Residual Hull Girder Strength of Asymmetrically Damaged Ships

— Influence of Rotation of Neutral Axis due to Damages —

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Summary

This paper discusses the influence of the rotation of the neutral axis on the residual hull girder strength of asymmetrically damaged ships under longitudinal bending. Progressive collapse analysis of the damaged cross section is performed applying the Smith's method for the biaxial bending problem. An explicit expression of the location of the neutral axis including its rotation is given as a function of biaxial curvatures. The procedures of the progressive collapse analysis of the cross section under biaxial bending are presented for several loading and constraining conditions. A series of progressive collapse analysis of bulk carriers and double-hull tankers having collision damages at the side structures is performed. The residual hull girder strength of damaged ships considering the rotation of the neutral axis is found to be reduced from that obtained by constraining the rotation. The reduction rate of the ultimate strength is investigated for different damage location and extent. For a sagging condition, simple formulae to estimate the residual strength and the reduction rate due to the rotation of the neutral axis are proposed using the elastic cross-sectional properties and critical member strength. The effectiveness of the formulae is examined through a comparison with the progressive collapse analysis. The proposed analysis method and formulae can be utilized for the rational assessment of the residual hull girder strength of damaged ships.

1. Introduction

Ship's hulls may suffer accidental damages such as collision, grounding, fire etc., which may threaten the safety of ships and surrounding environment. In order to enhance the safety of ships and reduce the associated risks, the International Maritime Organization required in the Goal Based Standard (GBS) for bulk carriers and tankers the assessment of the residual hull girder strength in the specified damaged condition¹⁾.

Ultimate hull girder bending strength is the most fundamental strength to ensure the safety of ships not only in the intact condition but also in the damaged condition. Many studies have been performed on the assessment of the residual hull girder strength of damaged ships. Paik et al.²⁾ developed a procedure to identify the possibility of the hull girder failure after collision and grounding damages based on the closed-form formulae of the ultimate hull girder strength and section modulus after damages. Wang et al.³⁾ proposed a similar approach based on the section modulus. Notaro et al.⁴⁾ carried out full nonlinear FE assessment of the hull girder capacity in intact and damage conditions. They found that the effect of damage extent in vertical and transversal direction is more critical than its longitudinal direction, and that

the damage varies the location of the neutral axis including higher stresses in proximity in the damage areas.

Another approach widely employed for the prediction of the residual hull girder strength is the Smith's method⁵⁾, known as the incremental-iterative approach in the IACS Common Structural Rules⁶⁾. In the Smith's method, a hull girder is divided into longitudinal elements composed of a stiffener and attached plating, which are assumed to act independently. Assuming that the cross section remains plane and considering the nonlinear load end-shortening behaviors of each element, the bending moment-curvature relationship of the cross section is obtained. The translation of the neutral axis of the cross section due to the progressive failure of structural elements is considered. The ultimate bending capacity is defined as the peak value of the bending moment-curvature relationship of the cross section.

When the cross section is geometrically and mechanically symmetric with respect to the centerline and subjected to vertical bending moment, the neutral axis is always horizontal and moves only vertically during the collapse process. When the cross section is asymmetrically damaged, the neutral axis rotates and the problem needs to be treated as a biaxial bending problem. The external loads in the heeled condition after suffering damages require the biaxial bending calculation also. The existing studies on the ultimate strength of hull girders under biaxial bending considered the rotation and translation of the neutral axis in a reasonable way⁷⁻¹²⁾. Most of them employed a trial-and-error procedure to detect the position of the neutral axis and some the iterative approach using the Newton-Raphson method¹⁰⁾. Smith and Pegg expressed the biaxial bending moment-curvature relationship in term of secant moduli. The post-ultimate strength behavior could not be obtained in the case of prescribed biaxial moment loading, resulting in the difficulty in determining the

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ultimate capacity¹²⁾.

The aim of the present study is to investigate the influence of the rotation of the neutral axis on the residual hull girder strength of asymmetrically damaged ships under predominantly vertical bending. This effect may be one of the key factors in the assessment of the residual hull girder strength¹³⁾. The tangential moduli formulation of the biaxial bending-moment curvature relationship, derived by Smith⁵⁾, is applied to the damaged cross section including the effect of unloading at the yielded region. An explicit expression of the position of the neutral axis is given. The procedures of the progressive collapse analysis of the cross section under biaxial bending are presented for several loading and constraining conditions. Applying the developed system, a series of progressive collapse analysis of bulk carriers and double-hull oil tankers having collision damages at the side structures is performed, and the reduction of the residual strength due to the rotation of the neutral axis is investigated. For a sagging condition, a simple formula to estimate the residual strength and its reduction rate due to the rotation of the neutral axis is proposed using the elastic cross-sectional properties and the critical member strength.

2. Method of Progressive Collapse Analysis

2.1 Relationship of Cross-Sectional Force and Deformation

An asymmetrically damaged cross section of a ship's hull girder is considered as shown in Fig.1. The neutral axis where the bending stress and strain are zero rotates from the horizontal plane depending on the position and extent of the damage and the applied bending moment.

The horizontal and vertical coordinates, y and z , with the origin O at the keel are defined. Assuming that the cross section remains plane, the axial strain $\varepsilon_i(y_i, z_i)$ at the i -th structural element caused by the horizontal curvature ϕ_H and the vertical curvature ϕ_V can be expressed as

$$\varepsilon_i(y_i, z_i) = \varepsilon_0 + y_i\phi_H + z_i\phi_V \quad (1)$$

where ε_0 is the axial strain at the origin O . The axial stress σ_i corresponding to the axial strain ε_i can be obtained from the average stress-average strain relationship, Fig. 2, calculated in advance for each element. The average stress-average strain relationship considering the effect of buckling and yielding is generally a nonlinear function of strain ε_i and here expressed as

$$\sigma = f_i(\varepsilon) \quad (2)$$

where $f_i(0)=0$. The axial force P , the vertical bending moment M_V , and the horizontal bending moment M_H on the cross section can be obtained by integrating stress σ_i over the intact part of the cross section as

$$P = \sum_{i=1}^N \sigma_i A_i \equiv 0 \quad (3)$$

$$M_H = \sum_{i=1}^N \sigma_i y_i A_i \quad (4)$$

$$M_V = \sum_{i=1}^N \sigma_i z_i A_i \quad (5)$$

where N is the number of intact elements and A_i is the cross-sectional area of the i -th element.

The essential condition to determine the location of the neutral axis is the zero-axial force condition given by Eq. (3). Substituting Eq. (1) and (2) into Eqs. (3) to (5), a set of nonlinear simultaneous equations with respect to the axial strain, ε_0 , and the curvatures, ϕ_H and ϕ_V , are obtained. The neutral axis on the y - z plane can be expressed by the following equation of the straight line on the y - z plane.

$$\varepsilon_0 + y\phi_H + z\phi_V = 0 \quad (6)$$

2.2 Incremental Relationship of Biaxial Bending Moment and Curvature

To solve the nonlinear equations of (3) to (5), an incremental approach is employed as proposed by Smith⁵⁾. Denoting the tangential axial stiffness of the i -th element given by the slope of the average stress-average strain relationship by D_i (see Fig.2), the incremental relationship of axial stress and axial strain can be expressed as

$$\Delta\sigma = D_i\Delta\varepsilon \quad \left(\because D_i = \frac{df_i}{d\varepsilon} \right) \quad (7)$$

Using Eqs. (1) and (7), the incremental form of Eqs. (3) to (5) can be given as

Fig. 1 Asymmetrically damaged cross section.

Fig. 2 Average stress-average strain relationship.

$$\begin{Bmatrix} \Delta P = 0 \\ \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} \bar{D}_{AA} & \bar{D}_{AH} & \bar{D}_{AV} \\ \bar{D}_{HA} & \bar{D}_{HH} & \bar{D}_{HV} \\ \bar{D}_{VA} & \bar{D}_{VH} & \bar{D}_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_0 \\ \Delta \phi_H \\ \Delta \phi_V \end{Bmatrix} \quad (8)$$

where

$$\begin{aligned} \bar{D}_{AA} &= \sum_{i=1}^N D_i A_i, & \bar{D}_{AH} &= \bar{D}_{HA} = \sum_{i=1}^N D_i y_i A_i \\ \bar{D}_{HH} &= \sum_{i=1}^N D_i y_i^2 A_i, & \bar{D}_{AV} &= \bar{D}_{VA} = \sum_{i=1}^N D_i z_i A_i \\ \bar{D}_{VV} &= \sum_{i=1}^N D_i z_i^2 A_i, & \bar{D}_{HV} &= \bar{D}_{VH} = \sum_{i=1}^N D_i y_i z_i A_i \end{aligned} \quad (9)$$

Eq. (8) can be simplified by using the variables with respect to the centroidal position of the instantaneous neutral axis at the current incremental step as in the following:

The expression of the axial force increment ΔP of Eq. (8) can be rearranged in the form

$$\begin{aligned} \Delta P &= \bar{D}_{AA} \Delta \varepsilon_0 + \bar{D}_{AH} \Delta \phi_H + \bar{D}_{AV} \Delta \phi_V \\ &= \sum_{i=1}^N D_i (\Delta \varepsilon_0 + y_i \Delta \phi_H + z_i \Delta \phi_V) A_i \\ &= \sum_{i=1}^N D_i \{ \Delta \varepsilon_G + (y_i - y_G) \Delta \phi_H + (z_i - z_G) \Delta \phi_V \} A_i \end{aligned} \quad (10)$$

where

$$\Delta \varepsilon_G = \Delta \varepsilon_0 + y_G \Delta \phi_H + z_G \Delta \phi_V \quad (11)$$

y_G and z_G are the coordinates of the point G in Fig. 3 and $\Delta \varepsilon_G$ is the axial strain increment at the point G. When y_G and z_G are given by Eqs. (12) and (13),

$$y_G = \left(\sum_{i=1}^N y_i D_i A_i \right) / \left(\sum_{i=1}^N D_i A_i \right) \quad (12)$$

$$z_G = \left(\sum_{i=1}^N z_i D_i A_i \right) / \left(\sum_{i=1}^N D_i A_i \right) \quad (13)$$

Fig. 3 Instantaneous neutral axes for the analysis of incremental relationship of bending moments and curvatures.

Eq. (10) can be simplified as

$$\Delta P = \left(\sum_{i=1}^N D_i A_i \right) \Delta \varepsilon_G \quad (14)$$

It is known from Eq. (14) that under pure bending (i.e. when $\Delta P=0$), no axial strain is induced at the point G regardless of curvature increments. This means that the point G is on the instantaneous neutral axes as shown in Fig. 3. By replacing y_i and z_i in Eq. (9) by $(y_i - y_G) + y_G$ and $(z_i - z_G) + z_G$, respectively, and using $\Delta \varepsilon_G$ of Eq. (11), Eqs. (8) and (9) can be written in the form

$$\begin{Bmatrix} \Delta P = 0 \\ \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{AA} & 0 & 0 \\ 0 & D_{HH} & D_{HV} \\ 0 & D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_G \\ \Delta \phi_H \\ \Delta \phi_V \end{Bmatrix} \quad (15)$$

where

$$\begin{aligned} D_{AA} &= \sum_{i=1}^N D_i A_i, & D_{HV} &= D_{VH} = \sum_{i=1}^N D_i (y_i - y_G)(z_i - z_G) A_i \\ D_{HH} &= \sum_{i=1}^N D_i (y_i - y_G)^2 A_i, & D_{VV} &= \sum_{i=1}^N D_i (z_i - z_G)^2 A_i \end{aligned} \quad (16)$$

The incremental relationship of the biaxial bending moments and curvatures expressed in terms of their vertical and horizontal components are therefore given by

$$\begin{Bmatrix} \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{HH} & D_{HV} \\ D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \phi_H \\ \Delta \phi_V \end{Bmatrix} \quad (17)$$

The η and ζ axes in Fig. 3 indicate the instantaneous neutral axes at the current incremental step, i.e. the principal axes of the cross section having the tangential stiffness. The rotation angle θ of the η and ζ axes about the centroid G can be obtained by the way given in Appendix. Using the components of the bending moment and curvature with respect to the η and ζ axes, Eq. (17) can be decoupled as

$$\begin{Bmatrix} \Delta M_\eta \\ \Delta M_\zeta \end{Bmatrix} = \begin{bmatrix} D_{\eta\eta} & 0 \\ 0 & D_{\zeta\zeta} \end{bmatrix} \begin{Bmatrix} \Delta \phi_\eta \\ \Delta \phi_\zeta \end{Bmatrix} \quad (18)$$

where

$$D_{\eta\eta} = \sum_{i=1}^N D_i \zeta_i^2 A_i, \quad D_{\zeta\zeta} = \sum_{i=1}^N D_i \eta_i^2 A_i \quad (19)$$

It should be noted that y_G , z_G and θ vary at each incremental step with the spread of the buckling and yielding in the cross section.

Eq. (18) gives a simpler expression of the incremental relationship of biaxial bending moments and curvatures than Eq. (17). However, for dealing with the bending moments and curvatures about vertical and horizontal axes, Eq. (17) is more convenient. In addition, the location of the neutral axis with respect to the total stress and strain distributions can be easily obtained by substituting the cumulated values of ϕ_H , ϕ_V and ε_0 into Eq. (6). Eq. (17) is therefore used in this study.

Smith (1977) employed the pure incremental procedure (with no iteration) in each load step, taking the sufficiently small increment of the curvature. The same incremental procedure is employed in the present analysis.

3. Procedures of Progressive Collapse Analysis

The residual strength of asymmetrically damaged hull girders

under predominantly vertical bending is analyzed applying the prescribed vertical curvature with and without allowing the rotation of the neutral axis. The residual strength interaction relationship of the damaged hull girders under biaxial bending is also analyzed under prescribed curvature or moment control. The procedures of the analyses are given in the following.

Case 1: Pure Vertical Bending

The vertical bending moment is applied to the cross section with no constraint on the horizontal curvature. In this case, the horizontal curvature as well as the vertical curvature is induced under the condition of $M_H=0$. The incremental equation to be solved is

$$\begin{Bmatrix} 0 \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{HH} & D_{HV} \\ D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \phi_H \\ \Delta \phi_V^0 \end{Bmatrix} \quad (20)$$

where the superscript '0' indicates a prescribed value. The solutions are

$$\Delta \phi_H = -\frac{D_{HV}}{D_{HH}} \Delta \phi_V^0, \quad \Delta M_V = \left(D_{VV} - \frac{D_{VH} D_{HV}}{D_{HH}} \right) \Delta \phi_V^0 \quad (21)$$

The residual hull girder strength is calculated from the peak value of the $M_V - \phi_V$ curve.

Case 2: Constrained Vertical Bending

The vertical bending moment is applied to the cross section with the horizontal curvature constrained. Only vertical curvature is increased, and thus no rotation of the neutral axis takes place. This loading condition can be simulated by increasing the vertical curvature ϕ_V under the condition of $\phi_H=0$, that is,

$$\begin{Bmatrix} \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{HH} & D_{HV} \\ D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} 0 \\ \Delta \phi_V^0 \end{Bmatrix} \quad (22)$$

The solutions are

$$\Delta M_H = D_{HV} \Delta \phi_V^0, \quad \Delta M_V = D_{VV} \Delta \phi_V^0 \quad (23)$$

ΔM_H is the horizontal bending moment necessary for constraining the horizontal curvature. The residual hull girder strength is calculated from the peak value of the $M_V - \phi_V$ curve. Comparing Case 1 and Case 2, the influence of the rotation of the neutral axis on the residual vertical bending strength can be examined.

Case 3: Biaxial Bending under Prescribed Curvature Control

The vertical and horizontal curvatures are applied to the cross section with the prescribed ratio of $\Delta \phi_H / \Delta \phi_V$.

$$\begin{Bmatrix} \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{HH} & D_{HV} \\ D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \phi_H^0 \\ \Delta \phi_V^0 \end{Bmatrix} \quad (24)$$

Case 2 is the special case of Case 3 where $\Delta \phi_H^0 / \Delta \phi_V^0 = 0$. Because of the progressive collapse of structural elements and the resulting change in the stiffness parameters D , the bending moment ratio M_V / M_H changes nonlinearly even when the applied curvature ratio $\Delta \phi_H / \Delta \phi_V$ is constant, as shown in Fig. 4. The residual strength is obtained by detecting the $M_V - M_H$ point at the maximum distance from the origin, as shown by the hollow circle in Fig. 4.

Case 4: Biaxial Bending under Prescribed Moment Control

The vertical and horizontal bending moments are applied to the cross section with the prescribed ratio of $\Delta M_H / \Delta M_V$. The curvature, ϕ_V or ϕ_H , is taken as a controlling parameter. When ϕ_V is employed, the incremental equation to be solved is

$$\begin{Bmatrix} \alpha \Delta M_V \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{HH} & D_{HV} \\ D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta \phi_H \\ \Delta \phi_V^0 \end{Bmatrix} \quad (25)$$

where $\alpha = \Delta M_H / \Delta M_V$. The solutions are

$$\Delta M_V = \frac{\Delta M_H}{\alpha} = \frac{D_{HH} D_{VV} - D_{HV}^2}{D_{HH} - \alpha D_{VH}} \Delta \phi_V^0 \quad (26)$$

$$\Delta \phi_H = \frac{\alpha D_{VV} - D_{HV}}{D_{HH} - \alpha D_{VH}} \Delta \phi_V^0$$

When the prescribed ratio of $\Delta M_H / \Delta M_V$ is constant, M_V and M_H change linearly as shown in Fig. 4. The residual strength can be directly obtained by the peak value of M_V or M_H . The reduction of the post ultimate capacity can be also calculated. The complicated procedures of detecting the ultimate capacity, required in Case 3 or in the incremental-iterative approach using secant-moduli¹¹⁾, are not needed.

Fig. 4 Loci of the biaxial bending moment.

The analysis procedures common for all cases are summarized as

- (1) Subdivide the cross-section into elements composed of stiffener and attached plating.
- (2) Derive the average stress-average strain relationship of individual elements, Eq. (2), considering the influences of buckling and yielding.
- (3) Calculate the tangential stiffness of individual elements D_i , Eq. (7), from the average stress-average strain curve at the present strain.
- (4) Calculate the centroidal position of the instantaneous neutral axis y_G and z_G using Eqs. (12) and (13).
- (5) Evaluate the flexural stiffness of the cross section with respect to the instantaneous neutral axis, Eq. (17).
- (6) Calculate the unknown increments of curvature and/or bending moment under specified condition, Eqs. (20)-(26).
- (7) Calculate the strain increment in individual elements from the curvature increment and then the stress increment using the slope of average stress-average strain curve.
- (8) Add the obtained increments of curvature, bending moment as well as strains and stresses of the elements to their cumulative values.
- (9) Calculate the position of the neutral axis with respect to the cumulative values of stress and strain, Eq. (6).

(10) Proceed to the next incremental step.

4. Progressive Collapse Analysis

The progressive collapse analysis of the ship's hull girder having damages at the top side structure is performed including the biaxial loading case. The program code, HULLST, developed by Yao and Nikolov¹⁴⁾, is used for the analysis. The solution procedures of Case 1 and Case 4 are introduced to HULLST, in which only the prescribed curvature methods, Case 2 and Case 3, were available. In HULLST, the average stress-average strain relationship of stiffened panel elements is calculated by a semi-analytical method. The accurate applicability of HULLST elements has been widely recognized through extensive benchmark studies including the ISSC work¹⁵⁾. The applied curvature increment is determined so that the maximum axial strain increment at the deck or the bottom plating is less than 1/50 of the yield strain. It has been confirmed that smaller curvature increment gives almost the same moment-curvature relationship.

In the progressive collapse behavior of ship's hull girder in bending, unloading due to a shift of the neutral axis may take place in the yielded region, and this behavior needs to be simulated. When unloading took place in the elements that yielded in tension, the tangential stiffness D_t in Eq. (7) is changed to the elastic modulus of the material.

Two single-hull bulk carriers, B1 and B2, and one double-hull oil tanker, T2, is taken as the subject ships as shown in Table 1. Ship B1 is designed according to Pre-IACS/UR and built in 1987, while ship B2 according to CSR-B having larger cross section and scantling sizes. Ship T2 is designed according to CSR-T.

Table 1 Subject ships.

Ship	B1	B2	T2
L (mm)	217,000	219,000	234,000
B (mm)	32,236	32,240	44,000
D (mm)	18,300	19,900	21,200
Design criteria	Pre-IACS/ UR	IACS/ CSR-B	IACS/ CSR-T

The damage of the top-side part of the cross section, as required in the IACS Draft Harmonized CSR¹³⁾, is assumed. The vertical damage extent is taken as 10%, 20%, 40% and 70% of the ship depth (70% is specified in HCSR), and the horizontal damage extent is taken as B/16. The cross section of Ship B1 is illustrated in Fig. 5 and that of Ship T2 in Fig. 6. The damaged part is completely removed from the stiffness calculation.

The vertical bending moment-vertical curvature relationship, $M_V-\phi_V$, obtained for B1, B2 and T2 with 70% damage are shown in Figs. 7-9. The positive moment means the hogging condition. Case 1 considers the rotation of neutral axis, while Case 2 does not. It is found that Case 2 generally gives larger ultimate strength than Case 1 because the horizontal curvature is constrained. The effect is largest in Ship B1, smaller in Ship B2 and smallest in Ship T2. For Ship B1, Case 1 strength is 7.3% smaller than Case 2 strength in the sagging condition and 7.8% in the hogging condition. The difference between B1 and B4 is due probably to the difference in the depth to breadth ratio, member scantlings etc.

According to the result of Ship T2, the effect of rotation of neutral axis on the residual hull girder strength of double-hull tanker is relatively small when only the outer shell is damaged.

Fig. 5 Bulk Carrier (Ship B1)

Fig. 6 Oil Tanker (Ship T2)

Fig. 7 Vertical bending moment-vertical curvature

Fig. 8 Vertical bending moment-verticle curvature relationship (Ship B2, 70% damage)

Fig. 9 Vertical bending moment-verticle curvature relationship (Ship T2, 70% damage)

Fig. 10 Stress distribution and neutral axis (Ship B1, 70% damage, Case 1)

Fig. 11 Stress distribution and neutral axis (Ship B1, 70% damage, Case 2)

Fig. 10 shows the stress distribution of Ship B1 with 70% damage obtained by Case 1 analysis. The results at the points A to F in Fig. 7 are plotted together with the neutral axis, Eq. (6). The triangles in the figures indicate the elements that collapsed in tension and the circles those in compression. In the hogging condition (A-C), initial yielding occurs at the deck part of the damaged side and spreads toward the undamaged side. Then, buckling takes place at the bottom part of the undamaged side, which is at the larger distance from the inclined neutral axis than the damaged side, and the ultimate strength is attained. Beyond the ultimate strength, the load carrying capacity of the buckled bottom members decreases as illustrated in Fig. 2. To satisfy the zero-axial force condition for a whole cross section, the stress at the deck side decreases also. As a results, the neutral axis moves upward on the undamaged side. In the sagging condition (D-F), buckling occurs at the deck of the damaged side first and spreads toward the undamaged side. The ultimate strength is attained when the topside tank of the damaged side including the tank bottom panels fully buckled. No yielding occurs in the bottom part. Beyond the ultimate strength, the neutral axis moves downwards, i.e. towards the bottom part where the elastic stiffness remains.

Fig. 11 shows the stress distributions obtained by Case 2 analysis. Since the rotation of the neutral axis is constrained, the stress distribution and the spread of failures are always symmetric. This is unreasonable. In the hogging condition (a-c), the ultimate strength is attained when the upper part of the cross section is almost fully yielded and the bottom part reached the ultimate compressive strength (point b). Beyond point b, the load carrying capacity of the buckled bottom part decreases, leading to the unloading at the upper side and deck parts. This is why a rapid reduction of the load carrying capacity is observed at point b in Fig. 7. Such behavior is not observed in Case 1. When unloading is neglected, the rapid reduction does not take place as shown by the dotted line b-c' in Fig. 7.

The vertical bending moment-vertical curvature relationships of Ship B1 with the different extents of damage obtained by Case 1 analysis are shown in Fig. 12. The residual hull girder strength is significantly reduced even by the 10% damage. The reduction due to further extent of damage is less significant..

After ships suffered the damage due to collision or grounding, a heeling generally takes place, and the hull girder is externally subjected to the biaxial bending. The ultimate strength interaction relationship is therefore necessary for the residual hull girder strength assessment. There are two basic approaches to obtain the residual strength interaction curves; the proportional curvature method (Case 3) and the proportional moment method (Case 4). Both methods are applied to the B1 and T2 with 70% damage.

The residual strength interaction curve obtained by Case 4 analysis is plotted by the solid line in Fig. 13. The horizontal moment, M_H , and vertical moment, M_V , are increased with a constant ratio. On the other hand, the chain dotted lines show the loci of M_H and M_V obtained by Case 3 analysis. The horizontal curvature ϕ_H and vertical curvature ϕ_V are increased with a constant ratio. The ratio M_H/M_V varies along the path with the change of the tangential stiffness of the cross section. It is found that the solid curve obtained by Case 4 analysis gives an accurate envelope of the loci of M_H and M_V and thus both analysis methods

is found to give almost identical interaction curve when the unloading of yielded element is taken into account. Case 4 is considered to be more useful than Case 3 for the generation of the residual strength interaction curve, since the applied moment ratio can be easily controlled.

It should be noted in Fig. 13 that the residual strength of Case 1 is given by the intercept A, and that of Case 2 by the loci of the thin solid lines. Clearly, Case 2 gives the larger capacity than Case 1. This indicates the influence of constraining or allowing the rotation of the neutral axis. The dashed line is the residual strength interaction curve of the intact cross section. The reduction of the capacity is found to be larger when the bending stresses caused by the biaxial moment act in the same direction at the damaged part.

Fig. 12 Effect of damage extent on the residual hull girder strength (Ship B1, CASE 1)

Fig. 13 Residual strength interaction curve (Ship B1, 70% damage)

Fig. 14 shows the residual strength interaction relationship of

Ship T2, and Fig. 15 the axial stress distribution at the ultimate strength under pure vertical bending moment obtained by Case 12 analysis. The ultimate strength is attained when almost full breadth of intact deck part failed in compression or in tension. The rotation of neutral axis is relatively smaller than B1. This is due to the presence of the intact inner side panel.

Fig. 14 Residual strength interaction curve (Ship T2, 70% damage)

Fig. 15 Stress distribution and neutral axis (Ship T2, 70% damage, Case 1)

5. Estimate of Residual Hull Girder Strength in the Sagging Condition

Based on the observation of the progressive collapse behavior of the bulk carriers and tankers with the top-side damage, an attempt is made to estimate the residual hull girder strength in the sagging condition using the elastic cross-sectional properties and the critical member strength.

For the elastic cross section, the bending stress at the i -th element, σ_i , and the bending moment-curvature relationship are respectively expressed as

$$\sigma_i = E \{ (y_i - y_G) \phi_H + (z_i - z_G) \phi_V \} \quad (27)$$

and

$$\begin{Bmatrix} M_H \\ M_V \end{Bmatrix} = \begin{bmatrix} EI_{HH} & EI_{HV} \\ EI_{VH} & EI_{VV} \end{bmatrix} \begin{Bmatrix} \phi_H \\ \phi_V \end{Bmatrix} \quad (28)$$

where E is Young's modulus; y_G and z_G are the centroidal coordinates of Eqs. (12) and (13) with D_i of E ; and I_{HH} , I_{VV} , I_{HV} and I_{VH} are the moment of inertia of the cross section given by

$$I_{HH} = \sum_{i=1}^N (y_i - y_G)^2 A_i, \quad I_{VV} = \sum_{i=1}^N (z_i - z_G)^2 A_i$$

$$I_{HV} = I_{VH} = \sum_{i=1}^N (y_i - y_G)(z_i - z_G) A_i \quad (29)$$

Under the pure vertical bending moment of Case 1 ($M_H=0$), the horizontal and vertical curvatures are given by

$$\begin{Bmatrix} \phi_H \\ \phi_V \end{Bmatrix} = \frac{1}{E(I_{HH}I_{VV} - I_{HV}^2)} \begin{Bmatrix} -I_{HV}M_V \\ I_{HH}M_V \end{Bmatrix} \quad (30)$$

Substituting Eq. (30) to Eq. (27), the bending stress σ_i is given by

$$\sigma_i = \frac{-(y_i - y_G)I_{HV} + (z_i - z_G)I_{HH}}{I_{HH}I_{VV} - I_{HV}^2} M_V \quad (31)$$

The terms including the cross moment of inertia, I_{HV} , represent the effect of the rotation of the neutral axis.

Here, it is assumed that the residual hull girder strength in the sagging condition M_V^u is attained when a critical member at the location of (y_C, z_C) reached its ultimate strength, σ_C^u , namely

$$M_V^u = \frac{I_{HH}I_{VV} - I_{HV}^2}{-(y_C - y_G)I_{HV} + (z_C - z_G)I_{HH}} \sigma_C^u \quad (32)$$

On the other hand, when the rotation of neutral axis is fixed (Case 2), the ultimate strength corresponding to Eq. (32) is given by

$$M_V^u|_{\text{Case2}} = \frac{I_{VV}}{z_C - z_G} \sigma_C^u \quad (33)$$

The reduction rate of the residual strength due to the rotation of the neutral axis in the framework of the proposed approximate approach is given by the ratio of Eq. (32) to Eq. (33) as

$$\frac{M_V^u}{M_V^u|_{\text{Case2}}} = \frac{I_{HH}I_{VV} - I_{HV}^2}{-(y_C - y_G)I_{HV} + (z_C - z_G)I_{HH}} \frac{z_C - z_G}{I_{VV}} \quad (34)$$

when the location of the critical member is assumed to be the same in Case 1 and Case 2.

Fig. 16 shows a comparison of the residual strength obtained by Eq. (32) and the result of the progressive collapse analysis. Two locations of the critical deck elements are considered as shown in Figs. 5 and 6.

B1, B2: L_1 at the center line, and L_2 at the hatch coaming on the damaged side

T2: L_1 at the center line, and L_2 at the distance of $B/4$ from the damaged side shell

For the location L_1 of bulk carriers, the ultimate strength of the critical element is evaluated by using that of the element at L_2 .

The residual strengths of B1, B2 and T2 obtained for the four different damage extents are summarized in Fig. 16. It is found

that Eq. (32) gives an estimate of the residual strength which is in good agreement with the result of the progressive collapse analysis. For bulk carriers, the critical element at the location L_2 gives a better estimate of the residual strength. This is consistent with the observed collapse behavior in which the ultimate strength is attained when the topside tank region of the damaged side almost fully failed. The location L_1 cannot well take account of the effect of the horizontal curvature induced by M_V , resulting in a slight overestimate of the strength. In the case of tankers, the location L_1 gives a better estimate of the residual strength than L_2 . This is also consistent with the failure behavior of T2 in which the ultimate strength is attained when the deck part almost fully failed.

Fig.17 compares the reduction rates of the residual strength due to the rotation of the neutral axis, obtained by Eq. (34) and the progressive collapse analysis. The influence of the rotation of the neutral axis is larger for a larger damage extent in general. For the case of subject ships and damages under consideration, the influence is larger in bulk carriers than in tankers. Eq. (34) gives a relatively good estimate of the reduction rate. It can be a good basis of a rational expression of the influence of the rotation of the neutral axis on the reserved hull girder strength, as required in ship structural rule¹³⁾. More systematic analyses are definitely needed to develop the formula having larger applicability in ship types and damaged cases.

Fig. 16 Comparison of residual hull girder strength M_V between the simplified method and the progress collapse analysis

Fig. 17 Comparison of reduction ration of residual hull girder strength due to the rotation of neutral axis between the simplified method and the progress collapse analysis (Case 1/Case 2)

6. Conclusions

Residual hull girder strength of asymmetrically damaged ships has been analyzed by the Smith's method using the newly presented incremental procedures for the biaxial bending collapse analysis. The influence of the rotation of the neutral axis due to asymmetric damages on the residual hull girder strength has been discussed. A simple method to estimate the residual hull girder strength of damaged ships under the sagging condition has been proposed. The following conclusions can be drawn:

- 1) The rotation of the neutral axis has a significant influence on the residual hull girder strength of asymmetrically damaged ships.
- 2) For the subject ships with the specified top-side damages, the effect of the rotation of the neutral axis on the residual hull girder strength is about 8% at maximum and smaller for smaller damage extent. The reduction rate depends on the damage extent and location.
- 3) The residual hull girder strength interaction relationship obtained by the proportional moment loading (Case 4) is in good agreement with that obtained by the proportional curvature loading (Case 3).
- 4) The effect of the damage on the residual hull girder strength is larger when the bending stresses caused by the biaxial moment act in the same direction at the damaged part.
- 5) The residual hull girder strength of asymmetrically damaged ships under the sagging bending moment can be predicted using the elastic cross-sectional properties and the critical member strength with a reasonable accuracy.

In the present study, the residual strength of a damaged cross section has been studied. For the case of actual damages of the finite length in the longitudinal direction, the rotation of the neutral axis at the damaged part is constrained by the adjacent intact parts, and thus its effect is considered to be smaller than that observed in the present study. The consideration of the effect of the damage length remains as a future work.

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Appendix: Rotation of Instantaneous Neutral Axes

The rotation angle, θ , of the instantaneous neutral axes about the centroid G in Fig. 3 can be obtained as follows:

Using the coordinates η and ζ , the axial strain increment at the i -th element is expressed by

$$\Delta \varepsilon_i(\eta_i, \zeta_i) = \zeta_i \Delta \phi_\eta + \eta_i \Delta \phi_\zeta \quad (\text{A1})$$

where $\Delta \phi_\eta$ and $\Delta \phi_\zeta$ are the curvature increments in the η and ζ directions, respectively. The bending moment increments about η and ζ axes are

$$\begin{aligned} \Delta M_\eta &= \sum_{i=1}^N \zeta_i \Delta \sigma_i A_i = \left(\sum_{i=1}^N D_i \zeta_i^2 A_i \right) \Delta \phi_\eta + \left(\sum_{i=1}^N D_i \zeta_i \eta_i A_i \right) \Delta \phi_\zeta \\ \Delta M_\zeta &= \sum_{i=1}^N \eta_i \Delta \sigma_i A_i = \left(\sum_{i=1}^N D_i \zeta_i \eta_i A_i \right) \Delta \phi_\eta + \left(\sum_{i=1}^N D_i \eta_i^2 A_i \right) \Delta \phi_\zeta \end{aligned} \quad (\text{A2})$$

When the η and ζ axes are the principal axes of the cross section having the tangential stiffness, D_i the coupled term in Eq. (17) vanishes.

$$\sum_{i=1}^N D_i \zeta_i \eta_i A_i = 0 \quad (\text{A3})$$

Substituting the following relationship of the coordinate transformation

$$\begin{aligned} \eta &= (y - y_G) \cos \theta + (z - z_G) \sin \theta \\ \zeta &= (z - z_G) \cos \theta - (y - y_G) \sin \theta \end{aligned} \quad (\text{A4})$$

Into Eq. (A3), the angle of rotation, θ , is obtained as

$$\theta = \frac{1}{2} \tan^{-1} \frac{2D_v}{D_{HH} - D_{VV}} \quad (\text{A5})$$

The bending moment-curvature relationship with respect to the η and ζ axes is given by Eqs. (18) and (19).