term \( \mathbf{F}_{\text{Elia}} \) is the nodal stiffness forces for a single element of riser; \( \mathbf{K}_\text{Elia} \) is the elastic stiffness defined by Eq. 7; \( \mathbf{K}_\text{Gia} \) is the geometric stiffness matrix as shown by Eq. 8; \( \pi_i \), \( \pi_{i+1} \) and \( \theta_i \) are the \( i \)-th nodal displacement in the local coordinate as shown by Fig. 7.

\begin{equation}
\mathbf{K}_\text{Elia} = \frac{EI}{L^3} \begin{bmatrix}
\frac{\Delta L^2}{2} & 0 & 0 & -\frac{\Delta L^2}{2} & 0 & 0 \\
12 & 6L & 0 & -12 & 6L & 0 \\
4L^2 & 0 & -6L & 2L^2 & 0 & 0 \\
\frac{\Delta L^2}{2} & 0 & 0 & \frac{\Delta L^2}{2} & 0 & 0 \\
SYM & 12 & -6L & 4L^2 & 0 & 0 \\
\end{bmatrix}
\end{equation}

\begin{equation}
\mathbf{K}_\text{Gia} = \frac{T_{\text{Elia}}}{30L} \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
36 & 3L & 0 & -36 & 3L & 0 \\
4L^2 & 0 & -3L & -L^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
SYM & 36 & -3L & 4L^2 & 0 & 0 \\
\end{bmatrix}
\end{equation}

Equation 7 represents the elastic stiffness matrix for the \( i \)-th element of riser in local coordinates. Eq. 8 features the geometric stiffness matrix for the \( i \)-th element in local coordinates. In both equations, \( E \) is the Modulus of Elasticity for the riser material in [Pa]; \( I \) is Second Moment of Area of the pipe’s cross section in [m\textsuperscript{4}]; \( L \) is the element length in [m]; \( A \) is the pipe’s cross sectional area in [m\textsuperscript{2}]; and \( T_{\text{Elia}} \) is the Effective Tension defined by Eq. 9.

\begin{equation}
T_{\text{Elia}} = (T + A_0 \cdot P_0 - A_1 \cdot P_1 - \rho_f \cdot A_1 \cdot U^2)
\end{equation}

In the Eq. 9, \( T_{\text{Elia}} \) is the Effective Tension; \( T \) is the axial tension in [N]; \( A_0 (A_1) \) is the external (internal) cross sectional area of the pipe; \( P_0 (P_1) \) is the external (internal) pressure acting on the riser pipe wall; and \( U \) is the internal fluid velocity.

The Structural Damping Properties. The structural damping (the hydrodynamic viscous damping is NOT included in this term) is a property that is difficult to be deduced. Thus the global damping system is estimated using the Rayleigh’s Coefficients\textsuperscript{15), viz.,

\begin{equation}
\mathbf{B} = a_1 \cdot \mathbf{M} + a_2 \cdot \mathbf{K}
\end{equation}

In the above equation, \( \mathbf{B} \) is the global damping matrix; \( \mathbf{M} \) is the global consistent mass matrix; \( \mathbf{K} \) is the global stiffness matrix; the Rayleigh’s Coefficients \( a_1 \) and \( a_2 \) are defined by Eqs. (11) & (12), respectively. The matrices \( \mathbf{M} \) and \( \mathbf{K} \) are obtained by the assembly of the respective single element’s matrices and their proper coordinate transformation.

\begin{equation}
a_1 = 2 \cdot \frac{\zeta_1 \omega_1 - \zeta_2 \omega_2}{\omega_1 - \omega_2}
\end{equation}

\begin{equation}
a_2 = 2 \cdot \frac{(\zeta_1 \omega_1) - (\zeta_2 \omega_2)}{\omega_1^2 - \omega_2^2}
\end{equation}

where \( \omega_1 \) and \( \omega_2 \) are the angular frequencies of the first and second riser’s predominant vibration modes, respectively; \( \zeta_1 \) and \( \zeta_2 \) are the damping rate for \( \omega_1 \) and \( \omega_2 \), respectively.

The External Loads. The right term of Eq. (3) corresponds to the external loads; in the case of risers, it means mainly the hydrodynamic forces. This term also corresponds to the right term of Eq. (2) that is deduced from the Morison Equation\textsuperscript{16}.

The Boundary Conditions. The boundary conditions are necessary because the assembled stiffness matrix \( \mathbf{K} \) is usually a singular matrix\textsuperscript{14). Thus the boundary conditions information, in our case, it means the node’s position that are known before the calculation, must be taken in account for the calculation of the inverted matrix \( \mathbf{K} \).

In this work, the boundary condition is applied only to the top node, which has a prescribed trajectory, the same trajectory of the pipe’s top end described during the experiment, as shown by Fig. 6.

For the boundary conditions of prescribed trajectories, it is necessary to use the penalty method\textsuperscript{19}. In such method, the penalty (a “large” number) is added to the stiffness matrix’s element in the leading diagonal that is corresponded to the DOF which is desired to applied a prescribed boundary conditions. Such penalty is assumed as \( \tau = 1 \times 10^6 \) in our program.

\begin{equation}
\begin{bmatrix}
a_1,1 + \tau & a_1,2 & \ldots \\
a_2,1 & a_2,2 & \ldots \\
\vdots & \ddots & \ddots \\
a_{n,n} & \ldots & a_{n,n} \\
\end{bmatrix} \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n \\
\end{pmatrix} = \begin{pmatrix}
p_n + \tau \\
f_2 \\
\vdots \\
f_n \\
\end{pmatrix}
\end{equation}

For example, Equation (13) has a system of equation composed by a generic stiffness matrix. In this example, a prescribed boundary condition was added in the first DOF, namely, \( x_1 \). The penalty \( \tau \) was added to the respective element \( a_{1,1} \) in the leading diagonal. The force vector’s relative component is composed by the sum \( p_n + 1 \tau; \)
where $p_n$ is the prescribed value of $x_1$, which can prescribe a trajectory varying its value for each $n$-iteration of the time.

When we solve the above system of equation for a given $n$-iteration, the response for the first DOF will be $x_1 \simeq p_n$. The error between $x_1$ and $p_n$ depends of how large is the penalty. The penalty is an empiric value that shall be “larger” enough than the sum of the other terms that compose the same row/column, and also not too large because a over large penalty can make the matrix ill-conditioned.\(^{21}\)

The Internal Pressure is an important simulation parameter, used for example, during the calculation of the Effective Tension (Eq. 9). In this work, beside the hydrostatic internal pressure, it is also assumed the effect of frictional pressure drop due to internal flow.

The internal pressure is calculated for each node of the discrete riser. For the Effective Tension (Eq. 9) calculation, the element’s internal pressure is assumed as the pressure average of the element’s nodes. The pressure of the first node is an input parameter, then the pressure of the next node is calculated as shown by Eq. (14). After, the calculation is repeated for all nodes.

$$P_{n+1} = P_n - \Delta P \quad (14)$$

In above equation, $P_n$ is the pressure of the $n$-th node (the pressure of the top node $P_1$ is an input parameter); $\Delta P$ is the pressure drop due to internal flow.

In According to Economides et al., the internal pressure drop can be estimated summing three different terms, namely,

$$\Delta P = \Delta P_{PE} + \Delta P_{KE} + \Delta P_{PF} \quad (15)$$

where $\Delta P$ is the pressure drop along the pipe; $\Delta P_{PE}$ is the pressure drop due to potential energy change defined by Eq. (16); $\Delta P_{KE}$ is the pressure drop due to kinetic energy change; and $\Delta P_{PF}$ is the frictional pressure drop defined by Eq. (17). The pressure drop related to kinetic energy is vanished because, in this work, it is assumed that the fluid is incompressible and the internal diameter is constant all along the pipe, then there is no variation of the kinetic energy of the fluid.

$$\Delta P_{PE} = g \cdot \rho_f \cdot \Delta z \quad (16)$$

$$\Delta P_{PF} = \frac{2 \cdot f_f \cdot \rho_f \cdot U^2 \cdot \Delta L}{D_f} \quad (17)$$

Where $g$ is the gravitational acceleration; $\rho_f$ is the density of the internal fluid; $\Delta z$ is the variation of the height; $f_f$ is the Fanning Friction Factor; $U$ is the internal fluid velocity; $\Delta L$ is the variation of the length of the pipe’s element; $D_f$ is the internal diameter.

The Fanning friction factor is defined, in laminar flow, as,

$$f_f = \frac{16}{Re} \quad (18)$$

where $Re$ is the Reynolds Number.

The Fanning friction factor, in turbulent flow, can be estimated using the Chen Equation, viz.

$$\frac{1}{\sqrt{f_f}} = -4 \log \left\{ \frac{\epsilon}{3.7065} \cdot \frac{5.0452}{Re} \cdot \log \left( \frac{e^{1.1098}}{2.8257} + \left( \frac{7.149}{Re} \right)^{0.8981} \right) \right\} \quad (19)$$

where $\epsilon$ is the Relative Roughness of the pipe’s internal surface, which can be found in the literature. $Re$ is the Reynolds Number.

4. Results

4.1 Data Analysis

Although the Vibration Induced by Vortices could be observed in the experimental analysis, no discussion about this phenomenon is included in this work. Only the results in the In-Line directions will be featured.

Both the experimental and numerical results were obtained in the time domain. But in order to compare such results, it was decided to calculate, for each one of the measurement stations, the Fourier Coefficients, namely:

$$a_f = \frac{1}{N\pi} \int_0^{2\pi N} g_i(t) \cos 2\pi f t \, dt \quad (20)$$

$$b_f = \frac{1}{N\pi} \int_0^{2\pi N} g_i(t) \sin 2\pi f t \, dt \quad (21)$$

where $a_f$ and $b_f$ are the Fourier coefficients; $N$ is the number of the time series’ cycles that shall be analyzed; $g_i(t)$ is the time series of the $i$-th station; $f$ is the frequency in hertz that shall be analyzed what, in this case, means the excitation frequency; and $t$ is time in second.

Using the Fourier coefficients, it is possible to calculate the amplitude (Eq. 22) and phase (Eq. 23 of a signal’s component at the excitation frequency, namely,

$$A_f = \sqrt{a_f^2 + b_f^2} \quad (22)$$

$$\varphi_f = \begin{cases} \arctan \left( \frac{a_f}{b_f} \right), & \text{if } b_f > 0; \\ \pi + \arctan \left( \frac{a_f}{b_f} \right), & \text{if } a_f \geq 0, b_f < 0; \\ -\pi + \arctan \left( \frac{a_f}{b_f} \right), & \text{if } a_f < 0, b_f < 0; \\ \frac{\pi}{2}, & \text{if } a_f > 0, b_f = 0; \\ -\frac{\pi}{2}, & \text{if } a_f < 0, b_f = 0; \\ 0 & \text{if } a_f = 0, b_f = 0. \end{cases} \quad (23)$$

where $A_f$ and $\varphi_f$ are the amplitude and the phase; and $a_f$ and $b_f$ are the Fourier coefficients defined by Eqs. 20 and 21. The function defined by Eq. 23, that is also called atan2 in the most of the mathematical computer libraries, is an extension of the arctangent function. The arctangent
results is defined only in the interval $[-\frac{\pi}{2},\frac{\pi}{2}]$. In the other hand, the function \texttt{atan2} (Eq. 23) extends the results to the interval $[-\pi,\pi]$.

### 4.2 Comparing Results

Following, we will compare experimental and numerical results. The experiment was carried out using the internal flow rate as a parameter. Unhappily, it was not possible to measure the pressure at the pipe’s inlet during the experiments. Thus we decided to make the simulation using the inlet pressure as simulation’s parameter for each experimental case, namely, for each internal flow rate.

**Simulation parameter: Inlet Pressure.** Figure 8 shows both experimental and numerical results for the Case 1 (no internal flow and top oscillation with frequency of 0.215 Hz), which is defined by the Table 3. Despite the numerical simulation was calculated in the time domain, the numerical and experimental results will be compared by their amplitude and phase, which were calculated as shown by Eqs. 22 and 23, respectively.

Further, this figure is composed by two graphs: the amplitude response named Normalized Amplitude on the left, and the phase response on the right. The amplitude graph was normalized by the amplitude response of the highest measurement station analyzed. In addition, it was assumed the highest measurement station’s phase as zero reference. In both Normalized Amplitude and Phase graphs, the vertical axis corresponds to the vertical position in meters. The vertical position zero means the highest measurement station analyzed what does not mean the pipe’s top end. The Deep Sea Basin’s Visual Measurement System has a blind area at the top. About 1.5 m of the pipe’s top portion could not be analyzed during the experiment analysis. Thus the same pipe’s top portion was neglected during the numerical analysis.

The experimental responses are represented by squares in both amplitude and phase response, as shown by Fig. 8. In the same figure, the solids lines represent the numerical responses. In this case, the calculation was carried using the Inlet Pressure (the internal pressure at the pipe’s top end) as parameter. The Inlet Pressure is used as initial condition to calculate the internal pressure all along the pipe.

In Figure 8, we can see a gap around 3.5 m of the experimental response. Sometimes, during the experiment, the Visual Measurement System missed some measurement station as such in this case. Despite the unmeasured stations, it is possible to observe, in the experimental results, that the top and lower parts are vibrating in opposite phase (about 180°) with a single node around 5 m. This shape is resembles the second modal shape of a vibrating cantilever beam.

For the numerical response, it does not have a good agreement with the experimental. The amplitude response for the numerical analysis is higher at the pipe’s lower portion; and the phase between top and bottom is about 120°. But we can observe the impact of the inlet pressure (and consequently the internal pressure) on the dynamic behavior. The amplitude has the tendency to decrease and the phase has the tendency to increase, when the inlet pressure increases.

Following, Figure 9 shows the amplitude and phase response for the Case 2. Again, the experimental response is missing the results of two measurement stations: 5.5 m and 6.75 m. In this case, the numerical response for normalized amplitude has a better agreement with the experimental results. As observed in the prior case, the amplitude response, specially at the pipe’s lower portion, has the tendency to decrease and the phase has the tendency to increase when the inlet pressure increases.

Next, the results for the Case 3 are shown by Fig. 10. The amplitude response differs between the numerical and experimental results. The experimental response has the highest amplitudes at the upper portion of the pipe while the numerical response has the highest amplitudes at the pipe’s lower portion. In the phase response, the upper and lower portion have good agreement between numerical and experimental responses; the difference is concerning mainly to the position of the node. The experimental phase is located around 5.5 m, and the numerical node is located around 4 m.

The last case is presented by Fig. 11. In this case, there are a good agreement for both amplitude and phase re-
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**Fig. 9** Case 2: internal flow rate 0.169 L/s & top oscillation 0.215 Hz. The *Inlet Pressure* is used as parametric input.

**Fig. 10** Case 3: internal flow rate 0.675 L/s & top oscillation 0.215 Hz. The *Inlet Pressure* is used as parametric input.

**Fig. 11** Case 4: internal flow rate 1.013 L/s & top oscillation 0.215 Hz. The *Inlet Pressure* is used as parametric input.

Responses. The inlet pressure has the same effect as observed in the prior cases, namely, the inlet pressure decreases the amplitude and increases the phase at the pipe’s lower portion.

**Simulation parameter: Modulus of Elasticity.** Following, a numerical parametric analysis was carried out in order to verify the effect of such parameters on the riser’s mechanical behavior.

The first parameter is the *Modulus of Elasticity* $E$. Figure 12 shows the results for an internal flow rate of 1.013 L/s and a top oscillation frequency of 0.215 Hz (the same conditions of the Case 4 in Table 3). Such results are also compared with the experimental results of the Case 4.

We can observe that the response for the lowest Modulus of Elasticity, namely, $E = 0.5 \text{ GPa}$ has the highest elongation. The elongation can be observed for the length the respective response in both normalized amplitude and phase responses. Further, no significant variation can be observed especially for the highest Modulus of Elasticity responses.

**Simulation parameter: Drag Coefficient.** Next, the Hydrodynamic’s *Drag Coefficient* $C_D$ is used as parameter for the results shown by Fig. 13. For the normalized amplitude response, the variation of $C_D$ has same effect on the amplitude at the lower part of the riser. For the phase response, no significant variation is observed.
Fig. 12 Parametric Analysis: the Modulus of Elasticity $E$ @ internal flow rate 1.013 L/s & top oscillation 0.215 Hz.

Fig. 13 Parametric Analysis: the Drag Coefficient $C_d$ @ internal flow rate 1.013 L/s & top oscillation 0.215 Hz.

Fig. 14 Parametric Analysis: the lower Weight's mass @ internal flow rate 1.013 L/s & top oscillation 0.215 Hz.

Simulation parameters: sinker's properties. The last two parametric analyses concern to the weight located at the pipe's lowest end. This weight is composed by a plate of about 7500 mm$^2$ of area and 4.5 kg of mass. Then the first analyzed parameter is the weight’s mass (Fig. 14). We can observe in Fig. 14 that the numerical response for weight’s mass of 2.0 kg has the shortest length as expected. Further, the 2.0 kg response differs from the others, but, beyond the elongation due to the higher tension caused by the weight’s increasing mass, no significant variation can be observed in the responses for 4.6 kg, 6.0 kg, and 7.0 kg.

The last parametric analysis concerns about the variation of the lower weight’s drag coefficient as shown by Fig. 15. We can observe that the drag coefficient has some impact at the middle portion of the riser, but no significant response variations at the pipe’s extremities.

5. Conclusion

In this work, the effect of the internal flow effect on the riser’s mechanical behavior was investigated comparing numerical and experimental results.

The numerical simulation program was coded using the effects of the riser’s internal pressure and internal flow velocity on the Geometric Stiffness Matrix$^{10, 11}$. Because the lack of the internal flow pressure information form the experiment, the simulations were carried...
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References


