A Study on Random Field Model for Representation of Geometry of Corroded Plates and Estimation of Stochastic Properties of Their Strength

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Summary

In this study, as an alternative to uniform corrosion model, the surface geometry of the corroded plate is represented by the random field model (Karhunen-Loeve Expansion Method). Capability of the random field model to represent the random properties of corroded surfaces is studied. The random characteristics of the real corroded surface are investigated based on the measurement data of the real corroded specimen. The random characteristics of minimum cross sectional area of the generated plates with random field corrosion are estimated by PCE (Polynomial Chaos Expansion) method as well as Montecarlo simulation to evaluate the availability of PCE to estimate stochastic properties of strength of corroded plate. By using PCE, the numbers of samples can be reduced significantly to get the accurate results and this effect is favorable to time consuming analysis. Finally, the ultimate tensile strength of the plates with random field of corrosion is obtained by non-linear three dimensional finite element analyses (FEA). Based on FEA results, random properties of the ultimate tensile strength of the plates are obtained using PCE.

Key Words: Random Field of Corrosion, Strength of Corroded Plates, Polynomial Chaos Expansion Method

1. Introduction

Corrosion is a process of uncertain nature, and the estimation of the strength of the corroded plate is very important for safety assessment of ship structures. Because of uncertain nature governed by many variables, only probabilistic models can describe the corrosion process itself and its effect on the strength of structural components.

Many publications have proposed the probabilistic model of corrosion wastage (Guo et al ¹⁾, Paik et al ²⁾ and Yamamoto & Ikegami ³⁾). Past researches investigated the time varying ultimate strength of ship structures considering uniform corrosion (Guo et al ¹⁾ and Paik et al ⁴⁾). Hart et al ⁵⁾ considered the probabilistic modeling of the ultimate strength by using the uniform corrosion wastage model represented by exponential distribution. However in the real field, the corroded surfaces do not have uniform thickness but having fluctuated geometry. This may affect to the ultimate strength estimation. For this reason, it is important to consider the geometry of the corroded surface to estimate its effect on the strength reduction.

Some studies have been carried out in the past to predict the ultimate strength of the corroded plate considering the geometry of the surface with pitting corrosion (Paik et al ⁶⁾, Nakai et al ⁷⁾ and Rabuil & Sumi ⁸⁾). Recently, Teixeira & Guedes Soares ⁹⁾¹⁰⁾ modeled the corroded surface by a random field model where the spatial variability of corrosion wastage is represented. And they proposed the probabilistic model for the ultimate strength of corroded plate represented by random field derived by Montecarlo simulation where large numbers of analysis are necessary.

The main objective of this study is to investigate the stochastic properties of ultimate strength of the corroded plate considering the real fluctuated geometry with practical

computation cost. To fulfill this objective, the corroded plate is represented by random field model (Karhunen Loeve Expansion method ¹¹⁾) by which model, the hypothetical corroded surfaces are generated. Then the ultimate tensile strength of the generated plates is estimated by non-linear three-dimensional finite element analysis (FEA). Based on the FEA results, the probabilistic modeling of the ultimate tensile strength of the plate represented by random field model is estimated by using PCE (Polynomial Chaos Expansion ¹²⁾) method.

2. Random Field Representation of Corroded Plates

In this chapter, the hypothetical corroded surfaces having random properties of corrosion wastage of 27 years old ship are generated using random field model (K-L expansion method) and the random characteristics of those surfaces are investigated.

2.1 Probabilistic Characteristics of Corrosion Wastage

Here, the probabilistic characteristics of corrosion wastage for the targeted ship year which will be used in generation of the hypothetical corroded surfaces are firstly obtained based on Paik et al ²⁾. From the statistical analysis of measurement data, Paik et al ²⁾ obtained the probabilistic characteristics of corrosion rate for plate elements in different location of single hull tankers. By using these data for bottom plate, the probabilistic characteristic of thickness reduction for 27 years old is estimated as Table 1.

Table 1 Probabilistic characteristics of corrosion wastage for 27 years old tanker.

Corrosion State	Mean, μ_{org}	Variance, var_{org}
Severe	3.465 mm	0.564
Average	1.184 mm	0.953

Note: Average corrosion represents all of the measurement data and severe corrosion represents the data of 95% above band ²⁾.

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By using average corrosion wastage model, the corroded surfaces which can represent the various corrosion conditions (severe, average and slight) can be generated by random field model while using severe corrosion wastage model, we can generate the corroded surfaces which represent severe corrosion condition.

2.2 Random Field Representation of Corroded Surface

For random field representation with K-L expansion method¹¹⁾, it is assumed that the probabilistic nature of the corroded surface is to follow normal distribution having a mean and variance (μ_{org} and var_{org}) which are obtained in previous section. Also it is assumed that covariance function, $C(\mathbf{x}_1, \mathbf{x}_2)$ is known for the corroded surface which represents the correlation property between two points, $\mathbf{x}_1$ and $\mathbf{x}_2$ of the discretized random field. Then the random field of that corroded surface can be represented by a set of uncorrelated random variables, ξ_i using K-L expansion method as in Eqn. (1) and Fig. 1.

$$\mathbf{v} = \boldsymbol{\mu} + \sum_{i=1}^N \xi_i \sqrt{\lambda_i} \boldsymbol{\phi}_i \quad (1)$$

Fig. 1 Schematic diagram of random field of corroded plate.

where $\mathbf{v} (v_1, v_2, \dots, v_N)^T$ is a vector of random variables having size N (numbers of discretized points) which represents thickness diminution at each point, x_i as shown in Fig. 1. $\boldsymbol{\mu}$ is the mean vector of the original random field which represents average thickness diminution, and it is assumed in this study that all of its components have same constant value, μ_{org} . The second term of the right hand side of Eqn. (1) represents the random fluctuation of the surface around the mean value. λ_i and $\boldsymbol{\phi}_i$ are eigenvalues and normalized eigenvectors of the covariance matrix $\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2)$. ξ_i is a set of N uncorrelated random variables having zero mean and unit variance.

The correlation of these random variables can be specified by the covariance matrix $\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2)$ in which its components can be obtained from the assumed covariance function, $C(\mathbf{x}_1, \mathbf{x}_2)$. It is noted that though Fig. 1 is drawn as one dimensional (x) for explanation purpose, we discretized two-dimensional (x - y) surface into square grid. Choice of covariance function is a practical consideration and the accuracy of the random characteristics of generated surface depends on the form of covariance function. In this study, based on the measurement of real corroded plate shown in chapter 3, the covariance function of random field is assumed to be a function of the distance between two points and to follow the exponential form as follows;

$$C(\mathbf{x}_1, \mathbf{x}_2) = \sigma^2 \exp \left[-\frac{|d|}{l_c} \right] \quad (2)$$

where σ is the standard deviation of the random field and σ^2 is var_{org} . d is the distance between $\mathbf{x}_1$ and $\mathbf{x}_2$. l_c is a parameter commonly known as the correlation length. If l_c is small, only closely adjacent points are correlated, and for large l_c , correlation exists to a large distance. It is noted that l_c should be appropriate one in order for the generated random field to be the representative of the realistic situations. In the present paper, the surfaces are generated by using different l_c and the random properties of those surfaces are investigated. The following procedure is used for generation of surfaces which can represent the random properties of corroded surface.

- (1) Firstly the targeted plate is discretized into N points using 1 mm mesh.
- (2) Secondly, the components of covariance matrix, $\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2)$ are obtained from the assumed correlation function as Eqn. (2) for the discretized plate.
- (3) Thirdly eigenvalues (λ_i) and eigenvectors ($\boldsymbol{\phi}_i$) of the covariance matrix are computed.
- (4) Finally, uncorrelated random variables (ξ_i) are numerically generated and multiplied to the eigenvalues and eigenvectors as in Eqn. (1) to generate random surface ($\mathbf{v}$).

According to Eqn. (1), when all of the eigenvalues and eigenvectors (all random variables (N)) are used, the generated random field can accurately represent to the original random properties.

However when the random field model is applied to the simulation to estimate the strength, the computational effort is proportional to the numbers of random variables to be used. Therefore, it is desirable to use small numbers of random variables to represent the random field of corroded surface which corresponds to fewer terms in K-L expansion equation. So, in this study, the surfaces are also generated by the reduced numbers of uncorrelated random variables (M) out of N , which is corresponding to the M largest eigenvalues, and the accuracy of those surfaces are investigated.

According to the procedure explained above, a series of hypothetical corroded surfaces ($N_s=500$) having dimension of 105x30 mm are generated for each condition of l_c and M . Total numbers of discretized points of the plate, N is 3286. The random properties of corrosion wastage are assumed to be average corrosion condition as shown in Table .1.

In the following sections, the effect of correlation and the effect of reduced numbers of random variables for the random properties of generated surfaces are investigated. For that purposes the following parameters are changed in surface generations.

- (1) Six l_c , 200, 105, 60, 30, 9.6 and 5mm are used to study the effect of l_c on the random properties of the generated surfaces.
- (2) For each l_c , $M=1, 2, 3, 4, 5, 10, 20, 30$ and 3286 (N) numbers of random variables are used to study the effect of reduced numbers of random variables on the accuracy of the generated surfaces.

Some of the generated surfaces which represent the reduction of plate thickness due to corrosion process represented by the random field are shown in Fig. 2. From this figure we can see that high fluctuation occurs when all random variables are used which is similar to the surface of real corroded plate.

Fig. 2 Representation of thickness reduction due to corrosion for $l_c = 200\text{mm}$ by using different M = (a) 3286 (b) 1000 (c) 30 and (d) 10.

2.3 Effect of Plate Size and Correlation Length

As defined in Eqn. (2), the covariance function of the random field is in terms of distance between random variables and so that the plate size might effect on the random properties of the corroded surfaces. In this section, to investigate the effect of plate size, a , which is taken as the shorter length of the plate and correlation length, l_c (correlation parameter of the random variables of the random field) on the random properties of generated surfaces, the statistical characteristic of the generated surfaces for different a/l_c are investigated based on the parameters defined in Eqns. (3)-(6). Firstly statistical characteristics (mean and variance) of each generated surface are calculated by using Eqns. (3) and (4).

$$\bar{v}_j = \frac{\sum_{i=1}^N v_{ij}}{N}, \quad (j=1,2,\dots,N_s) \quad (3)$$

$$\text{var}_j = E[(v_{ij} - \bar{v}_j)^2], \quad (j=1,2,\dots,N_s) \quad (4)$$

where, v_{ij} is the corrosion diminution of the generates surface at i -th discretized point of j -th surface. N_s is the number of generated surfaces.

Then the mean of these two terms for all surfaces in each case are defined as $\text{mean}(\mu_{\text{surface}})$ and $\text{mean}(\text{var}_{\text{surface}})$ and calculated using Eqns. (5) and (6).

$$\text{mean}(\mu_{\text{surface}}) = \frac{\sum_{j=1}^{N_s} \bar{v}_j}{N_s} \quad (5)$$

$$\text{mean}(\text{var}_{\text{surface}}) = \frac{\sum_{j=1}^{N_s} \text{var}_j}{N_s} \quad (6)$$

These parameters represents average stochastic property of one generated surface, and it can be considered that the effect of a/l_c on the random characteristic of the generated surfaces can be evaluated by these computed parameters. The calculated $\text{mean}(\mu_{\text{surface}})$ is nearly the same with μ_{org} for all a/l_c as shown in Fig. 3. However, we can see in Fig. 3 that the variance of generated surfaces is smaller than the var_{org} for small a/l_c even though all of the random variables are used. With increasing a/l_c , the variances become close to the var_{org} . From that point we can

say that when l_c is large compared to the plate size (a), the generated small surface cannot represent to the original random properties and it will have smaller variance.

Fig. 3 Size effect on the generated surface for different a/l_c (all random variables ($M=3286$) are used).

2.4 Effect of Reduced Numbers of Random Variables Used in Random Field Representation

In this section, the effect of reduced numbers of random variables (M) on the accuracy of the generated surfaces is studied. Firstly, the mean and variance of each discretized point, i ($i=1, 2, \dots, N$) for all generated surfaces in each case are estimated using Eqns. (7) and (8).

$$\bar{v}_i = \frac{\sum_{j=1}^{N_s} v_{ij}}{N_s}, \quad (i=1,2,\dots,N) \quad (7)$$

$$\text{var}_i = E[(v_{ij} - \bar{v}_i)^2], \quad (i=1,2,\dots,N) \quad (8)$$

And then the mean of total discretized points are defined as $\text{mean}(\mu_{\text{grid}})$ and $\text{mean}(\text{var}_{\text{grid}})$ and they are calculated using Eqns. (9) and (10). These parameters represent the average stochastic property of each grid point and different from Eqns. (5) and (6). These indexes should be equal to μ_{org} and var_{org} regardless of the ratio a/l_c when all of the random variables are used.

$$\text{mean}(\mu_{\text{grid}}) = \frac{\sum_{i=1}^N \bar{v}_i}{N} \quad (9)$$

$$\text{mean}(\text{var}_{\text{grid}}) = \frac{\sum_{i=1}^N \text{var}_i}{N} \quad (10)$$

Fig. 4 Accuracy vs M for different a/l_c .

Here also, it is found that the computed mean of each grid point ($\text{mean}(\mu_{\text{grid}})$) is nearly the same as μ_{org} for all conditions. So

that the accuracy of generated surfaces are discussed by comparing the $mean(var_{grid})$ with var_{org} using the normalized value as shown in Fig. 4. The calculated variance of the generated surfaces using all random variables become the same as the var_{org} for all a/l_c (though it is not shown in figure for all random variables). It is possible to say that the proposed random field model can represent the original random properties when all random variables are used.

However the accuracy of the generated surfaces becomes lower as decreasing M . For the same M , the accuracy decreases as increasing a/l_c . And also we can see in Fig. 4 that for small a/l_c in which l_c is large compared to the plate size, the accuracy of the generated surfaces by using all random variables and reduced random variables are not so different. For example if $a/l_c = 0.2$, the normalized variance is close to 1.0 if the number of random variables (M) is greater than 5. From this fact, we can conclude again that for the plate having large l_c compared with the plate size (a), we can use small numbers of random variables to represent the original random properties.

3. Measurement of Real Corroded Surfaces

In this section, random characteristics of corroded surface are investigated from real corroded specimen, which is part of a ship NAKHODKA (Yao et al.¹³) as shown in Fig. 5. The specimen has general corrosion and the size of corroded area is 105 mm x 250 mm. Each side of the specimen are named Surface A and Surface B. Corroded surfaces are measured by using laser sensor, LJ-G080 (Keyence) and desktop NC milling machine, SG01, which is used only to move the specimen. The measurement was divided into 9 areas as shown in Fig. 6. The measurement intervals are set as 0.14625 mm both on X and Y directions. Random characteristics of different plate sizes (small (105mm x 30mm) and large (105mm x 250mm)) are calculated from the measurement data. Small one represents the data from one of the 9 small surfaces, and large one includes whole corroded area made by combining of 9 small surfaces.

It should be noted that the original thickness of this specimen is unknown. So the height in our measurement data does not mean the thickness reduction but the geometry of the corroded surface because the height will change depending on the position of laser sensor.

Fig. 5 Picture of specimen (Surface B).

Fig. 6 Specimen size and measurement image.

3.1 Probabilistic Characteristic of Corroded Surface

In general, corrosion condition changes (from pitting to general corrosion) as the ship ages. As progress of corrosion, the shape of probability density function of the corrosion diminution varies from exponential to normal distribution (Yamamoto¹⁴).

As shown in Fig. 7, we obtained the probability density function of the height for two surfaces, A and B of the corroded specimen. It can be seen that the probability density function of Surface B well agrees to have normal distribution. Therefore, we will mainly discuss about the random characteristics of the surface B which has normal distribution. The standard deviation of large surface and mean of that of 9 small surfaces on Surface B are shown in Table 2. We can see the standard deviation of small surface is smaller than that of large surface.

(a) Surface A

(b) Surface B

Fig. 7 Probability density functions of large surface.

Table 2 Comparison of standard deviation between large and small surface (surface B).

Large surface	Mean of small surfaces
0.4299 mm	0.3643 mm

3.2 Autocovariance Function of Corroded Surface

Autocovariance function for the specimen is calculated in this section. It is assumed that the height of two points at same distance (d) has same correlation and autocovariance function can be defined as shown in Eqn. (11);

$$\rho(d) = \frac{E[(v(\mathbf{x}_1) - \mu)(v(\mathbf{x}_2) - \mu)]}{\sigma^2} \quad (11)$$

where, $v(\mathbf{x}_i)$ is the height at $\mathbf{x}_i$, d is the distance between $\mathbf{x}_1$ and $\mathbf{x}_2$, μ and σ are the mean and standard deviation of the height of a surface.

The computed autocovariance function for large surface and mean of 9 small surfaces are shown in Fig. 8 as dotted line. Then the correlation length (l_c) is estimated by least square method so that Eqn. (2) fit to the obtained autocovariance function. In Fig. 8, the two solid lines are the exponential autocovariance functions (Eqn. (2)) using estimated l_c for large surface and small surfaces. We can see l_c calculated from large surface is larger than that calculated from small surfaces.

From this result, we can again conclude that there is possibility that stochastic properties calculated from such small specimen to be smaller than that of real corroded panel on ship.

Fig. 8 Calculated autocovariance function and estimated l_c .

4. Estimation of Real Correlation Length

In this part of the study, the correlation length, l_c of the real large plate in actual field is estimated based on the calculated l_c of real corroded specimen in the previous chapter. In order to estimate the real l_c of the corroded plate in actual field, the random corroded surfaces having the size 105 mm x 30 mm generated in section 2.2 for different l_c with all (3286) numbers of random variables are used. It is assumed that the targeted plate has the random characteristics of corrosion diminution as shown in Table 1 for severe corrosion condition.

4.1 Real Correlation Length Estimation from Viewpoint of Correlation Length

For the generated surfaces for each assumed l_c , the correlation length of each generated surface is calculated by using the same method as section 3.2. The calculated l_c for 500 surfaces has distribution as shown in Fig. 9 as an example for assumed $l_c(9.6$ mm). The calculated correlation lengths at 5%, mean and 95% of the distribution is plotted in Fig. 10 for each assumed l_c . The l_c of the real corroded specimen calculated in section 3.2($l_c=9.616$ mm) is plotted also in the same figure. With assumption that the l_c of real corroded specimen to have within these three conditions (5%, mean and 95% of distribution), the range of real l_c is estimated as shown in Fig. 10 and Table 3. The same procedure is used for calculation of real l_c for average corrosion condition as shown in Table 3.

It can be concluded that from viewpoint of correlation length, the range of l_c is 8-17-450 mm and 7-15-2000 mm for severe and average corrosion condition.

Fig. 9 Probability density function of l_c of 500 generated surfaces for assumed $l_c=9.6$ mm (severe case).

Fig.10 Estimation of real correlation length (l_c) from correlation length of generated surfaces (severe case).

Table 3 Range of real correlation length, l_c (mm).

Severe Corrosion	Average Corrosion
7.22-17-439.4*	7-15-2000*

*Extrapolated value of correlation length.

5. Estimation of the Stochastic Properties of Minimum Cross Sectional Area

The purpose of this study is to estimate the stochastic properties of strength of the corroded plate by using PCE method. Before estimation of stochastic properties of strength, in order to evaluate the availability of PCE method, the stochastic properties of the minimum cross sectional area which can be regarded as reference index of strength is estimated by PCE in this chapter.

5.1 Estimation of Stochastic Properties of Minimum Cross Sectional Area of Corroded Plate by PCE Method

5.1.1 Polynomial chaos expansion (PCE) method

To estimate the stochastic properties of response (strength of plates) with input random variables such as material properties, geometric properties (random field of corrosion in this study) as shown in Fig. 11, Montecarlo simulation is one approach where large numbers of numerical simulations are necessary.

Response surface method is another approach, in which the unknown response function is approximated as a polynomial function of input random variables by using a smaller number of simulations. Polynomial coefficients are determined by least square method to fit the actual responses (simulation results) to the approximated response function. By using the obtained simple equation of the response, it is possible to estimate the stochastic properties of the response by Montecarlo simulation.

Fig. 11 Estimation of stochastic properties of response.

Polynomial Chaos Expansion (PCE) is a special kind of response surface for random variables by which the stochastic properties of the responses can be easily obtained by using the orthogonal properties of the polynomial. The elementary concepts of PCE method used in this study are based on Seung-kyum et al⁽¹²⁾. A simple definition of the response surface, $u(\xi)$ by using PCE for a set of Gaussian input random variables is the sum of the product of the coefficients, a_i and polynomial, Ψ_i as follows;

$$u(\xi) = \sum_{i=0}^p a_i \Psi_i(\xi) \quad (12)$$

where $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ is a set of Gaussian random variables. $\Psi_i(\xi)$ is a set of multidimensional Hermite polynomials of order p . For example, Hermite polynomials for 2nd order PCE with two random variables, $\xi = (\xi_1, \xi_2)^T$ are;

$$\Psi_0 = 1, \quad \Psi_1 = \xi_1, \quad \Psi_2 = \xi_2,$$

$$\Psi_3 = \xi_1 \xi_2, \quad \Psi_4 = \xi_1^2 - 1, \quad \Psi_5 = \xi_2^2 - 1.$$

And, $a_i(a_0, a_1, a_2, a_3, \dots, a_p)$ are constants to be determined. In any case, for an m -th order of PCE with n random variables, the total number of PCE coefficients is given by Eqn. (13).

$$p+1 = \frac{(m+n)!}{m!n!} \quad (13)$$

Assume that n_s times of simulations are carried out and u_I is computed response for the n numbers of input random variables $\xi_I = (\xi_{1I}, \xi_{2I}, \dots, \xi_{nI})^T$, ($I=1, 2, \dots, n_s$) at I^{th} simulation. Then by using Eqn. (12), the matrix form to determine the coefficients is obtained as follows;

$$\hat{\mathbf{Y}} = \mathbf{X} \hat{\mathbf{a}} \quad (14)$$

where $\hat{\mathbf{Y}}$ is a $n_s \times 1$ vector of the responses, $\mathbf{X}$ is a $n_s \times p$ matrix of the Hermite polynomials and $\hat{\mathbf{a}}$ is a vector of undetermined coefficients having a dimension of $p+1$ as follow;

$$\hat{\mathbf{Y}} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n_s} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} \Psi_0(\xi_1) & \Psi_1(\xi_1) & \dots & \Psi_p(\xi_1) \\ \Psi_0(\xi_2) & \Psi_1(\xi_2) & \dots & \Psi_p(\xi_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_0(\xi_{n_s}) & \dots & \dots & \Psi_p(\xi_{n_s}) \end{bmatrix}, \hat{\mathbf{a}} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} \quad (15)$$

Generally, the least square method is used to obtain the coefficients vector $\hat{\mathbf{a}}$ from known responses as Eqn. (16).

$$\hat{\mathbf{a}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \hat{\mathbf{Y}} \quad (16)$$

After estimating the unknown coefficients of the response surface (Eqn. (12)), by using the orthogonal properties of the Hermite polynomials as Eqn. (17), the stochastic properties of the response (mean (μ_u), variance (σ_u^2) and correlation coefficients of each random variable to the response ($C[u, \xi_i]$) can be calculated simply using Eqns. (18)-(20);

$$\Psi_0 = 1, \quad E[\Psi_i] = 0, \quad E[\Psi_i \Psi_j] = E[\Psi_i^2] \delta_{ij} \quad (17)$$

$$\mu_u = E[u] = a_0 \quad (18)$$

$$\sigma_u^2 = \sum_{i=1}^{\infty} a_i^2 E[\Psi_i^2] \quad (19)$$

$$C[u, \xi_i] = E[u - \mu_u, \xi_i] = \frac{a_i}{\sigma_u} \quad (20)$$

where $E[\cdot]$ indicates the expected value operation and δ_{ij} is the Kronecker delta. Seung-kyum et al.¹²⁾ reported that minimum required numbers of samples (n_s) to determine the coefficients by least square method, is two times of PCE coefficients ($n_s=2(p+1)$) if LHS (Latin Hypercubic Sampling) method is used in simulation of input random variables.

5.1.2 Stochastic properties of minimum cross sectional area

When considering tensile strength, it seems to be reasonable to assume that ultimate strength is proportional to the minimum cross sectional area. Based on this consideration, it is assumed that the ratio of minimum cross sectional area can be regarded as a reference index of the ultimate strength reduction of the plate with random field of corrosion as Eqn. (21);

$$R_{area} = \left(\frac{A_{min}}{A_o} \right) \quad (21)$$

where A_o is the original cross sectional area of the intact plate and A_{min} is minimum cross sectional area of the corroded plate. And the stochastic properties of minimum cross sectional area of the corroded plate are estimated by PCE using the following procedure.

(1) Firstly, corroded surfaces for the targeted plates having the parameters shown in Table 4 are generated by the proposed random field model as explained in chapter 2. It is noted that, based on the estimated real correlation length in section 4.1, l_c , is taken as 30 mm for random field representation. 10 numbers of random variables ($M=10$) is used in surfaces generation based on Fig. 4. The required numbers of PCE coefficients ($p+1$) are determined using Eqn. (13) as 66 and 286 numbers for 2nd and 3rd order PCE respectively. The numbers of samples (generated surfaces), n_s based on LHS are 132 and 572 for 2nd and 3rd order of PCE respectively.

(2) Secondly, minimum cross sectional area of the n_s number of generated plates are calculated to determine the coefficients, $\hat{\mathbf{a}}$ by using Eqn. (16). Then random characteristics of minimum cross sectional area of the generated corroded plates are calculated by Eqns. (18) and (19).

Table 4 Values of parameters for targeted corroded plate.

Parameters		Values
Plate size	Plate dimension	(40 x 100 x t_o) mm
	Original thickness, t_o	10mm
	Numbers of discretized points, N	4141
Random field properties	Correlation length, l_c	30 mm
	a/l_c	1.33
	Mean, μ_{org}	3.465 mm
	Variance, var_{org}	0.564 mm
	Numbers of random variables, M	10

As a comparison with PCE method, Montecarlo simulation is carried out. For Montecarlo simulation, 10^4 surfaces are generated by random field model and the stochastic properties of the minimum cross sectional area of those surfaces are investigated.

The stochastic properties R_{area} of the surfaces estimated by PCE method and Montecarlo simulation are compared in Table 5 and we can see that the stochastic properties of R_{area} estimated by using 2nd and 3rd order PCE are same as to that estimated by Montecarlo simulation. We can conclude that by using PCE method, the stochastic properties can be obtained with a few numbers of samples which will be favorable in computing the response by expansive analysis. Moreover to reduce the calculation cost, 2nd order PCE is enough for this case.

And also we can estimate the correlation of each random variable ($\xi_1, \xi_2, \dots, \xi_{10}$) to that of response (R_{area}) based on PCE method by using Eqn. (20) as shown in Fig. 12. We can see that the first random variable has the largest correlation to R_{area} and 3rd and 8th random variables have larger correlation than the other random variables.

Table 5 Stochastic properties of R_{area} estimated by PCE method and Montecarlo simulation.

	2 nd Order PCE (132 samples)	3 rd Order PCE (572 samples)	Montecarlo simulation (10 ⁴ samples)
Mean	0.597	0.595	0.596
Standard deviation	0.052	0.052	0.049

Fig.12 Correlation of each random variable to R_{area} .

5.2 Effects of Numbers of Random Variables Used

In estimation of the response or strength by time-consuming analysis such as non-linear FEM, it is desirable to use smaller numbers of random variables. However as we have observed in section 2.4, by using reduced numbers of random variables (M) in random field representation, the accuracy of random properties becomes lower.

Therefore the effects of M on the accuracy of stochastic properties of the maximum cross sectional area of thickness reduction, $A_{max(M)}$ are investigated in this section by using Montecarlo simulation. Table 6 shows the stochastic properties of maximum cross sectional area of thickness reduction of the generated plate having the random properties as shown in Table 4 by using different numbers of reduced random variables ($M=10, 20, 40$) and all random variables. As we expected that by using reduced number of random variables (M), the mean values of the maximum cross sectional area of thickness reduction are smaller than that of using all random variables, while the standard deviation is nearly same.

By using the data of mean value of Table 6, the formula to estimate the exact accuracy with reduced random variables (M) can be proposed by the fitted curve of Fig. 13. In this figure, horizontal axis is the accuracy of standard deviation of random field with M random variables (square root value of accuracy of variance shown in Fig. 4). Vertical axis is the normalized value of the mean of maximum cross sectional area of thickness reduction using M random variables to that of using all random variables, $A_{max(all)}$. From the relation of $A_{max(all)}$ and $A_{min(all)}$ (the minimum cross sectional area using all random variables) as shown in Fig. 14 (a), we can estimate $A_{min(all)}/A_0$ (which is the reference index of ultimate tensile strength) by using the procedure shown in Fig. 14(b).

(i) From the generated surfaces using M random variables, $A_{max(M)}$ is obtained.

(ii) And by using the fitted curve of Fig. 13, $A_{max(M)}$ is corrected to $A_{max(all)}$.

(iii) Then considering the original thickness of the plate, $A_{min(all)}$, which is the remained minimum cross sectional area of the corroded plate can be computed by subtracting $A_{max(all)}$ from the original cross sectional area, A_0 .

(iv) Then R_{area} using all random variables, $(A_{min(all)}/A_0)$ is obtained.

As an example, mean of the strength reduction using M ($=10$) is corrected to that using all random variables as shown in Table 7. The corrected value is almost same as the mean of R_{area} obtained by using all random variables. In this table, we can see that if original thickness is very small like 10 mm, the strength reduction is very severe. We can also confirm that the mean of R_{area} is small compared with considering as uniform thickness reduction as shown in the last column of Table 7.

It can also be considered that the strength reduction estimated by PCE using all random variables can also be estimated from that of using M random variables by using such kind of process.

Table 6 Stochastic properties of maximum cross-sectional area of thickness reduction.

Number of random variables(M)	Mean(mm ²)	Standard deviation(mm ²)
10	$A_{max(10)}=161.55$	19.52
20	$A_{max(20)}=164.16$	19.82
40	$A_{max(40)}=165.36$	19.46
4141(all random)	$A_{max(all)}=167.60$	19.63

Fig.13 Estimation of maximum cross-sectional area of thickness reduction using all random variables (when $a/l_c=1.33$). Fig.14 (a) Relation of A_0 , $A_{max(all)}$ and $A_{min(all)}$ (b) Procedure to estimate R_{area} using all random variables.Table 7 Stochastic properties of R_{area} .

Original thickness (mm)	Mean of R_{area} ($M=10$)	Mean of $*R_{area}$	Standard Deviation of R_{area}	**Mean (uniform corrosion)
10	0.596	0.581	0.049	0.653
15	0.731	0.721	0.033	0.769
20	0.798	0.791	0.024	0.827

*Corrected value to that of using all random variables.

**Mean value considering uniform thickness reduction.

6. Estimation of the Stochastic Properties of the Strength of Corroded Plates

In this chapter, the ultimate tensile strength of the generated plates with random field of corrosion are obtained by non-linear finite element analysis by Marc/Mentat¹⁵. Based on the FEA

results, the stochastic properties of the ultimate tensile strength of corroded plates are estimated by using PCE method.

6.1 Ultimate Tensile Strength of Plates with Random Field of Corrosion

Firstly to verify the result of non-linear finite element analysis, the basic problem shown in Mobesher et al¹⁶⁾ was analyzed in this study with the uniaxial extension of a rectangular plate without corrosion. The properties and true stress-strain curve of elasto-plastic material used in the analysis is as shown in Fig. 15 and Table 8¹⁶⁾. We proved that the ultimate strength calculated by Marc/Mentat well agree to that of Mobesher et al¹⁶⁾.

Then a series of non-linear finite element analysis are carried out for the 42, 72 and 132 numbers of corroded surfaces generated using three different numbers of random variables (M), 5, 7 and 10. The size and random properties generated plates are as shown in Table 4. The numbers of surfaces (analyses), n_s are decided by the minimum requirement numbers of samples, $2(p+1)$ (See section 5.1.1) for each number of random variables used for 2nd order PCE.

For FEM model of the plates with random field corrosion, one-sided corrosion is assumed with a linear variation of the thickness between the nodes as shown in Fig. 16 (b). The size of the mesh is $1 \times 1 \times t_i/3$ (mm) in x, y and z direction, where t_i is the remaining thickness at a certain location. The material model and imposed boundary conditions used are as shown in Fig. 15 and Fig. 16 (a).

Fig. 15 True stress-strain relation of material.

Table 8 Material properties used in non-linear FEM.

Yield Strength	Tensile Strength	Young's Modulus	Poisson's ratio
440 MPa	523MPa	204.5 GPa	0.3

Fig.16 Schematic diagram of FEM model (a) boundary conditions imposed (b) assumed one-sided corrosion with original thickness, 10mm.

The ultimate strength of the corroded plate is represented by the strength reduction, R as follow:

$$R = \frac{F}{F_0} \quad (22)$$

where F and F_0 are the ultimate tensile strength of the corroded and intact plate defined as the maximum value of the load-deflection curve obtained from non-linear FEM analysis. The ultimate tensile strength reduction of some of the generated surfaces are shown in Fig. 17 and 18. We can see that the strength reduction varies for different surface generations and in most cases, the strength reduction of the plate with random field corrosion is higher than the one obtained for uniform corrosion.

Fig. 17 Variation of normalized load ("load at each step" / F_0) for different surfaces.

Fig. 18 Variation of strength reduction of 132 surfaces ($M=10$, $t_0=10\text{mm}$).

6.2 Stochastic Properties the Ultimate Strength of Corroded Plates

Based on the FEA results of previous section, stochastic properties of the ultimate tensile strength reduction of the corroded plates (R) are estimated by 2nd order PCE as shown in Table 9. As a comparison, the stochastic properties of R_{area} estimated in section 5.1 are also shown.

In this table, we can see that the almost same random characteristics of R_{area} and R , and also as M become larger, the mean value of strength reduction becomes smaller. Therefore if we use reduced numbers of random variables in estimating the strength of corroded plate, we should correct it to that of using all numbers of random variables. For correction purpose, we can use the same fitting curve proposed in section 5.3.

Table 9 Stochastic properties of R and R_{area} by PCE.

M	$R=F/F_0$		$R_{area}=A_{min}/A_0$	
	Mean	Std	Mean	Std
5	0.605	0.047	0.609	0.049
7	0.599	0.048	0.602	0.049
10	0.594	0.054	0.596	0.049

In Fig. 19, we can also see the tendency of correlation of each random variable to R is same to that of R_{area} (shown in Fig. 12). This means that it is possible to regard the minimum cross sectional area as the reference index of the tensile strength.

Fig. 19 Correlation of each random variable to R .

7. Conclusions

In this study, the corroded plate is represented by random field model (K-L expansion method), and the stochastic properties of their strength are estimated by PCE method. The following conclusions can be made from the study.

- (1) Random field model (K-L expansion method) can be used to represent the spatial variability of corroded plate.
- (2) When the correlation length, l_c (correlation parameter of random field) is large compared with the plate size (a), the generated surface cannot represent the original random characteristics. It will have smaller variation than the original one.
- (3) For same accuracy, the plate having small plate size compared with correlation length can be represented by a smaller number of random variables. This effect is favorable to the reliability analysis in which the computational effort can be minimized by using a few random variables. On the other hand, if reduced number of random variables are used to estimate the random property of large plates, direct application of the random field model may cause inaccurate results.
- (4) PCE method can be used to estimate the stochastic properties of the strength by using non-linear FEM with a few numbers of analyses compared with Montecarlo simulation.
- (5) The ultimate tensile strengths of the plates with random field corrosion are investigated. We can conclude that the strength reduction of the plate with random field corrosion is higher than the one obtained for uniform corrosion.
- (6) The stochastic properties of strength reduction are estimated by PCE method. If we use reduced numbers of random variables in surface generation with random field model, the mean value of strength reduction should correct to that value using all numbers of random variables using the proposed formula.
- (7) It can be regarded that the stochastic properties of the corroded surface is dependent on the progression of corrosion and the size of the plate. In this study, thickness reduction of the corroded surfaces is assumed to have general corrosion following normal distribution. For other types of corrosion (such as pitting corrosion) following non-normal distribution, we should study the proper method to represent the corroded surfaces with random field model. And the size of the targeted plate is chosen to fit the specimen scale of the laboratory test and the available computation level of the personal computers. To investigate the effect of plate size on the stochastic properties of the corroded surface and on their ultimate strength is the next work of this study.
- (8) In addition to general corrosion, ship structures are suffering

severe localized corrosion. As future work, local corrosion effect should be taken into account in investigation of ultimate strength.

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