Nominal Wake Fluctuation due to Waves
— Volume Mean and Distribution Based on CFD —

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Summary

The KVLCC2 tanker in fully-loaded condition free to heave and pitch at Froude number Fr=0.142 in three head wave-ship length ratios (0.6, 1.1 and 1.6) is simulated by CFDSHIP-IOWA V4.5. The detailed phenomena of nominal wake behaving in waves are studied by analyzing the velocity and vorticity distribution, and vector field on the propeller plane. The periodic change with phase lag for the volume average axial velocity and circulation are observed. The axial velocity distribution is also decomposed by Fourier analysis. Two vortex systems appear: bilge vortex shedding from the hull body and secondary vortex shedding around the shaft. Based on the results, the propulsion design with higher efficiency, more proper engine margin and lower gas emission are expected.

1. Introduction

The tanker KVLCC2 in fully-loaded condition advancing at design Froude number Fr=0.142 in calm water and in regular head waves is investigated. As reported in Sadat-Hosseini et al.1), CFD (computational fluid dynamics) and EFD (experimental fluid dynamics) results had good agreement on the RAQs (response amplitude operators), added resistance and detailed flow field in tank-fixed coordinate. CFDSHIP-IOWA V4.5 was used for viscous flow simulation in fixed and free surface condition. The EFD measurement was conducted in Osaka University towing tank for free surface condition. In the present work, the detailed phenomena of nominal wake behaving in waves are analyzed for wave-ship length ratio \( \lambda/L \)=0.6, 1.1 and 1.6 based on CFD results in propeller-fixed coordinate and fixed surge condition. A periodic change with phase lag of the velocity field on propeller plane is observed. Based on our results, the propulsion design with higher efficiency, a more appropriate engine margin and lower gas emission are expected.

The studies of added resistance have been focused on ship bow for a long history1). The blunt bow shape generally suffers larger added resistance1). The peak of the added resistance happens near the maximum bow relative motion, and wave length around one ship length, and heave natural frequency1). Since the modern ships are always driven by the propeller operating at stern in waves, the importance to understand the propeller performance in waves has been stressed. Few studies considered propulsion in waves. McTaggart2) measured the added resistance for a towed and self-propelled FFG-7 frigate model at Fr=0.3 in waves and concluded that the self-propulsion would not influence the added resistance much.

Nakamura et al.3) conducted the self-propulsion experiment in irregular head waves for a single screw high speed container. The inflow velocity at the propeller plane was measured by a ring type wake meter. The propeller performance also was calculated by blade element theory using the measured inflow velocity. The added resistance and RAQs in irregular waves could be estimated by the linear superposition of the data from regular waves.

Kashiwagi et al.4) developed an analysis system based on EUT (enhanced unified theory) to estimate the propeller performance in waves. The RAQs in frequency domain, relative height, added resistance, steady lateral force and yaw moment were predicted firstly. By using those results with a wave spectrum, the ship speed loss in irregular waves could be calculated. Good prediction was presented compared with the data from experiment, speed trail and actual voyages.

However, very few studies discussed the ship wake field the propeller works inside in waves in detail. The wake profile and behavior would influence the propeller performance definitely. Tsukada et al.5) measured the unsteady ship wakes in regular waves by a five Pitot tube system for future CFD validation. The ship was towed in heave and pitch free, and motion fixed condition in wave length \( \lambda/L \)=1 and 0.5, and a forced pitch condition in calm water. The changes of wake fraction and mean circulation in one encounter period were presented. The wake factor increases because of larger ship motions in longer waves. In short waves, the main influence is from incident wave number. The wake factor change due to motion free in one encounter period could be explained by the superposition of the motion fixed and forced pitch result.

Ueno et al. (2013) conducted the free running test for a 4m long container ship model (CB=0.65) with rudder in regular and irregular waves at Fr=0.158 and 0.223. The wake velocity was measured by vane-wheel current meters. The thrust and torque measured by dynamometer in waves varied by time. The thrust and torque were estimated from the inflow velocity and ship motions in waves provided by a strip method. The measurement and estimate of relative longitudinal flow velocity were compared*

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and discussed. The effective wake coefficient in regular waves is higher than that in calm water for all wave lengths $\lambda/L=0.4\sim3.0$. The difference increases as heading angle increases from 0°-180°.

2. Ship geometry and test condition

As presented in Sadat-Hosseini et al.\textsuperscript{1)}, the experiments and simulations conducted for full hull KVLCC2 appended with propeller dummy hub in fully loaded condition. The main particulars of the EFD and CFD models are shown in Table 1. The model ship is a 1/100 scale model of a 320m long real ship. Froude number $Fr=0.142$ which is 15.5 knots (7.954m/s) design speed of the full scale ship. Reynolds number $Re=2.546\times10^6$. The wave lengths cover the wide range (calm water and $\lambda/L=0.18\sim2.0$). Wave steepness $h/\lambda\leq0.30$ is considered. Wave amplitude $A=0.03$m corresponds to $A/L=0.009375$. For the present study, ship motion is 2DOF (degree-of-freedom): free to heave and pitch with fixed surge. Three wave lengths are chosen: $\lambda/L=0.6$ represents short waves, $\lambda/L=1.1$ has the maximum added resistance and $\lambda/L=1.6$ represents long waves.

Table 1 KVLCC2 model geometry.

| Length between perpendiculars | $L_{W2}$ [m] | 3.200 |
| Beam | $B_{WU}$ [m] | 0.580 |
| Draft | $d$ [m] | 0.208 |
| Displacement | $\Psi$ [m$^3$] | 0.313 |
| Longitudinal center of buoyancy | LCB [%Lyp] | 3.48 |
| Vertical Center of Gravity | $K_{Gz}$ (m) | 0.186 |
| Radius of gyration | $K_{xy}$ (m) | 0.4B |
| | $K_{x}$ (m) | 0.25$L_{pp}$ |
| | $K_{z}$ (m) | 0.25$L_{pp}$ |
| Propeller plane location | $x/L$ | 0.98 |
| Propeller center position | $z/L$ | -0.04 below C.G. |
| | | -0.0469 under $z=0$ |
| Propeller radius | $r_p/L$ | 0.0154 |

3. CFD Method

The CFDSHIP-IOWA V4.5 is a dynamic overset, block structured URANS (unsteady Reynolds-averaged Navier-Stokes) solver coupling with 6DOF motion solver specifically designed for ship applications. For this study, the absolute inertial earth-fixed coordinates and SST $k$-$\omega$ turbulence model using no wall function are employed. The free surface is modeled by a single-phase level-set method. The location of the free-surface is given by the ‘zero’ value of the level-set function $\phi$, positive in water and negative in air.

The computational domain extends from $-0.41<x/L<2.35$, $0<y/L<1$, $-0.97<z/L<0.23$, as shown in Fig.1. The ship axis is aligned with $x$ with the bow (FP) at $x=0$ and the stern (AP) at $x=1$. The $y$ axis is positive to starboard with $z$ pointing upward. The undisturbed free surface at lies on $z=0$.

Several types of boundary condition are required, as listed in Table 2. The half of the ship and flow field is modeled due to the symmetric conditions of the test cases. Thus, $y$-symmetric condition is specified on $y=0$. The same boundary condition is also used for $y=1$. The far field boundary conditions are imposed on the top and bottom of background. The no-slip condition is applied on the solid surfaces. The wave boundary conditions calculated from the linear potential flow solution are used for inlet and outlet of the domain.

As presented in Sadat-Hosseini et al.\textsuperscript{1)}, the total numerical uncertainty was 3.46% for verification. For validation, the heave and pitch motion amplitude showed 4.21% and 7.06% error. CFD uncertainties are 4.7M which are decomposed on 40 CPUs for parallel processing. Since no wall function is applied in this study, the grid size on the solid surface is designed small enough to capture the boundary layer and turbulence. The grid size normal to the hull surface is $1\times10^6$ cells.

In the simulation, the ship motions (2DOF: heave and pitch) were solved in ship coordinate and the flow field was solved by URANS in earth coordinate. Thus, in the present analysis the geometry $(x,y,z)$ and velocities $(u,v,w)$ were transformed from ship/earth coordinate to propeller coordinate.

Table 2 Boundary conditions.

<table>
<thead>
<tr>
<th>Type/ Location</th>
<th>$u$</th>
<th>$v$</th>
<th>$w$</th>
<th>$p$</th>
<th>$k$</th>
<th>$\omega$</th>
<th>$\phi$</th>
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<tr>
<td>Wave inlet/outlet</td>
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<td>0</td>
<td>$w_p$</td>
<td>$p_f$</td>
<td>$10^7$</td>
<td>0</td>
<td>$\zeta_p$</td>
</tr>
<tr>
<td>Y-symmetric</td>
<td>$\psi_u$</td>
<td>$\psi_v$</td>
<td>$\psi_w$</td>
<td>$\psi_k$</td>
<td>$\psi_{\omega}$</td>
<td>$\psi_{\phi}$</td>
<td></td>
</tr>
<tr>
<td>Two sides of background</td>
<td>$w_u$</td>
<td>$w_v$</td>
<td>$w_w$</td>
<td>$w_k$</td>
<td>$w_{\omega}$</td>
<td>$w_{\phi}$</td>
<td></td>
</tr>
<tr>
<td>Bottom of background</td>
<td>$w_u$</td>
<td>0</td>
<td>$w_v$</td>
<td>$w_w$</td>
<td>$w_k$</td>
<td>$w_{\omega}$</td>
<td>$w_{\phi}$</td>
</tr>
<tr>
<td>Far field #1</td>
<td>$w_u$</td>
<td>0</td>
<td>$w_v$</td>
<td>$w_w$</td>
<td>$w_k$</td>
<td>$w_{\omega}$</td>
<td>$w_{\phi}$</td>
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<td>$w_w$</td>
<td>$w_k$</td>
<td>$w_{\omega}$</td>
<td>$w_{\phi}$</td>
<td></td>
</tr>
<tr>
<td>No-slip hull/shell</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$Re_{Fr2}$</td>
</tr>
</tbody>
</table>

$w_p = w(x,z,t) = a_w q e^{i\omega t} \cos(kz-\alpha_xt)$
$w_p = w(x,z,t) = a_w q e^{i\omega t} \sin(kz-\alpha_xt)$
$P_{fr} = \frac{p(x,z,t) - a_w q e^{i\omega t} \cos(kz-\alpha_xt) - a_w^2 q^2 e^{2i\omega t} \phi}{2}$
$\zeta_p = \psi_p q e^{i\omega t}$

Fig. 1 Computational domain and boundary conditions
4. Results

4.1 Vortex Behavior in Nominal Wake

As reported in Sadat-Hosseini et al.1, the total numerical uncertainty was 3.46% for verification. For validation, the heave and pitch motion amplitude showed 4.21% and 7.06% error. CFD predicted the added resistance with 6% error at the peak. The major error occurred for the wave length less than λ/L=0.5. The figures for the validation result are listed in the Appendix.

The main interest of the present work is to understand the vortex behavior of the nominal wake on the propeller plane (x/L=0.98). Fig. 2 shows the result in calm water: the contour flooding for vorticity Ω, contour lines for axial velocity U/U∞ and horizontal and vertical velocity vectors (U/U∞, w/U∞). Two sources of vortices appear: the bilge vortex and secondary vortex under the shaft. The bilge vortex is generated from the ship hull body and then shedding into the propeller plane. It locates around medium boundary layer (U/U∞=0.3-0.5) and covers a large hook or U-shape region in the upper part of propeller plane. It is a counter-clockwise rotating vortex producing positive vorticity with the magnitude Ω<+64. On the other hand, the secondary vortex is induced by the bilge vortex near the shaft surface. Thus, against the rotation of the bilge vortex it has negative vorticity corresponding to the clockwise rotation. It locates beneath the shaft inside the low speed area (U/U∞=0.1) and lower part inside around 0.5 propeller radius. Compared to the bilge vortex it occupies smaller region but with higher vorticity magnitude Ω<+80. The low speed area is the extension of the boundary layer developed along the ship bottom. It is inside the propeller radius with a dense contour distribution of the axial velocity changing from 0.0 to 0.8. Outside the propeller radius, the stern upward flow velocity can be seen clearly here. It is in the outer layer of the boundary layer (U/U∞=0.8 or 0.9). Those phenomena are caused by the three dimensional curved surface of the ship stern. Basically, the flow is extremely non-uniform inside the propeller radius.

Fig. 2 Vortex behavior of nominal wake in calm water

In order to understand the above-mentioned phenomena in waves, for λ/L=1.1 Fig. 3 shows the ship vertical motion responses in one encounter period. Based on the same plot layout of Fig. 2 the flow field solutions for one-quarter encounter period are plotted in Fig. 4. For phase, the crest of the incident waves presents at x/L=0.98 at t/Te=0, i.e. the cosine waves ζ=0.98. Time is non-dimensionalized by the encounter period T. The displacement of the propeller plane d=0.98, i.e. vertical stern motions, is the resultant of heave z and pitch θ motions: z=(0.98L-XCG)sin(θ). XCG is the longitudinal center of gravity of the ship. Basically d=0.98 follows ζ=0.98. Ship pitches its bow up (increasing θ) and stern down (decreasing d) when wave crest is passing the stern (tough is around XCG). Meanwhile, around t/T=0.25 later the ship sinks the most during the stern downward movement and wave crest is above XCG. Thus, around 180° (t/T=0.5) phase lag between ζ=0.98 and θ can be seen, and around 270° (t/T=0.75) between ζ=0.98 and z. Corresponding to d=0.98 curve, at t/T=0 the stern is near the highest position; at t/T=0.25 it is moving down; at t/T=0.5 it is near the lowest position; at t/T=0.75 it is moving up.

Fig. 4 Vortex behavior in one encounter period for λ/L=1.1 (wave crest at x/L=0.98 at t/T=0.0)

According to Fig. 4 and d=0.98 curve, the bilge vortex moves up and down relative to the vertical stern motions. The vorticity magnitude becomes larger than the calm water one (Ω>80). In addition to the periodic vortex movement, the wake shape, i.e. boundary layer thickness, and stern upward flow velocity changes periodically as well. For example, at t/T=0.25 as stern moving down with a high speed, large downward vectors above the hub and near upper y=0 plane can be observed. The boundary layer and bilge vortex region become very narrow in y-direction and the stern upward flow velocity becomes small. The secondary vortex is not only induced by the bilge vortex but also is shedding around the shaft caused by the vertical stern motions. When the stern moves up, a very low speed area beneath the shaft develops and extends downward deeply even larger than...
one propeller radius, e.g. \( v/T_e = 0.5 \) to 0.75. Meanwhile the bilge vortex is below and next to the shaft. When stern moves down such as \( v/T_e = 0.25 \), the other set of secondary vortex forms on the side of or above the shaft: the vortex with \(-\omega_k\) will turn to shedding up and induce another small vortex with \(+\omega_k\). Also, the bilge vortex moves above the shaft.

\[ \Gamma ( \text{wave crest at } x/L=0.98 \text{ at } t/T_e=0) \]

The circulation for calm water, \( \lambda/L = 0.6, 1.1 \) and 1.6 is presented in Fig. 5 for one encounter period, \( t/T_e = 0.0 \) is for the incident wave crest at \( x/L=0.98 \) (propeller plane). As the discussion for Fig. 3, the vorticity shows: \(+\omega_k\) for the bilge vortex covers larger area with lower magnitude and \(-\omega_k\) for the secondary vortex occupies smaller area with higher magnitude. Thus, the positive and negative vorticity would almost cancel each other in Eq. (1) as shown in Fig. 5. The circulations are very small positive values suggesting the bilge vortex’s contribution is slightly larger as it covers larger area. The average circulation \( \bar{\Gamma} \) is \( 13.245 \times 10^{-3} \) for \( \lambda/L = 0.6 \) which is very close to calm water value \( \bar{\Gamma} = 13.144 \times 10^{-3} \). \( \bar{\Gamma} = 12.136 \times 10^{-3} \) for \( \lambda/L = 1.1 \) and \( \bar{\Gamma} = 12.581 \times 10^{-3} \) for \( \lambda/L = 1.6 \) are slightly smaller than calm water one. For \( \lambda/L = 0.6 \), \( \Gamma \) oscillation is simple harmonic with a 180 degree phase lag. It implies that the bilge vortex movement in short waves is small and the upward stern flow follows the wave orbital movement, i.e. at \( v/T_e = 0.5 \) the wave crest vertical velocity \( w \) would reach a maximum. The wake pattern of \( \lambda/L = 0.6 \) is just like Fig. 2 only with magnitude oscillation but the vortices maintain in the same place. However, for long waves, the bilge vortex moves tremendously relative to the large ship motions. As shown in Fig. 4 (\( \lambda/L = 1.1 \)), the high \(+\omega_k\) region at least appear twice in one period, e.g. the bilge vortex with \( \omega_k \) on the upper part of propeller plane) and 0.75 (lower part of propeller plane). Also, they happens in the different instant while the incident wave crest propagates. This phenomenon causes the higher harmonic component (non-simple harmonic curve) and phase lag in \( \Gamma \) oscillations in Fig. 5. For \( \lambda/L = 1.6 \) the phase lag is around \( v/T_e = 0.2 \) but for \( \lambda/L = 1.6 \) it shifts to \( v/T_e = 0.3 \). The 2nd harmonic component can be seen at the curve crest and decreasing slope. The oscillation amplitude becomes larger for both longer waves. It indicates that larger ship motions cause more intense periodic change for bilge vortex.

### 4.2 Orbital Velocity

Base on the linear wave theory in deep water condition, potential flow (PF) orbital velocity \( u' \) (axial component) and its amplitude \( u'_{PF} \) at propeller plane \( x/L = 0.98 \) under a certain water depth \( z \) are

\[ u'(t) = \frac{\partial \phi}{\partial x} = A \omega e^{kz} \cos(kx' - \omega_k t)/U_0 \]  \hspace{1cm} (2)

\[ u'_{PF} = A \omega e^{kz}/U_0 \]  \hspace{1cm} (3)

where \( \omega_k \) is encounter frequency, \( k = 2\pi/\lambda \) is wave number, and \( \omega \) = \( \sqrt{gk} \) is wave frequency. Also, \( L = L_{pp} = 3.2m \), \( A = 0.03m \), \( g = 9.81m/s^2 \), \( x' = x/L-0.98 \), \( z/L = -0.0469 \) which is the initial location of the propeller center under the undisturbed free surface \(z = 0\).

The axial velocities along several lateral positions at the \( z \) position of propeller center, are compared in Fig. 6. The velocity \( u'+1 \) is for PF (orbital velocity plus free-stream velocity) and \( u/U_0 \) for CFD. The \( v/T_e = 0.0 \) is for the propeller under incident wave crest. The far field flow would behave as potential flow and linear waves. The same conclusion could be drawn in the comparison of the amplitude of far field \( u'_{PF} \) and \( u'_{CFD} \) in Table 3.
u'_{CFD} = u/U_0 - 1 \text{ (axial velocity minus free-stream velocity) at } y=2B \text{ for } \lambda/L=0.6 \text{ and 1.6. For the case with maximum added resistance (} \lambda/L=1.1), u'_{CFD} \text{ is obtained at } y=4B \text{ since the disturbance is still strong at } y/L=2B. \text{ Overall, the difference } E1 \text{ between } u'_{CFD} \text{ and } u'_{PF} \text{ is less than 4%. The orbital velocity amplitude (} -0.1U_0 \text{) decreases as wave lengths increases (wave frequency decreases). In Fig. 6, while the flow field is closer to the ship hull, the mean value of axial velocity decreases due to the viscosity near the solid surface. On the other hand, for longer wave length } \lambda/L=1.1 \text{ and 1.6 the axial velocity amplitude increases with a certain phase lag. They are the response to the pressure gradient between inner and outer layer of the boundary layer in waves. Also, the } 2^{nd} \text{ harmonic component (non-simple harmonic oscillation) is observed for } \lambda/L=1.1 \text{ and 1.6 clearly. It is believed to be induced by the bilge vortex movement, especially for the curves } y/L=0.01 \text{ which are inside the propeller radius (} r_p/L=0.0154 \text{). In Fig. 2 (calm water), the bilge vortex is with the hook-shape velocity profile in the upper part of propeller plane. In Fig. 4 (} \lambda/L=1.1), \text{ the hook-shape pattern at least appear twice in one period, e.g. } t/Te=0.0 \text{ (upper part of propeller plane) and 0.75 (lower part of propeller plane). By the same conclusion in section 4.1 (time series) and later 4.5 (Fourier analysis), the bilge vortex moves in one encounter frequency vertically relative to the propeller position due to the ship motions. So, the velocity region related to the bilge vortices also moves vertically in the propeller plane. It is the cause of the higher frequency component. The strength and shape of vorticity field change due to the harmonic frequency.}

4.3 Volume Mean Velocity

By integrating the axial velocity distribution inside the propeller radius, the volume average nominal wake velocity } u_N \text{ (nominal wake factor } l-w_c) \text{ could be computed as}

\[
u_N = \frac{1}{\pi(r_p^2-r_0^2)} \int_{r_0}^{r_p} udA
\]

where } r_p \text{ and } r_0 \text{ are the radius of propeller and hub, respectively.}

A linear method to estimate } u_N \text{ is also proposed to compare with the CFD result. If calm water } u_{N-calm} \text{ is given, } u_N-\text{linear in waves could be estimated by}

\[
u_N-\text{linear}(t) = u_{N-calm} + u'_{PF} \cos(\omega_CT)
\]

Herein, CFD calm water } \bar{u}_N \text{ is used for } u_{N-calm}. \text{ PF orbital velocity amplitude } u'_{PF} \text{ are obtained from Eq. (3) and Table 3.}

The CFD } u_N \text{ for one encounter period is showed in Fig. 7 for } \lambda/L=0.6, 1.1 \text{ and 1.6 compared with calm water value and the linear method. The } t/Te=0 \text{ is for the incident wave crest at } x/L=0.98. \text{ The harmonic components of } u_N \text{ are also listed in Table 4. It shows the time average } \bar{u}_N \text{ (mean value) in waves would be higher than the calm water value, confirmed by the other CFD literatures as well. Osahi et al.} \text{ used SURF to study the same condition and geometry with the present work. Sadat-Hosseini et al.} \text{ reported } \omega_c \text{ for KCS container ship in } \lambda/L=1.37 \text{ predicted by STAR-CCM+. The difference becomes larger for } \lambda/L=1.1 \text{ and smaller than those in shorter (} \lambda/L=0.6 \text{) and longer wave (} \lambda/L=1.6). } \bar{u}_N \text{ of } \lambda/L=0.6 \text{ is lower than } \lambda/L=1.6 \text{ value because of smaller ship motions. It corresponds to the trend of added resistance which reaches the peak at } \lambda/L=1.1, \text{ and has lower values in shorter and longer wave region).}

For amplitudes, } u'_{PF} \text{ of } \lambda/L=0.6 \text{ is } 50\% \text{ (} E2 \text{ in Table 4) larger than } |u'_{PF} | \text{ (CFD 1st harmonic amplitude) but both have similar and very small phase lag (Fig. 7) because of smaller ship motions. As reported, the added resistance in short waves mainly is caused by diffraction. Instead, for longer waves } |u'_{PF} | \text{ is larger than } u'_{PF} \text{ (} E2=-1\% \text{ for } \lambda/L=1.1 \text{ and -10\% for 1.6) and their phase lags deviate much more (around 0.25T, in Fig. 7). As mentioned in section 4.2, the trend of velocity amplitude near the hull is different from the far field orbital velocity: as the wave length increases } u'_{PF} \text{ and } u'_{CFD} \text{ decreases but } |u_{N}| \text{ increases. Also, their } 2^{nd} \text{ harmonic components are observed clearly in Fig. 7 and Table 4. } |u_{N}|_2 \text{ is around 40% of } |u_{N}|_1 \text{ for } \lambda/L=1.1 \text{ and 1.6; for } \lambda/L=0.6, \text{ it is only less than 3%. In longer waves, the added resistance is dominated by larger ship motions which generate larger bilge vortex movement and more disturbances to the flow field. In conclusion, } u_N \text{ only could be re-constructed conditionally by the linear method: the PF orbital amplitude can be used for long waves and calm water value for the mean value in short waves.}

![Fig. 7 Volume average nominal wake axial velocity](image)

Table 4 Components of volume average velocity (CFD).

<table>
<thead>
<tr>
<th>\lambda/L</th>
<th>Calm</th>
<th>0.6</th>
<th>1.1</th>
<th>1.6</th>
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</thead>
<tbody>
<tr>
<td>\bar{u}_N</td>
<td>0.4124</td>
<td>0.4391</td>
<td>0.5176</td>
<td>0.4780</td>
</tr>
<tr>
<td>\bar{</td>
<td>u_{N}</td>
<td>}</td>
<td>0.06191</td>
<td>0.1013</td>
</tr>
<tr>
<td>E2(%)</td>
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<td>-9.36</td>
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<tr>
<td>\varepsilon(deg)</td>
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<td>54</td>
<td></td>
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</tr>
<tr>
<td>\varepsilon(deg)</td>
<td>-46</td>
<td>179</td>
<td>166</td>
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</table>

\[ u^*E2(%) = 100 \times (u'_{PF} - |u_{N}|)/u'_{PF} \]
4. 4 Vertical velocity component

By integrating vertical velocity \( w/U_0 \), as Eq. (4) did, the volume average vertical velocity \( w_N \) is shown in Fig. 8 for one encounter period for three wave lengths. The oscillations are nearly simple harmonic, except for the peak of \( \lambda/L=1.1 \) curve. As the trend of \( |w_N|_1 \) (Table 4), the \( w_N \) amplitude increases as \( \lambda/L \) increases. \( |\bar{w}_N|_1 \) and \( w_N \) amplitudes are similar as well for each \( \lambda/L \) but their values are closer to \( u'v' \) (\(-0.1U_0\)) only for long waves. With the different conclusion to \( \bar{w}_N \), \( w_N \) mean values are very close to calm water ones which is a positive value very close to zero. It indicates in general upward flow velocity is only very slightly larger than the downward flow one inside the propeller radius. As mentioned in section 4.1, outside the propeller radius the stern flow mainly is upward. Also, the near 90 degree phase lag almost follows the linear wave orbital movement. For the cosine wave crest at \( t/T_e=0 \), the vertical components of the orbital velocity would be a minus sine curve.

![Wave crest at \( x/L=0.98 \) at \( t/T_e=0 \)](image)

Fig. 8 Volume average vertical velocity in one encounter period

4.5 Fourier Analysis

Fig. 9 is the harmonic components of axial velocity decomposed by Fourier analysis inside the propeller radius for \( \lambda/L=0.6, 1.1 \) and 1.6 (from left to right). Herein, propeller center is at \( z/L=0.04 \), i.e. C.G. (center of gravity) is at \( z=0 \) and incident wave crest at \( x/L=0.0 \) when \( t/T_e=0 \).

For mean value component, only shorter wave \( \lambda/L=0.6 \) has similar pattern to the calm water one (Fig. 2), e.g. the hook-shape pattern, because of small ship motions. For longer waves, the mean value distribution show higher values generally. It is related to ship motions in waves. In long waves, the outer flow with high velocity would come into the propeller radius when ship stern is moving down with high speed, e.g. \( u/U_0=0.9 \) at \( t/T_e=0.25 \) in Fig. 4. It is supported by the conclusion for Table 4 and Fig. 7: \( \bar{w}_N \) of \( \lambda/L=0.6 \) is closer to the calm water value, and it is higher for longer waves. The largest 1st harmonic amplitude is observed under the shaft for longer waves \( \lambda/L=1.1 \) and 1.6. It corresponds to the low speed area extending beneath the shaft induced by the larger ship motions in long waves. It extends once in one encounter period as seen in Fig. 4. The location of 2nd harmonic component is mainly related to the bilge vortex movement for \( \lambda/L=1.1 \) and 1.6. For \( \lambda/L=0.6 \), because of small ship motions the low speed area under the shaft behaves similar to the calm water one. Only the bilge vortex moves in one encounter frequency slightly up and down in waves and rotates in a higher frequency, so it is governing both harmonics. For the phases, all cases and components are very different in detail. Generally, one \( \pi \) phase lag is located around the upper part of propeller plane, except for the 2nd harmonic of \( \lambda/L=0.6 \). It might be related to the timing of the bilge vortex moving in and out of the upper part of the propeller radius, e.g. \( t/T_e=0.0 \) to 0.25 in Fig. 4.

5. Conclusions

The nominal wake at propeller plane in waves is analyzed in detailed. The bilge vortex moving relative to ship motions and the secondary vortex around the shaft with a low speed area are observed. Both vortex systems rotate in different directions and are dominated in different area. Based on the analysis for circulation, volume averaged axial velocity, Fourier analysis, the mean value would be different from clam water value. The global and local values oscillate by time with phase lag and 2nd harmonic component which is related to the bilge vortex movement and its development in waves. The amplitude of the volume average axial velocity would be different with the potential flow orbital velocity, especially for short waves.

<table>
<thead>
<tr>
<th>( \lambda/L=0.6 )</th>
<th>( \lambda/L=1.1 )</th>
<th>( \lambda/L=1.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value</td>
<td>Mean value</td>
<td>Mean value</td>
</tr>
<tr>
<td>1st harmonic amplitude</td>
<td>1st harmonic amplitude</td>
<td>1st harmonic amplitude</td>
</tr>
<tr>
<td>2nd harmonic amplitude</td>
<td>2nd harmonic amplitude</td>
<td>2nd harmonic amplitude</td>
</tr>
<tr>
<td>Heave phase</td>
<td>Pitch phase</td>
<td>Heave phase</td>
</tr>
<tr>
<td>Pitch phase</td>
<td>Heave phase</td>
<td>Pitch phase</td>
</tr>
<tr>
<td>Added resistance</td>
<td>Added resistance</td>
<td>Added resistance</td>
</tr>
</tbody>
</table>

Fig. 9 Fourier analysis on axial velocity distribution

References

Acknowledgments

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References


Appendix

A.1 CFD validation results

Fig. A1 Ship motion response and added resistance in waves for fully-loaded KVLCC2 in waves at Fr=0.142 (OU: Osaka University, Japan; INSEAN: Istituto Nazionale per Studi ed Esperienze di Architettura Navale, Italy; NTNU: Norwegian University of Science and Technology, Norway) (\(f_e\): encounter frequency; \(f_{n\text{,heave}}\): natural frequency of heave motion; \(f_{n\text{,pitch}}\): natural frequency of pitch motion)