The Yawing of Ships caused by the Oscillation amongst Waves.

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Though this phenomenon has not so far been studied by any investigators, it would have perhaps been noticed by seamen of keen observation, that a boat amongst waves sets itself parallel to wave crests in one case and perpendicular to them in the other case; this being manifested most markedly when the boat rides on breaking rollers on a sea shore. Even an ocean liner of the largest size, when she meets with huge waves, would be affected in a similar manner, though it has been left unnoticed; partly on account of the reason that huge waves are always associated with heavy wind, and partly because a ship, apart from this yawing, cannot keep a straight course without steering all the time.

The first opportunity which occurred for the author to notice this phenomenon was during his experiments on the rolling of model ships; in his experiments model ships were held parallel to waves by a small constraining force. In the course of the experiments it happened that, for a certain range of the period of waves the model remained always in the same position, while for the other range of the period it had a strong tendency to yaw in one direction or the other. This fact reminded the author of the behaviour of small fishing boats amongst breaking rollers on a beach, and he thought that the yawing might be attributed to the oscillation of the boats on waves.

To study the general feature of such a motion on waves, a model ship was released from the constraint and was left to itself on waves. It was found that for a certain period of waves, the model yawed to set itself parallel to the line of the crest, and for another period, perpendicular to it. The author thought that such a motion was caused
by the action of gyrostatic couple due to the rotational motion in two different planes at right angles—namely pitching and rolling. In view of this the author made a mathematical solution which is shown in Appendix. As described there, the mathematical solution tells us that—if the author's view is right—a ship when riding obliquely to a series of waves, is set to yawing by the gyrostatic couple which is developed by the "forced" rolling and pitching. As can be seen from equation (4) and from what follows from it, in some cases the gyrostatic couple acts constantly in one direction to turn the ship in a definite direction until she reaches a stable position; while in other cases the gyrostatic couple set in during a half oscillation is counteracted by the same during the next half oscillation, and in consequence the ship is not turned in a definite direction, but makes to-and-fro yawing only. This can also be seen from Fig. 3 which well explains how gyrostatic couple is developed by the forced rolling and pitching. This diagram also tells that the difference in the nature of the yawing motion described above can solely be traced back to the phase relation of the forced oscillations and the motion of waves.

As described in Appendix, five different cases may be mentioned:—

First case. When the period of waves is shorter than that of the pitching, a ship yaws until her length becomes parallel to the line of the crest of waves.

Second case. When the period of waves synchronizes with that of the pitching, the ship does not yaw in a constant direction, but remains in any direction whatever.

Third case. When the period of waves is intermediate between that of the pitching and the rolling, the ship is turned until her length becomes square to the line of the crest.

Fourth case. When the period of waves synchronizes with that of the rolling, the ship remains in any direction whatever, just as in the second case.

Fifth case. When the period of waves is longer than that of the rolling, the ship is turned until her length becomes parallel to the line of the crest as in the first case.

To testify this with an experiment a small model ship was set to oscillation amongst waves in a tank of water which is fitted up to produce waves of any desired period and in which previous rolling experiments were made. Cinematographs showing the motion of the model ship were taken, a part of them being shown in Fig. 1. The particulars
of the experiment are as follows:

The complete periods of the oscillations of the model ship throughout the experiment are:

Pitching..................0.76 sec.
Rolling....................1.17 sec.

(1) Period of waves........0.65 sec. This corresponds with the first case. The stable position of the ship is parallel to the line of the crest of waves.

(2) Period of waves........0.75 sec. This corresponds with the second case. The ship has no stable position.

(3) Period of waves.......0.95 sec. This corresponds with the third case. The stable position of the ship is square to the line of the crest of waves.

(4) Period of waves........1.17 sec. This corresponds with the fourth case. The ship has no stable position.

(5) Period of waves.......1.23 sec. This corresponds with the fifth case. The stable position of the ship is parallel to the line of the crest of waves.

It will be seen that the experiment accords very well with the theory.

From what is described above, it will be seen that a ship, when set to oscillation amongst waves, yaws and in general finally assumes a definite direction relative to the waves; and also that it may be attributed to the gyrostatic couple caused by the joint action of the pitching and rolling motions.

Another conceivable source of such yawing will be the unsymmetrical distribution of wave pressure around a ship, that is inevitable with a floating body having an oblong from like a ship. With a view to eliminating such a doubtful source the author repeated a similar experiment with a semi-spherical model in which mass is unsymmetrically distributed in order that it may represent a ship. It will hardly be necessary to say that with such a form symmetry of wave pressure can surely be expected.

The period of oscillation of the semi-spherical model in water was as follows:

The period of oscillation about an axis in the direction of the least moment of inertia...............0.83 sec.

The same about an axis in the direction of the largest moment of inertia .......
It was found that the behaviour of this model amongst waves was similar to that of the ship-shaped model. Thus, the yawing motion of a ship under consideration may most probably be attributed to the gyrostatic couple due to the rolling and pitching motions.

Whatever be the cause of the yawing motion, the following facts which will naturally follow from this phenomenon, are especially worth mentioning.

(1) When a ship rides on waves nearly synchronous to her period of rolling, two slightly different cases—namely, the period of waves a little shorter in one case and a little longer in the other case than the natural period of her rolling—which might seemingly bring forth the same effect on the rolling of the ship, will actually result in quite a different state of rolling. In the former case which corresponds to Case 3, the ship, even if left to herself, will be directed square to the waves and relieved from heavy rolling; while in the latter case which corresponds to Case 5, the ship will ultimately be exposed to waves abeam and subjected to heavy rolling.

Unfortunately owing to the lack of the systematic analysis of the actual rolling and pitching oscillations of ships, the calculation of the actual value of the yawing moment will be impossible, as it is produced only by the forced portion of general mixed-up oscillations. However, theoretical calculations, one of which is shown below, tell us that the yawing moment is generally of such a small magnitude that it is under easy control of a rudder.

A numerical calculation of theoretical yawing moment.

Assumed data are as follows:—

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period of waves ( \frac{2\pi}{p} )</td>
<td>12.5 secs.</td>
</tr>
<tr>
<td>Natural period of rolling ( \frac{2\pi}{\omega} )</td>
<td>12 secs.</td>
</tr>
<tr>
<td>Natural period of pitching ( \frac{2\pi}{\omega'} )</td>
<td>6 secs.</td>
</tr>
<tr>
<td>Displacement of ship ( D )</td>
<td>5,000 tons</td>
</tr>
<tr>
<td>Metacentric height ( h )</td>
<td>2 ft-0 in.</td>
</tr>
</tbody>
</table>
Coefficient of damping of rolling \((k) = 0.02\).

"" "" "" pitching \((k') = 0.07\).

Maximum wave slope \((\varphi) = 6\) degrees.

Angle which length of ship makes with line of crest of waves \((\alpha) = 45\) degrees.

Substituting these values into

\[
I \frac{d^2 \theta}{dt^2} = \frac{1}{4} I \frac{\omega^2 \omega'^2 \varphi \sin 2\alpha}{\sqrt{\omega'^2 - p'^2} \sqrt{\omega^2 - p^2} + k^2 \varphi^2 + p^2 \cos (\beta' - \beta)} \ldots \ldots \text{(see Eqn. 4)}
\]

we have

\[
I \frac{d^2 \theta}{dt^2} = 356 \text{ ft. tons.}
\]

If the speed of the ship is 12 knots, turning moment of the same amount as this yawing moment would be produced with a helm angle of four degrees or so.

Therefore, so long as the rudder of a ship is intact, there is nothing to worry about in this yawing motion. It is only in the case of a mishap in the steering gear that this motion must not be overlooked.

(2) When a small boat overtakes or is overtaken by a large ship, the interaction between the two vessels will be augmented by the present yawing phenomenon; because, as soon as the small boat approaches the region of the divergent waves formed by the large ship, the former will be set to oscillation, and as the apparent period of the divergent waves with respect to the small boat is generally long, the boat will be turned to become parallel to the line of the divergent waves. Therefore, if not steered properly, the danger of collision will be increased thereby.

Appendix.

Mathematical Solution.

To solve exactly the three-dimensional motion of ship riding on waves obliquely to the line of the crest is a matter of the utmost difficulty, and for the purpose of studying merely the nature of the motion it has not special merit compared with an approximate
solution; a rough solution as described here will suffice for the aim of the present paper.

Now, referring to Fig. 2 let the centre of gravity $g$ of the ship be selected as the origin, and take co-ordinate axes as follows:—The longitudinal axis of the ship through $g$, which is fixed in the ship and moving with it, as the axis of $x$, the transverse line drawn horizontally through $g$ as the axis of $y$, and the line drawn perpendicular to both of the $x$ and $y$ axes as the axis of $z$. The positive directions of these axes are so chosen as to conform with the sense of rotation which we shall presently take—clockwise rotation as positive. Also, let $gx_0$ be a horizontal axis which is occupied by the $gx$ axis, when the ship is at rest.

Now let us suppose that waves of sinusoidal form and of maximum slope $\Phi$ run obliquely to the ship, the line of the crest making an angle $\alpha$ with the $gy_0$ axis. If the maximum slope $\Phi$ is small, it is evident that the slope in a vertical plane perpendicular of the $gx_0$ axis $\Phi \cos \alpha$ and the same in a vertical plane perpendicular to $gy$ is $\Phi \sin \alpha$; also, it can easily be shown that, when a ship in which both sides are, as usual, symmetrical with respect to her middle line plane, makes a small oscillation, the component righting levers in the transverse and longitudinal plane are $-\phi h$ and $-h'\psi$ respectively; in which $h$ is the height of the transverse metacentre, $h'$ the height of the longitudinal metacentre, $\phi$ the angle of rolling, and $\psi$ the angle of pitching.

Let another assumption be made that the moment of inertia of the ship about any axis which lies in the plane $yyz$ and passes through $g$, to be constant. By this assumption which will not be far from the truth with actual ships, the equation of motion admits of great simplification.

From what is described above, the dynamical equations,* referring to the moving axes as described above, of the motion of a ship on large waves (compared with the size of the ship) can at once be written down as follow:—

$$I_x \frac{d^2\phi}{dt^2} + K \frac{d\phi}{dt} + Dgh \phi = Dgh \phi \cos \alpha \cos \phi.$$

*See Routh's Advanced Rigid Dynamics, Art. 20.
in which

$I_x =$ moment of inertia of ship about $g_x$ axis.

$I =$ moment of inertia of ship about $g_y$ or $g_z$ axis.

$D =$ displacement of ship.

$K, K', K'' =$ coefficient of damping.

$\frac{2\pi}{P} =$ period of waves.

$\theta =$ angle of yaw.

$g =$ acceleration due to gravity.

Here let a few words be added with regard to the method of solution. More accurate solution than this can be obtained by using Euler's equation. However, with the use of the equations the solution becomes unduly complicated, notwithstanding the fact that the final result given by it is practically the same as that given by the present method. This is the reason why the author preferred the present much more simplified method.

Now, in the second of the above equation the second term which expresses "gyrostatic couple" may, as usual, on account of its smallness in magnitude may be discarded. In addition to this the term of resistance in the third equation may be put aside for the purpose of studying only the nature of the motion. Thus finally we have

\[
\begin{align*}
\frac{d^2 \phi}{dt^2} + k \frac{d\phi}{dt} + \omega^2 \phi &= A \cos pt \\
\frac{d^2 \psi}{dt^2} + k' \frac{d\psi}{dt} + \omega'^2 \psi &= B \cos pt \\
\frac{d^2 \theta}{dt^2} - n \frac{d\phi}{dt} \frac{d\psi}{dt} &= 0
\end{align*}
\]

where

\[
k = \frac{K}{I}, \quad k' = \frac{K'}{I}, \quad \omega^2 = \frac{Dgh}{I_x}, \quad \omega'^2 = \frac{Dgh'}{I}
\]
The first two equations are those of forced oscillation with damping, the solution of which is too well known to require any explanation. In the case where the resistance is small, and the oscillation starts from rest (i.e. \( \phi \) and \( \frac{d\phi}{dt} = 0 \) when \( t = 0 \)), we have, from the first equation, the following approximate solution:

\[
\phi = a \cos (\phi t - \beta) - b \cos (\omega t - \gamma) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)
\]

in which

\[
a = \frac{A}{\sqrt{(\omega^2 - \beta^2)^2 + k^2 \beta'}}, \quad \tan \beta' = \frac{k \rho}{\omega^2 - \beta^2},
\]

\[
b = a \sqrt{\cos^2 \beta' + \frac{P^2}{\omega^2} \sin^2 \beta'}, \quad \text{and} \quad \tan \gamma = \frac{P}{\omega} \tan \beta'.
\]

Needless to say, the first term of the right hand member of (2) is forced rolling and the second term free rolling.

Similarly, under the same condition we have, from the second equation,

\[
\psi' = a' \cos (\phi t - \beta') - b' \cos (\omega t - \gamma') \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)
\]

where

\[
a' = \frac{B}{\sqrt{(\omega'^2 - \beta'^2)^2 + k'^2 \beta'^2}}, \quad \tan \beta' = \frac{k' \rho}{\omega'^2 - \beta'^2},
\]

\[
b' = a' \sqrt{\cos^2 \beta' + \frac{P^2}{\omega'^2} \sin^2 \beta'}, \quad \text{and} \quad \tan \gamma' = \frac{P}{\omega'} \tan \beta'.
\]

Evidently the pitching motion, too, is composed of the forced and free ones.

Differentiating equations (2) and (3), we have

\[
\frac{d\phi}{dt} = -a \rho \sin (\phi t - \beta) + b \omega \sin (\omega t - \gamma)
\]

and

\[
\frac{d\psi}{dt} = -a' \rho \sin (\phi t - \beta') + b' \omega' \sin (\omega t - \gamma')
\]

Substituting these equations into the last one of equation (1), we have
This equation is too complicated to make any deduction therefrom. So that, let us study what this equation tells, applying it to several special cases.

(1) The case when the period of waves is shorter than that of pitching.

As in the case, \( p \) is larger than \( \omega' \), \( p > \omega' > \omega \) (evidently \( \omega' > \omega \), because the period of pitching is always less than that of rolling), and as already described, we have assumed that \( k \) is small; so that, from equations (2) and (3), we know that the following relation approximately holds:

\[
\beta \approx \gamma \approx \pi
\]

\[
\beta' \approx \gamma' \approx \pi
\]

Therefore, \( \beta' - \beta \) is a very small angle, and equation (4) becomes

\[
\frac{2}{n} \frac{d^2 \theta}{dt^2} = a'a'p^2 - a'a'p^2 \cos 2\omega' t - a'b'p' \omega' \cos (\omega - \omega') t - \cos (\omega + \omega') t
\]

\[-ab'p' \omega' \cos (p - \omega') t - \cos (p + \omega') t; + bb' \omega' \omega \cos (\omega - \omega') t - \cos (\omega + \omega') t]

In this equation the terms on the right hand side, except the first one give the oscillatory yawing motion of the ship and have not much importance on the present investigation. Now, remembering that \( a \) is the product of positive quantities and \( \cos a \), and \( a' \) the product of positive quantities and \( \sin a \), we may write the above equation as follows:

\[
\frac{d^3 \theta}{dt^3} = C \sin a \cos a + \text{terms containing cosines}
\]

in which \( C \) is a positive coefficient.

This equation tells that, if \( a \) is positive, the ship yaws positively and in the contrary
case makes negative yawing. Thus with all directions of waves the ship yaws in such a manner as to become parallel to the line of the crest, in the meantime yawing to-and-fro; only when ship's length is parallel to the line of the crest of waves, she makes no yawing in a constant direction. (apparently $\frac{d^2\theta}{dt^2} = 0$ also when $a = \frac{\pi}{2}$, but this position is evidently unstable).

(2) The case when the period of waves synchronizes with that of pitching $p = \omega' > \omega$).

By a similar reasoning as before, we have

\[ \beta = \gamma = \pi \]

\[ \beta' = \gamma' = \frac{\pi}{2} \]

Therefore, $\beta' - \beta$ is nearly equal to $-\frac{\pi}{2}$, and the first term on the right hand side of equation (4) is practically nil; so that, the equation takes the form:

\[ \frac{d^2\theta}{dt^2} = \text{only terms containing sines.} \]

Thus the ship has no tendency to yaw in a definite direction; she only yaws to the starboard in one instant and then to the port in the next instant.

(3) The case when the period of waves is intermediate between the periods of pitching and rolling ($\omega' > p > \omega$).

In this case the following relation will approximately hold:

\[ \beta = \gamma = \pi \]

\[ \beta' = \gamma' = 0 \]

So that, equation (4) becomes

\[ \frac{2}{n} \frac{d^2\theta}{dt^2} = -aa'p^2 + \text{terms containing cosines.} \]

or

\[ \frac{d^2\theta}{dt^2} = -C \sin a \cos a + \text{terms containing cosines.} \]

It will be seen that the motion is just contrary to case (1). The ship, in this case, yaws in such a direction as to set herself perpendicularly to the line of the crest of waves.
(4) The case when the period of waves synchronizes with that of rolling \((\omega' = p = \omega)\).

In this case we have approximately

\[ \beta = \gamma = \frac{\pi}{2} \]

\[ \beta' = \gamma' = 0 \]

Therefore, just as in Case (2), the first term of the right hand side member of equation (4) will practically be zero, and the equation becomes

\[ \frac{d^2 \theta}{dt^2} = \text{only terms containing sines.} \]

Thus the equation is just the same as in Case (2). So that, the ship yaws only to-and-fro and does not make any yawing in a definite direction.

(5) The case when the period of waves is longer than that of rolling \((\omega' > \omega > p)\).

In this case we have approximately

\[ \beta = \gamma = 0 \]

\[ \beta' = \gamma' = 0 \]

and equation (4) becomes

\[ \frac{2}{n} \frac{d^2 \theta}{dt^2} = a\alpha' p^2 + \text{terms containing cosines.} \]

or

\[ \frac{d^2 \theta}{dt^2} = C \sin \alpha \cos \alpha + \text{terms containing cosines.} \]

Thus the nature of yawing motion is just the same as that described in Case (1); the ship yaws to direct herself parallel to the line of the crest of waves.