THE APPROXIMATE FORMULA FOR LIFT DISTRIBUTION ON A RECTANGULAR AEROFOIL.

By **K. Wada**, *Member*.

Abstract

Assuming a very simple form of expression for the lift distribution along the span of a rectangular aerofoil, formulas for the transformation of aspect ratio and the wind tunnel boundary correction have been obtained which agree quite satisfactorily with experiments

Introduction.

An accurate knowledge of the lift distribution on an aerofoil is most essential in aeronautics from aerodynamical as well as from constructional standpoints. The problem involves an integral equation and its exact solution can hardly be expected in its general form. The simplest case to consider is that of a rectangular aerofoil with constant angle of incidence and chord across the span. The problem as such has been attacked first by Betz⁽¹⁾ and afterwards by Fuchs,⁽²⁾ Trefftz,⁽³⁾ Glauert,⁽⁴⁾ Gates⁽⁵⁾ and others. The results obtained by them are in more or less complicated forms and are not convenient in the application to practical problems, such as, in obtaining the transformation formula of aspect ratio or the correction formula for the wind tunnel interference.

The present author has obtained a very convenient formula of the lift distribution for an aerofoil having constant angle of incidence and chord. Though it is only a formula of approximation the comparison of the results deduced therefrom with those obtained by the more elaborate methods or by experiments seems to suggest its merit as a good first approximation.

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- (1) Betz: "Beiträge zur Tragflügeltheorie mit besonderer Berücksichtigung des einfachen rechteckigen Flügels." Göttingen Dissertation, (1919).
 - (2) Fuchs: "Beiträge zur Prandtl'schen Tragflügeltheorie," 'ZAMM' Bd. I, (1921) p. 106.
 - (3) Trefftz: "Prandtl'sche Tragflächen-und Propeller-Theorie." 'ZAMM' Bd. I (1921) p. 206.
 - (4) Glauert: 'The Element of Aerofoil and Airscrew Theory' p. 137. (1926)
 - (5) Gates: "An Analysis of a Rectangular Monoplane with Hinged Tips." 'R. and M.' No. 1175 (1928).

The Fundamental Equation.

An aerofoil is replaced by a bound line vortex and a system of trailing vortices in accordance with Prandtl's aerofoil theory.⁽¹⁾

The flow at any section of an aerofoil differs from the flow which would occur round the section in two dimensional motion owing to the influence of the trailing vortex system. The trailing vortex system gives rise to an induced velocity at a point of the aerofoil. The induced velocity is normal to the span of the aerofoil and to the direction of the motion. It will be denoted by w and will be assumed to be small in comparison with the velocity V of the general stream of air. The effect of the induced velocity is then equivalent to a reduction of the angle of incidence at the point of the aerofoil by w/V .

The lift coefficient is generally a function of the effective angle of incidence. If α is the geometrical angle of incidence at a point of the aerofoil, the effective angle of incidence α_e is

$$\alpha_e = \alpha - \frac{w}{V}. \quad (1)$$

Therefore the lift coefficient c_a of the aerofoil section at the point is

$$c_a = f_n \left(\alpha - \frac{w}{V} \right).$$

Expanding by Taylor's theorem and neglecting the terms of the higher power of w/V ,

$$c_a = c_{a\infty} - \frac{dc_{a\infty}}{d\alpha} \frac{w}{V},$$

where $c_{a\infty}$ denotes the lift coefficient of the aerofoil section in two dimensional flow or that corresponding to the geometrical angle of incidence α . The theory of an aerofoil⁽²⁾ in two dimensional flow has shown that $\frac{dc_{a\infty}}{d\alpha}$ is approximately equal to 2π for all practical aerofoil sections at small angle of incidence. Hence

$$c_a = c_{a\infty} - 2\pi \frac{w}{V}. \quad (2)$$

This is the fundamental equation connecting the lift coefficient and the induced velocity.

If Γ and Γ_∞ are the circulations round the aerofoil section at any point under consideration in the actual state and in the state of two dimensional motion respec-

⁽¹⁾ Prandtl: 'Tragflügeltheorie,' Göttingen Nachrichten, 1918 and 1919.

⁽²⁾ Glauert, *loc. cit.* p. 86.

tively and t the chord of the aerofoil, then expression (2) can also be written

$$\Gamma = \Gamma_{\infty} - \pi t w, \quad (2')$$

the relationship between the circulation and the lift coefficient being given from Kutta-Joukowski's equation⁽¹⁾ as follows:

$$\Gamma = \frac{1}{2} c_a V t. \quad (3)$$

The Approximate Expression for Lift Distribution.

It is assumed that the circulation Γ at any point of aerofoil, which is naturally a function of co-ordinate y along the span, (the origin being taken at the middle of the span) may be expressed as follows:

$$\Gamma = \Gamma_0 (1 - \eta^2)^m, \quad (4)$$

where Γ_0 is the circulation round the aerofoil section at the middle of span, η denotes y/b , in which b is the half-span of the aerofoil, and m is a positive number.

With the assumed constancy of the chord and the geometrical angle of incidence of the aerofoil section over the span, m may be considered to be a function of the aspect ratio only.

The approximate result obtained by Fuchs⁽²⁾ seems to suggest the following form for m :

$$2m = \frac{k \frac{\pi t}{b}}{1 + k \frac{\pi t}{b}}.$$

It has been found after trial that unity seems to be the most appropriate value for k . Hence

$$2m = \frac{\frac{\pi t}{b}}{1 + \frac{\pi t}{b}}. \quad (5)$$

It is evident at once that expression (5) satisfies conditions at the extreme cases. Namely, when the span of an aerofoil becomes very large, from (5) m tends to zero or, from (4), the distribution of circulation is getting uniform along the span as may be expected. On the contrary, if the chord of an aerofoil is getting very large, the

⁽¹⁾ Fuchs and Hopf: 'Aerodynamik' (1922) p. 63.

⁽²⁾ Fuchs, *loc. cit.* p. 113.

expression (5) gives $\frac{1}{2}$ as the limiting value for m , which agrees with the result obtained by Betz.⁽¹⁾

For intermediate cases the comparison is made in Fig. 1 between the results obtained from (4) and (5) and those by Fuchs⁽²⁾ and Trefftz,⁽³⁾ which have been obtained by more tedious methods. The agreement of the results is quite good and seems to be consistent with the view that the lift distribution may be expressed in a simple form as given by expression (4), at least within the limit of a good first approximation.

Further Results derived.

The lift of an elementary section of an aerofoil can be obtained from the circulation round it by Kutta-Joukowski's equation⁽⁴⁾

$$dA = \rho V \Gamma dy,$$

where dA denotes the lift of the elementary aerofoil section. Then the total lift is obtained very simply by evaluating the integral

$$\begin{aligned} A &= \rho V b \int_{-1}^1 \Gamma d\eta, \\ \text{or} \quad &= 2\rho V b \Gamma_0 \int_0^1 (1-\eta^2)^m d\eta. \end{aligned}$$

The integral can be easily evaluated in terms of Gauss' Π function, namely

$$A = \rho V b \Gamma_0 \sqrt{\pi} \frac{\Pi(m)}{\Pi\left(m + \frac{1}{2}\right)}, \quad (6)^{(5)}$$

where $\Pi(a)$ denotes Gauss' Π function.

The lift coefficient of the aerofoil is, therefore,

$$c_a = \frac{\sqrt{\pi}}{2} \frac{\Pi(m)}{2\Pi\left(m + \frac{1}{2}\right)} c_{a0}, \quad (6')$$

where c_{a0} is the lift coefficient of the aerofoil section at the middle of the span.

Betz⁽⁶⁾ has given in his paper the calculated value of the ratio of the mean circulation Γ_m over the aerofoil to the circulation round the middle section, namely the value of Γ_m/Γ_0 or, it is the same thing, of c_a/c_{a0} . The value of the same ratio

(1) Betz, *loc. cit.*

(2) Fuchs, *loc. cit.*

(3) Trefftz, *loc. cit.*

(4) Fuchs and Hopf, *loc. cit.*

(5) Jahnke-Emde: 'Funktionstafeln mit Formeln und Kurven.' p. 28.

(6) Betz, *loc. cit.*

has been calculated by means of formula (6') and it is compared with that due to Betz in Table I. The numerical value of Gauss' Π function can easily be obtained from any mathematical table.⁽¹⁾

Table I. Value of Γ_m/Γ_0 .

L	m	Betz	Wada	L	m	Betz	Wada
1	0.400	0.820	0.817	4	0.250	0.876	0.875
2	0.333	0.844	0.842	9	0.154	0.916	0.916

The agreement is quite satisfactory.

The induced velocity w_0 at the middle section of the aerofoil can be determined by evaluating the integral⁽²⁾

$$\begin{aligned}
 w_0 &= -\frac{1}{4\pi b} \int_{-1}^1 \frac{d\Gamma}{d\eta} \frac{d\eta}{\eta}, \\
 &= \frac{m\Gamma_0}{\pi b} \int_{-1}^1 (1-\eta^2)^{m-1} d\eta, \\
 &= \frac{m\Gamma_0}{2b\sqrt{\pi}} \left(m + \frac{1}{2}\right) \frac{\Pi(m)}{\Pi\left(m + \frac{1}{2}\right)}. \quad (7)
 \end{aligned}$$

From the fundamental equation (2')

$$\Gamma_0 = \Gamma_\infty - \pi t w_0,$$

whence

$$\Gamma_\infty = \left\{1 + \frac{t}{2b} \left(m + \frac{1}{2}\right) \frac{\Pi(m)}{\Pi\left(m + \frac{1}{2}\right)}\right\} \Gamma_0. \quad (8)$$

From (6') and (8) the relation between Γ_∞ and Γ_m can be written in the following form:

$$\Gamma_\infty = \left\{1 + \frac{t}{2b} \sqrt{\pi} \left(m + \frac{1}{2}\right) \frac{\Pi(m)}{\Pi\left(m + \frac{1}{2}\right)}\right\} \frac{2}{\sqrt{\pi}} \frac{\Pi\left(m + \frac{1}{2}\right)}{\Pi(m)} \Gamma_m \quad (9)$$

or

$$c_{a\infty} = \left\{1 + \frac{t}{2b} \sqrt{\pi} \left(m + \frac{1}{2}\right) \frac{\Pi(m)}{\Pi\left(m + \frac{1}{2}\right)}\right\} \frac{2}{\sqrt{\pi}} \frac{\Pi\left(m + \frac{1}{2}\right)}{\Pi(m)} c_a \quad (9')$$

⁽¹⁾ Jahnke-Emde, *loc. cit.* p. 31.

⁽²⁾ Glauert, *loc. cit.* p. 134.

The numerical value of $c_a/c_{a\infty}$ has been calculated by Betz⁽¹⁾ and also it has been determined by (9'). Table II gives the comparison between the two results.

Table II. Value of $c_a/c_{a\infty}$.

L	m	$\frac{2b}{\pi t}$	Betz	Wada
1	0.400	0.5	0.427	0.422
2	0.333	1.0	0.588	0.582
4	0.250	2.0	0.728	0.725
9	0.154	4.5	0.847	0.845

The fair agreement seems again provides confirmation of the assumption.

The Transformation Formula for the Aspect Ratio.

From the relation, which has been developed in the previous section, it is possible to establish the formula for the transformation of aspect ratio.

In formulas (9) and (9') the left-side member is independent of the value of the aspect ratio, being a function of the geometrical angle of incidence only. Hence if the aspect ratio were reduced from $1/\lambda_1$ to $1/\lambda_2$, the relation of the lift coefficients at a given value of the geometrical angle of incidence can easily be obtained from (9') in the following form:

$$c_{a2} = \frac{\frac{\Pi\left(m_1 + \frac{1}{2}\right)}{\Pi(m_1)} + \sqrt{\pi}\lambda_1\left(m_1 + \frac{1}{2}\right)}{\frac{\Pi\left(m_2 + \frac{1}{2}\right)}{\Pi(m_2)} + \sqrt{\pi}\lambda_2\left(m_2 + \frac{1}{2}\right)} c_{a1}, \quad (10)$$

where c_{a1} and c_{a2} denote the lift coefficients corresponding to aspect ratios $1/\lambda_1$ and $1/\lambda_2$ respectively.

Having obtained the formula of the transformation of aspect ratio, it has been applied in transforming the result of experiment, which was carried out on an aerofoil of the aspect ratio unity, to the aspect ratio five. The transformed value is compared in Table III and Fig. 2 with the result of experiment made on the same aerofoil section but of the aspect ratio five. The numerical values have been taken from the result of experiments carried out at the Aerodynamical Research Institute at Göttingen.⁽²⁾

⁽¹⁾ Betz, *loc. cit.*

⁽²⁾ Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, I. Lieferung (1923), p. 53.

The result is also compared with what has been obtained under the assumption of an elliptic distribution of lift. From Fig. 2 it may be seen that formula (10) gives much better agreement than that obtained with the usual assumption of an elliptic loading.

It may be remarked that the assumption of an elliptic loading gives rise to a constant induced velocity along the span of the aerofoil. Therefore the transformation of the aspect ratio can be effected by adjusting the angle of incidence for the same value of the lift coefficient. But in general the induced velocity is variable along the span and in the transformation the lift coefficient must be corrected for the same value of the geometrical angle of incidence.

Table III. Transformation of Aspect Ratio.

Experimental result. Aspect ratio 5		Experimental result. Aspect ratio 1		c_a transformed to aspect ratio 5 by (10)	α° corrected for aspect ratio 5 by the assumption of elliptic loading
angle of incidence α°	c_a	angle of incidence α°	c_a		
-8.9	-0.274	-9.0	-0.088	-0.185	-7.7
-6.0	-0.078	-6.0	-0.009	-0.019	-5.9
-4.5	0.032	-4.5	0.036	0.076	-5.0
-3.1	0.137	-3.0	0.075	0.158	-4.1
-1.6	0.229	-1.5	0.122	0.256	-3.3
-0.1	0.326	0	0.154	0.324	-2.2
1.3	0.432	1.5	0.203	0.426	-1.5
2.8	0.537	3.0	0.248	0.521	-0.8
4.3	0.635	4.5	0.295	0.620	0.2
5.7	0.734	6.0	0.353	0.742	0.9
8.7	0.939	9.0	0.450	0.946	2.4
11.6	1.091	12.0	0.572	1.201	3.7
14.6	1.154	15.0	0.688	1.446	5.0
17.6	1.132	18.0	0.820	1.722	6.0

The Wind Tunnel Interference.

The experiment in wind tunnel is naturally performed within a limited space of air flow. The effect of the boundary on the measured values of lift and drag experienced by an aerofoil is quite appreciable and it has been known that corrections must be applied to wind tunnel results before they can be applied to free air conditions. It is a usual procedure to assume an elliptic loading for the lift distribution of an aerofoil. It is the object of the present section to extend these results when the lift distribution is such as given by expression (4).

The problem shall be treated as two dimensional, assuming the trailing vortices

were extending from plus to minus infinity.

Dealing with the case of the wind tunnel with a circular jet, or so called Göttingen type, the boundary condition to be satisfied is that the pressure is constant over the surface of the jet. It can easily be shown that, in such case, the system of the trailing vortices must be so compensated to give the circumferencial velocity at the boundary is zero. This is done by introducing for every trailing vortex an image vortex of equal strength and sign at the inverse point with respect to the circle corresponding to the boundary of the jet.

The induced downward velocity at y_1 (Fig. 3) due to the image vortices at the inverse points y_A and y_B , corresponding to the trailing vortices at $\pm y$, is

$$\begin{aligned} dw_{y_1} &= \frac{1}{4\pi} \frac{d\Gamma}{dy} \left(\frac{1}{y_1 - y_B} - \frac{1}{y_1 - y_A} \right) dy \\ &= -\frac{d\Gamma}{dy} \frac{y}{2\pi R^2} \left(\frac{1}{1 - \xi^2} \right) dy, \end{aligned}$$

where $\xi = y_1 y / R^2$, denoting by R the radius of the circle.

From the assumption for Γ as given in (4) follows:

$$dw_{y_1} = \frac{mb\Gamma_0}{\pi R^2} \frac{\eta^2(1-\eta^2)^{m-1}}{(1-\xi_0^2\eta^2)} d\eta, \quad (11)$$

where $\xi_0 = y_1 b / R^2$.

If c_a denotes the lift coefficient corresponding to free air condition, it follows from (6') and (3)

$$\Gamma_0 = \frac{Vt}{\sqrt{\pi}} \frac{\Pi\left(m + \frac{1}{2}\right)}{\Pi(m)} c_a.$$

Hence

$$dw_{y_1} = c_a \frac{m\Pi\left(m + \frac{1}{2}\right)}{2\sqrt{\pi}\Pi(m)} \frac{FV}{F_0} \frac{\eta^2(1-\eta^2)^{m-1}}{(1-\xi_0^2\eta^2)} d\eta,$$

where F_0 is the cross sectional area of the jet and F is the total area of the aerofoil.

The total sum of the induced velocities at the point y_1 due to the images of the system of the trailing vortices corresponding to the assumed lift distribution may therefore be put in the form:

$$w_{y_1} = c_a \frac{m}{2\sqrt{\pi}} \frac{\Pi\left(m + \frac{1}{2}\right)}{\Pi(m)} \frac{FV}{F_0} \int_0^1 \frac{\eta^2(1-\eta^2)^{m-1}}{(1-\xi_0^2\eta^2)} d\eta. \quad (12)$$

Put for brevity

$$I = \int_0^1 \frac{\eta^2(1-\eta^2)^{m-1}}{(1-\xi_0^2\eta^2)} d\eta.$$

The integrand becomes infinity at $\xi_0^2\eta^2=1$ or at $y_1y/R=1$ and it is to be understood that its principal value has been taken.

The integral I can be evaluated as follows, expanding the denominator into an infinite series,

$$\begin{aligned} I &= \int_0^1 \eta^2(1-\eta^2)^{m-1} \sum_0^\infty \xi_0^{2n} \eta^{2n} d\eta \\ &= \frac{\Pi(m-1)}{2} \sum_0^\infty \frac{\Pi\left(n+\frac{1}{2}\right)}{\Pi\left(m+n+\frac{1}{2}\right)} \xi_0^{2n} \\ &= \frac{\sqrt{\pi}}{4} \frac{\Pi(m-1)}{\Pi\left(m+\frac{1}{2}\right)} \left\{ 1 + \frac{3}{2m+3} \xi_0^2 + \frac{3 \cdot 5}{(2m+3)(2m+5)} \xi_0^4 + \dots \right\} \end{aligned} \quad (13)$$

On substitution into (12) the induced velocity can be put

$$\begin{aligned} w_{y_1} &= c_a \frac{FV}{8F_0} \left\{ 1 + \sum_1^\infty \frac{(2n+1)(2n-1)\dots 5 \cdot 3}{(2m+2n+1)(2m+2n-1)\dots (2m+5)(2m+3)} \xi_0^{2n} \right\}, \\ \text{or} \quad &= c_a \frac{FV}{8F_0} \left\{ 1 + \sum_1^\infty p_n \xi_0^{2n} \right\}, \end{aligned} \quad (14)$$

abbreviating the notation by writing

$$p_n = \frac{(2n+1)(2n-1)\dots 5 \cdot 3}{(2m+2n+1)(2m+2n-1)\dots (2m+5)(2m+3)}.$$

The effect of the induced velocity due to the boundary may be considered to be equivalent to a reduction of the effective angle of incidence at the point by $\Delta\alpha$, which is given by

$$\Delta\alpha = -\frac{w_{y_1}}{V} = -\frac{c_a F}{8F_0} \left(1 + \sum_1^\infty p_n \xi_0^{2n} \right) \quad (15)$$

Let c_{ay_1} and $c_{ay_1}^b$ be the lift coefficients of the aerofoil section at y_1 corresponding to free and limited conditions of air flow. Then $c_{ay_1}^b$, obviously a function of the effective angle of incidence at the point, may be expressed

$$c_{ay_1}^b = c_{ay_1} + \frac{dc_{ay_1}}{d\alpha} \Delta\alpha. \quad (16)$$

But from (4) and (8)

$$\frac{dc_{ay_1}}{d\alpha} = \frac{2\pi}{1 + \frac{t}{2b} \sqrt{\pi} \frac{\Pi(m)}{\Pi\left(m-\frac{1}{2}\right)}} (1-\eta_1^2)^m. \quad (17)$$

On substitution into (16) the following relation is established:

$$c_{ay1}^b = c_{ay1} - \frac{2\pi}{1 + \frac{t}{2b} \sqrt{\pi} \frac{\Pi(m)}{\Pi(m - \frac{1}{2})}} \frac{F}{8F_0} (1 - \eta_1^2)^m \left(1 + \sum_1^\infty p_n \zeta_0^{2n} \right) c_a. \quad (18)$$

The total lift coefficient c_a^b or the lift coefficient of the aerofoil within the limited air flow can be expressed:

$$\begin{aligned} c_a^b &= \int_0^1 c_{ay1}^b d\eta \\ &= c_a \left\{ 1 - \frac{2\pi}{1 + \frac{t}{2b} \sqrt{\pi} \frac{\Pi(m)}{\Pi(m - \frac{1}{2})}} \frac{F}{8F_0} \int_0^1 (1 - \eta^2)^m \left(1 + \sum_1^\infty p_n \zeta^{2n} \eta^{2n} \right) d\eta \right\} \\ &= c_a \left[1 - \frac{2\pi \Pi(m)}{1 + \frac{t}{2b} \sqrt{\pi} \frac{\Pi(m)}{\Pi(m - \frac{1}{2})}} \frac{F}{8F_0} \left\{ \frac{\sqrt{\pi}}{2\Pi(m + \frac{1}{2})} + \sum_1^\infty p_n \frac{\Pi(n - \frac{1}{2})}{2\Pi(m + n + \frac{1}{2})} \zeta^{2n} \right\} \right], \end{aligned} \quad (19)$$

where $\zeta = b/R$ or $\zeta_0 = y_1 b/R^2 = \eta \zeta^2$.

But

$$\begin{aligned} &\sum_1^\infty p_n \zeta^{2n} \frac{\Pi(m) \Pi(n - \frac{1}{2})}{2\Pi(m + n + \frac{1}{2})} \\ &= \frac{\sqrt{\pi} \Pi(m)}{2\Pi(m + \frac{1}{2})} \frac{3}{(2m+3)^2} \left\{ 1 + \frac{3 \cdot 5}{(2m+5)^2} \zeta^4 + \frac{3^2 \cdot 5 \cdot 7}{(2m+5)^2 (2m+7)^2} \zeta^8 + \dots \right\} \end{aligned}$$

Hence from (19)

$$c_a = \frac{c_a^b}{1 - \frac{\pi F}{4F_0} \frac{\phi(\zeta)}{\frac{2}{\sqrt{\pi}} \frac{\Pi(m + \frac{1}{2})}{\Pi(m)} + \frac{t}{b} (m + \frac{1}{2})}}, \quad (20)$$

where $\phi(\zeta)$ denotes the following abbreviation

$$\phi(\zeta) = 1 + \frac{3}{(2m+3)^2} \zeta^4 + \frac{3^2 \cdot 5}{(2m+3)^2 (2m+5)^2} \zeta^8 + \frac{3^3 \cdot 5 \cdot 7}{(2m+3)^2 (2m+5)^2 (2m+7)^2} \zeta^{12} + \dots$$

Hence in order to convert the experimental result of a circular jet wind tunnel

to free air condition the former result must be multiplied by a numerical constant as determined from (20).

Comparison with Experimental Result.

In order to verify the result established in the previous section an experiment has been made under very large diameter-span ratio (b/R).

The test was carried out in the 1.5-meter wind tunnel of Aeronautical Research Institute, Tokyo Imperial University.

The aerofoil tested was Göttingen 420 section and of the following dimensions:

Span	143.3 cm.
Chord	30.0 cm.
b/R	0.956

It should be noted that the span is abnormally large compared with the diameter of the wind tunnel.

The lift coefficient c_a^b as determined from the test is tabulated in the second column of Table IV and in Fig. 4. The values reduced to free air condition by means of the correction formula (20) are given in the third column of Table IV and in Fig. 4.

The result is compared in Fig. 4 with that obtained at Göttingen⁽²⁾, which has been deduced from the test performed with low value of span-diameter ratio, namely, $b/R=0.44$. The agreement is quite satisfactory to justify the assumption (4).

In Fig. 4 is also shown the lift coefficient as deduced from the experiment under the assumption of an elliptic loading. It will be seen that the assumption seems to fail in such an extreme case.

In conclusion the writer wishes to acknowledge his indebtedness to his assistants Mr. K. Toyoda and Mr. T. Fukui for their labour in preparing experiments and calculations.

Table IV.

α°	-9	-6	-3	0	3	6	8	10	12	14	16	18	20	22
c_a^b	.001	.179	.357	.529	.702	.859	.968	1.069	1.143	1.182	1.231	1.266	1.284	1.272
c_a	.002	.215	.429	.635	.843	1.031	1.162	1.281	1.372	1.420	1.478	1.519	1.541	1.528

The Wind Tunnel Department,
The Aeronautical Research Institute,
Tokyo Imperial University.
October 1931.

⁽¹⁾ Report of the Aeronautical Research Institute, Tokyo Imperial University, Vol. V, No. 66.

⁽²⁾ Göttingen Ergebnisse. *loc. cit.* p. 96.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

討 論

○會長代理(藤島範平君) 唯今の御講演に對して御質問又は御意見のある方は御述べを願ひます。

○山本武藏君 實驗に用ひた span と wind tunnel の dia. との ratio が 0.956 であるとしめすと、trailing vortex は外に半分位出はしませんか。

○和田小六君 出ません。

○山本武藏君 span の中央部にて induced velocity w があるとする、それは vortex の centre から遠い所での流體運動の爲めである事勿論であるが、此 induced velocity を生ずる流體運動中其半分近くが風洞の外まで出て居る様に思ひますが。

○和田小六君 流れの境界に於て pressure が一定であると云ふ條件で boundary 自身の形は adjustable ですが、外に出るとは考へられないと思ひます。

○山本武藏君 一寸考へると自由にあるものと constrained flow の状態にあるものを考へて見ると其間に非常の差が起るべきが當然の様ですが、いかゞですか。

○和田小六君 其 effect が即ち境界修正となりますので、結果から見ると(4)式の方が從來の橢圓型分布より良いと思はれます。

○會長代理(藤島範平君) 他に別段御質問なくば私から一言御禮を申述べさせて戴きます。和田博士は從來よりの正員であります、實は先般造船協會の定款を改正しまして航空を本會の目的の一つに加へて、造船造機と並べて目的としました。從來からも航空關係の御方に御講演を願ひましたが、卑近な例を申す様ですが恰も借家で講演される様な状態でありましたが、今後は自邸で講演される様な譯でありまして、本講演は定款改正以來其意味での最初の講演でありまして、其點に於て有意義のものと思ひます。此後益々航空關係の御發表あらん事を願ひます。

和田君の此の有益なる論文を御發表下さつた事に對しまして、諸君と共に拍手して、感謝の意を表し度いと思ひます。(一同拍手)