New Rules for the Calculation of the Moment and the Moment of Inertia of a Plane Area about its Middle Ordinate.

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ABSTRACT

Simple and accurate rules similar to that of Tchebycheff. A numerical example is given for a sine curve and the result compared with those by Gauss', Tchebycheff's, Simpson's and Trapezoidal rules.

The method about to be explained is, as formerly suggested by the author, the extension of the Tchebycheff's. (Kōgaku Kwaishi No. 242. 1902.)

The Tchebycheff's rule, though very convenient to find a plane area, involves the multiplication of each ordinate with the leverage when its moment and the moment of inertia about the middle ordinate are to be calculated. The process is rather troublesome when several areas and consequently a large number of ordinates are to be dealt with, as in the case of the shipbuilding calculation.

According to the following method, the moment and the moment of inertia are given directly by the simple sum of the ordinates multiplied by a constant, the troublesome multiplication of each ordinate with the corresponding leverage being altogether eliminated, and, in addition to this remarkable simplification, the accuracy of the result is increased when compared with some of the existing rules.

Let

$$y = c_0 + c_1x + c_2x^2 + \dots + c_nx^n, \dots\dots\dots(1)$$

where $c_0, c_1, c_2, \dots, c_n$ are constants and n a positive integer, be the equation of the boundary curve. Then, if l denotes the half-length of the base, the moment M of the area about the middle ordinate is

$$M = \int_{-l}^l yx \, dx \dots\dots\dots(2)$$

Again let

$$\text{Moment of Inertia} = C(y_1 + y_2 + \dots + y_{\frac{n}{2}} + y_{-1} + y_{-2} + \dots + y_{-\frac{n}{2}}).$$

n	C	x_1/l	x_2/l	x_3/l
2	$l^3/3$	0.7746		
4	$l^3/6$	0.5815	0.9284	
6	$l^3/9$	0.5030	0.8133	0.9410

(For $n=8$ and higher values, the abscissae become imaginaries.)

As an example, the moment and the moment of inertia of the area bounded by the curve of sine between 0 and $\frac{\pi}{2}$ are calculated by several rules and the results compared with the exact values.

	Moment	Moment of inertia
Exact value	0.2146018	0.1876466
My rule	0.214611 (8 ordinates)	0.187648 (6 ordinates)
Gauss'	0.214602 (4 ,,)	0.187648 (4 ,,)
Tchebycheff's	0.214604 (9 ,,)	0.187651 (9 ,,)
Simpson's	0.214589 (11 ,,)	0.187612 (11 ,,)
Trapezoidal	0.218276 (11 ,,)	0.189617 (11 ,,)

Judging from this and several other examples, the present rules appear to give generally more accurate results than the Simpson's or the Trapezoidal rule, while the process of calculation is surprisingly simple if compared with the Gauss' and the Tchebycheff's.

討 論

○會長(今岡純一郎君) 御質問なり、御討論なりありませぬか、……………横田博士も、前回御講演の末廣博士と共に本協會に於ける權威ある學者として、常に協會の爲に御研究を御發表下され、今また新らしき計算式を御發表に相成りました。是に依つて吾々造船に従事する者は、多大の利便を得ることゝ思ひます。將來皆さんが此「ルール」を御使ひになつて、横田君の御勧めを實際に應用せられんことを望む次第であります。此御發表に對し拍手を以て御禮を申し上げたいと思ひます。

(拍手起る)