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Damping Resistances in Rolling, Pitching, and Vibration of a Ship in her Motion-ahead.

By **Katsutada Sezawa**, *KH., R.H., Member*,

Tokyo Imperial University⁽¹⁾.

Abstract.

In his previous paper, the author obtained the coefficient of vibration damping of a plate that is parallel to a water stream, and movable normal to the same stream, with a theory regarding the dynamic damping of the vibrations of a ship in motion-ahead. Although the vibration damping of any ship increases almost linearly with speed, since the vibrational force usually varies as the square of the same speed, the amplitudes of the ship vibrations are apparently augmented with speed increase. Since, however, no data on vibration test of an actual ship was given in the previous paper, there has remained some doubt as to whether or not the damping condition in a full-sized ship would obey the law that the author formulated.

In the present paper, with a view to confirming the results given in the previous paper, the author directs his attention to the results of the rolling experiments of the "Revenge" that was conducted about fifty years ago. Based on the data of his model experiments with a plate vibrating in a circulation water tank that were published in his previous paper and also reexamined in this one, he predicted the damping coefficient of the rolling of H.M.S. "Revenge" in her motion-ahead (free of the actual coefficient of the vessel itself), from which he finds that, notwithstanding the large differences between the conditions of the vibrating model plate and the conditions that attended the rolling experiments of the "Revenge," in oscillatory frequency as well as in displacement amplitude, the damping coefficient of rolling of the "Revenge" predicted from the data of model experiments fairly agrees with the same coefficient that had been actually observed. The author thus concluded that the data of his model experiments can straightway be applied to the vibrations of a full-sized ship—a ship with the frequencies and amplitudes within the range of those model plates in the author's experiments. In extending his investigation to the case of the pitching motion of a ship, the reason for the pitching of a ship in her motion-ahead in a calm sea being damped more quickly than in her stand-still condition was ascertained. Since, as a matter of fact, the problem is merely qualitative, it was impossible for the author to give accurate values of the dynamic damping coefficients under consideration.

⁽¹⁾ Upon the author's appointment by the government, in 1925, to engage in researches in seismology (and aeronautics), he found it impossible to interest himself in naval engineering to the extent that he would have liked to have done, with the result that the present paper, together with others he has recently published, on naval engineering, represent merely work that was done in his leisure in the way of recreation as also to repay his scientific debt to naval engineering, in which subject he once held a teaching post. For this reason he hopes that the reader will appreciate the circumstances under which these papers were written and consider mainly the essentials of the papers.

1. Introduction.

In the previous paper⁽²⁾, it was shown that the vibration damping of a ship increases with increase in speed of her motion-ahead, for which purpose a circulation water tank was used and the change in the coefficient of that vibration damping with speed was qualitatively ascertained. Upon reading the finished paper, the question arose whether or not in an actual ship, the vibration damping could really increase with increase in the speed of the vessel. Although the vibration amplitude of any ship may increase, apparently, with speed increase, since the vibrational force usually varies as the square of the speed, it is impossible to conclude that the vibration damping cannot increase with speed. Since the vibration damping that we have ascertained, increases almost linearly with speed, such a change in the vibration amplitude as is slower than the square of the ship's speed, is nothing more than an indication of the real existence of the vibration damping in question.

In the previous paper, it was stated that, since the condition that the vibration damping increases with the ship's speed is, as a matter of fact, well established, even though the Reynolds number is not the same, the vibration damping of an actual ship, as predicted from the result of our model experiments, does not differ much from real vibration damping, not at any rate in the order of magnitude. Since, however, there is no experimental data obtained specially for showing the vibration damping of a ship in her motion-ahead, it was not possible to examine and say to what case the problem could be extended. In the previous paper hope was expressed of extending the idea of the present vibration damping to the rolling and pitching of a ship in her motion-ahead, when Dr. Oota kindly called my attention to a paper⁽³⁾ of John's dealing with a similar problem. In examining that paper, it was a pleasure to find that White's result⁽⁴⁾ from the "Revenge" experiments could be applied directly to our investigation. Using the damping coefficient deduced from the data of our model experiments, without regard to Reynolds number and other dynamical conditions, it was nevertheless possible to ascertain that the increase of damping in rolling of the "Revenge" in her motion-ahead originated,

(2) K. Sezawa and W. Watanabe, "The Vibration Damping of a Ship in her Moving State," *Journ. Soc. Nav. Arch., Japan*, **63**, (1938), 156-170.

(3) A.W. Johns, "The Effect of Motion ahead on the Rolling of Ships," *T.I.N.A.*, **47** (1905), 186-198.

(4) Sir W. White, "Notes on Further Experiments with First-class Battleships," *T.I.N.A.*, **36** (1895), 127-150.

at least qualitatively, from the same cause as that in the vibration damping shown previously, from which there is the likelihood that our previous result is not only available for estimating the vibration damping of a full sized ship, but also could be extended to that of the case of rolling or pitching of the same ship.

2. Experimental Data for the Damping Coefficient.

From the vibration experiments with a rectangular plate, 40 cm long and 20 cm wide⁽⁵⁾, placed lengthwise in a circulation water tank, we obtained the damping force⁽⁶⁾

$$\{|k_0| + |k_1| V\} \dot{z}, \quad (1)$$

acting per unit area of a plate vibrating with velocity $\dot{z}$ normal to the direction of the stream, V being the velocity of that stream. If V , $\dot{z}$ be taken in cm/sec, the values of $|k_0|$, $|k_1|$ are

$$|k_0| = 8.35 \text{ gr. mass/cm}^2 \cdot \text{sec}, \quad |k_1| = 0.358 \text{ gr. mass/cm}^3, \quad (2)^{(7)}$$

respectively. The damping coefficient $|k_0|$ is not a dynamical coefficient, and the above value of $|k_0|$ depends particularly on the model condition of our previous experiments, while $|k_1|$, on the other hand, participates in the dynamic damping in every case. Since, in the case of an actual ship, there are such dampings as are caused by water friction, generation of pressure waves, generation of surface waves, and structural damping force; the static damping, that is, the damping that is present even in the case of a ship at anchor, should be considered separately⁽⁸⁾.

Although in the previous paper, we predicted vibration damping of an actual ship in her motion-ahead, yet for comparing its order of magnitude with that in any other case, the problem in question will now be briefly reviewed. When a ship of the form of parallelopiped is in flexural vibration, the equation of vibration of any cross section is

$$\rho A \frac{d^2 T}{dt^2} + b |k_1| V \frac{dT}{dt} + n^2 T = 0, \quad (3)$$

where $n^2 = m^4 EI$, the damping coefficient K_1 being $b |k_1| V / \rho A$. Since $\rho A / b$ is the

⁽⁵⁾ This relatively long plate was used for avoiding, as much as possible, the effect of trailing edge on the damping force.

⁽⁶⁾ *loc. cit.* (2).

⁽⁷⁾ There were small errors in the numerical calculation in the previous paper, though their effects on the vibration problem are very slight, qualitatively speaking.

⁽⁸⁾ K. Sezawa and W. Watanabe, "Damping Forces in Vibrations of a Ship," *Journ. Soc. Nav. Arch., Japan*, 591 (936), 99-120.

ship's mass, including the inertia mass of water, per unit area of the ship's bottom, $|k_1|V\dot{z}$ just corresponds to the damping force acting per unit area of the same bottom. Using the value of $|k_1|$ shown already, the dynamic damping coefficient K_1 at different speeds assumes the following values

Speed of the ship	$V_1(\text{km/h})=0$	3.6	18	36
Damping coefficient	$K_1(\text{sec}^{-1})=0$	0.0179	0.0896	0.179

J. Lockwood Taylor's experimental result⁽⁹⁾ shows that the damping of a certain ship in still water obeyed the law $e^{-0.066t}$, that is to say, $K_1=0.066$ (sec^{-1}), from which it is possible to conclude that vibration damping in still water is of the same order of magnitude as that of dynamic damping with a ship's speed of 6 or 7 knots.

As already shown in the previous paper, the value of the coefficient of dynamic damping $|k_1|$ was determined for a low water velocity range, and furthermore for the case of a relatively small plate. But, if change in dynamic damping with Reynolds number were to obey such a law as that resembling the change in frictional resistance with the same Reynolds number, it would then be possible for model experiments to apply to the case of a full-scale ship, at any rate in the qualitative sense. Since, as a matter of fact, the water velocity in our model experiments was by no means exceptionally low compared with the speed of an actual ship, it is likely that our experimental value could be extended to other cases. It should however be borne in mind that the vibration damping in our case, is not of the kind as frictional resistance of the ship's hull in her steady motion, but results from the momentum imparted by the relative motion of the hull normal to a uniform stream.

3. Further Experiments for Ascertaining the Effect of the Inertia Mass of Water on the Damping Coefficient.

Although in the previous paper the existence of the inertia mass of water vibrating with the model was examined, since the relation between the quantity of inertia mass and the dynamic damping of the same model has not yet been ascertained, the following additional experiments were made with the kind assistance of Messrs. W. Watanabe and Z. Yamada.

The same model as that used in the previous study was immersed to a depth of 20 cm from the free surface of the circulation water tank. The distance d of this plate from the bottom of the tank was 16 cm in one case and 10.7 cm in the

⁽⁹⁾ J. Lockwood Taylor, *T.I.N.A.*, 70 (1930), 162-193.

other. The methods of experiments were virtually similar to that described in the previous paper.

In every case, experiments in both free and forced vibrations were made for different water velocities, namely, $V=0, 0.50$ m/s, 0.80 m/s, 1.18 m/s, respectively, in the first case ($d=16$ cm) and $0, 0.50$ m/s, 0.80 m/s, 1.00 m/s, respectively, in the second ($d=10.7$ cm). The results of observation for forced vibrations in the two cases are shown in Figs. 1, 2. If θ is the amplitude of oscillation, N the vibrational frequency of the coil, f the maximum force due the current in the same coil, N_0 the resonance frequency of the mechanical system under experiment, C a constant, and K the damping coefficient of the same system, then the formula for estimating the damping coefficients of the vibration is expressed by

$$\theta = \frac{f \cos \{2\pi Nt - \tan^{-1} NK/2\pi(N_0^2 - N^2)\}}{C \{N_0^2 - N^2\}^2 + N^2 K^2 / 4\pi^2}^{\frac{1}{2}}. \quad (4)$$

Analysing the results in Figs. 1, 2 by means of (4), we get

(i) $K(\text{sec}^{-1}) = 0.81 + 0.026 V(\text{cm/sec})$ for the case $d=16$ cm,

(ii) $K(\text{sec}^{-1}) = 0.82 + 0.023 V(\text{cm/sec})$ for the case $d=10.7$ cm.

On the other hand, from the results of free vibration experiment, we get

(i) $K(\text{sec}^{-1}) = 1.00 + 0.0250 V(\text{cm/sec})$ for the case $d=16$ cm,

(ii) $K(\text{sec}^{-1}) = 1.02 + 0.0246 V(\text{cm/sec})$ for the case $d=10.7$ cm.

The part corresponding to the free vibration in the equation of motion is

$$(I_0 + I')\ddot{\theta} + (k_0 + k_1 V)\dot{\theta} + c_0 \theta = 0, \quad (5)$$

from which the damping coefficient of the model system becomes

$$K = (k_0 + k_1 V) / 2(I_0 + I'). \quad (6)$$

Since in the present case $c_0 = 46.8 \cdot 69 \cdot 10^7$ dyne-cm/rad. rotation and $N_0 = 4.1/\text{sec}$ (nearly), we get

$$I_0 + I' = c_0 / (2\pi N_0)^2 = 6.09 \cdot 10^6 \text{ gr.mass. cm}^2,$$

from which we have

(i) For the free vibration in case $d=16$ cm,

$$k_0 = 12.18 \cdot 10^6 \text{ gr.mass. cm}^2/\text{sec}, \quad k_1 = 3.06 \cdot 10^5 \text{ gr.mass. cm},$$

and for the forced vibration in the same case

$$k_0 = 9.85 \cdot 10^6 \text{ gr.mass. cm}^2/\text{sec}, \quad k_1 = 3.17 \cdot 10^5 \text{ gr.mass. cm};$$

(ii) For the free vibration in case $d=10.7$ cm,

$$k_0 = 12.4 \cdot 10^6 \text{ gr.mass. cm}^2/\text{sec}, \quad k_1 = 3.00 \cdot 10^5 \text{ gr.mass. cm},$$

and for the forced vibration in the same case

$$k_0 = 9.97 \cdot 10^6 \text{ gr.mass. cm}^2/\text{sec}, \quad k_1 = 2.80 \cdot 10^5 \text{ gr.mass. cm}.$$

Fig. 1. Resonance Curves for the Case that the Distance d of the Plate from the Bottom of the Tank is 16 cm.

Fig. 2. Resonance Curves for the Case that the Distance d of the Plate from the Bottom of the Tank is 10.7 cm.

In order to obtain the values of the coefficients of dynamic damping $|k_0|$, $|k_1|$, we use the relations

$$|k_0| = 3k_0/2bL^3, \quad |k_1| = 3k_1/2bL^3. \quad (7)$$

Substituting the values of k_0 , k_1 (the respective mean values of k_0 , k_1 obtained from free vibration and those obtained from forced vibration) in the relations (7) and putting $b=20$ cm, $L=40$ cm, we get the following results.

(i) When the distance between the plate and the bottom of the tank is $d=16$ cm,

$$|k_0| = 12.9 \text{ gr.mass/cm}^2 \cdot \text{sec}, \quad |k_1| = 0.362 \text{ gr.mass/cm}^3.$$

(ii) When the same distance is $d=10.7$ cm,

$$|k_0| = 13.09 \text{ gr.mass/cm}^2 \cdot \text{sec}, \quad |k_1| = 0.339 \text{ gr.mass/cm}^3.$$

From these results it will be seen that even should the condition of the inertia mass of the water be considerably changed, the change in the coefficient of dynamic damping is not overgreat. Since, furthermore, the values of $|k_1|$ given above do not differ much from the value shown in the previous paper, we shall use the previous results throughout in subsequent calculations.

4. Damping Resistance in the Rolling of a Ship in Motion-ahead.

The linear velocity in the rolling of a ship does not greatly exceed that in the vibration of the same ship. If it is possible to fit our experimental result from the

vibration of a small plate to such a case as that of rolling, then the experimental result in question should also be applicable to the case of the vibration of a full-sized ship.

The equation of rolling motion of a ship may be expressed in the form

$$I\ddot{\theta} + Mmg\theta + \{|k_0| + |k_1| V\} \Phi \dot{\theta} = 0, \quad (8)$$

where I , M , m , g , V are the mass moment inertia of the ship about the axis of rolling, the mass of the ship, the metacentric height, the acceleration due to gravity, and the ship's speed respectively. For a ship of rectangular cross-section, the value of Φ is

$$\Phi = \alpha \left\{ \frac{B^3 L_1}{12} + \frac{2(d_1^3 + d_2^3) L_2}{3} \right\} + 4\beta A R_1^2, \quad (9)$$

in which B is the breadth of the flat bottom part, L_1 the length of the same part; d_1 , d_2 are the depths of immersion of the wall-sided part above and below the level surface through the rolling axis; L_2 is the length of the same part, A the area of the outer or inner surface of every bilge keel, and R_1 the distance of the bilge keel from the rolling axis.

In the case of a rounded form of hull, Φ in (9) can be put in the form

$$\Phi = 4\beta A R_1^2. \quad (10)$$

Even in this case, the whole area of the bilge keel is not so effective as in the case of our model experiments, namely, the case of a plate vibrating between two side walls of the circulation tank. From this consideration, we shall take $\beta \approx 1/2$.

With a view to comparing our results with those of the "Revenge" experiments, we shall now use the data in White's paper⁽¹⁰⁾. The fourth rolling experiment of H.M.S. "Revenge" was conducted on February 13, 1895. The condition of this ship then was as follows: mean draught 27'11 $\frac{1}{4}$ ", displacement 14,620 tons, metacentric height 3.86', mean period of single (half) swing 7.75 sec ($= T_1$), weight of coal on board 1,782 tons, number of men for exciting free rolling 370, every bilge keel 200' long 3' deep, distance of every bilge keel from the centre of gravity 40 $\frac{1}{4}$ '. The midship of the same ship was fairly rounded, the effect of the wall and the bottom on the damping of rolling being thus neglected.

From the foregoing data, we have

$$\begin{aligned} I &= T_1^2 M g m / \pi^2 = 7.75^2 \cdot 1.46 \cdot 10^{10} \cdot 980 \cdot 117 / \pi^2 \\ &= 10 \cdot 17 \cdot 10^{15} \text{ gr. mass. cm}^2, \\ \Phi &= 4\beta A R_1^2 = 4.0 \cdot 5.5 \cdot 56 \cdot 10^5 \cdot 1219^2 = 1.65 \cdot 10^{12} \text{ cm}^4, \end{aligned}$$

⁽¹⁰⁾ Sir W. White, *loc. cit.* (4).

so that the damping coefficient of rolling due to dynamic damping alone is

$$K_1 = \frac{\Phi |k_1| V}{2I} = \frac{1.65 \cdot 10^{12} \cdot 0.358 V}{2 \cdot 10 \cdot 17 \cdot 10^{15}} \\ = \frac{0.0290 V}{10^3} \text{ sec}^{-1}, \quad (V \text{ in cm/sec}) \quad (11)$$

from which the damping coefficients of rolling at $V=10$ knots ($=514.9$ cm/sec) and 12 knots ($=617$ cm/sec) become

$$K_1 = 0.0149 \text{ sec}^{-1}, \quad K_1 = 0.0179 \text{ sec}^{-1}, \quad (12)$$

respectively.

5. Comparison of the Above Results with the Data of the "Revenge" Rolling Experiments.

White's paper gave the data of free rolling of the "Revenge" in the condition shown in the preceding section. Starting from the initial angle of roll, namely, 5 degrees, the inclining angles after successive finite numbers of swings for various ship speeds became

Case	(A)	(B)	(C)
Ship speed	Nil	10 knots	12 knots
After 4 (half) swings	2.95 deg.	2.35 deg.	2.2 deg.
.....8	1.95	1.12	1.05
.....12.....	1.45	0.55	0.45
.....16.....	1.15	0.20	0.25

It is possible to evaluate from this table the damping coefficient of every case. Let the damping coefficients in the respective cases be $K_0 + K_1$; K_0, K_1 being the parts contributed from the damping in still water and dynamic damping respectively. The mean values of the damping in the three cases then become

Case (A)	Case (B)	Case (C)
No speed	10 knots	12 knots
$K_0 = 0.0142 \text{ sec}^{-1}$	$K_0 + K_1 = 0.0244 \text{ sec}^{-1}$	$K_0 + K_1 = 0.0258 \text{ sec}^{-1}$

If the equation of rolling motion could be put in the same form as in (8), the form of the additive type, $K_0 + K_1$, of the damping coefficient of the rolling, is reasonable. From the above result, it is now possible to assume that the coefficients of dynamic damping in rolling of the "Revenge" at 10 knots and 12 knots were

$$K_1 = 0.0102 \text{ sec}^{-1}, \quad K_1 = 0.0114 \text{ sec}^{-1}, \quad (13)$$

respectively.

Comparing (12) with (13), it will be seen that the value of the dynamic damping coefficient of rolling of the "Revenge" that was predicted from the result of our previous model experiments is in the same order of magnitude as the one that is deduced directly from the data gained in the actual rolling experiments of the same ship. Even if it is assumed that β =unity, in lieu of $1/2$, the estimated value of K_1 is not far from that in actual experiments, not at any rate, in the order of its magnitude. If, furthermore, the value of β were assumed to be somewhat less than $1/2$, it would be possible to get the estimated value of K_1 almost equal to its experimental value, although such an agreement in the two values in K_1 is of no theoretical importance.

At all events, that the value of the dynamic damping coefficient as predicted from an idealized model experiment should have the same order of magnitude as the observed value in the rolling of an actual ship, is interesting. It now seems that the value of the dynamic damping coefficient found in model experiments is applicable to the vibration damping of a ship in her motion-ahead with somewhat less error and, furthermore, to the case of damping in the pitching of the same moving ship.

5. Damping Resistance in the Pitching of a Ship in Motion-ahead.

It is an actually observable fact that the pitching of a ship in her motion-ahead condition is damped more quickly than in her stand-still condition.

Taking damping in its dynamical sense only, the equation of pitching motion of a ship is expressed by

$$I'\ddot{\phi} + Mm'g\phi + |k_1| V\mathcal{P}\dot{\phi} = 0, \quad (14)$$

where I' , m' are the mass moments of inertia of the ship about her pitching axis and the longitudinal metacentric height respectively. If $l_1 + l_2$ be the length of the flat bottom part with varying breadth b ; $\mathcal{P}$ can be put $\gamma \int_{-l_1}^{l_2} bx^2 dx$, γ being a constant less than unity. If b does not vary along the ship's length and $\gamma=1$, we get $\mathcal{P} = b(l_1^3 + l_2^3)/3$.

Although the values of I' , m' in the case of the "Revenge" are not known, in order to get the approximate damping coefficient, we write

$$\begin{aligned} b &= 70' (= 212 \text{ cm}), \quad l_1 = l_2 = 100' (= 3,030 \text{ cm}), \\ I' &= 14,600^T \cdot 50'^2 = 0.337 \cdot 10^{17} \text{ gr.mass. cm}^2. \end{aligned}$$

Then

$$|k_1|\mathcal{F} = |k_1| \frac{b}{3} (l_1^3 + l_2^3) = 14 \cdot 0 \cdot 10^{12} \text{ gr.mass. cm,}$$

$|k_1|$ being $0 \cdot 358 \text{ gr.mass/cm}^3$, so that the damping coefficient of pitching due to dynamic damping alone is

$$K_1 = \frac{\mathcal{F}|k_1| V}{2I'} = \frac{14 \cdot 0 \cdot 10^{12} V}{2 \cdot 0 \cdot 337 \cdot 10^{17}} \text{ sec}^{-1}, \quad (V \text{ in cm/sec}) \quad (15)$$

from which the damping coefficients of pitching at $V=10$ knots and 12 knots become

$$K_1 = 0 \cdot 107 \text{ sec}^{-1}, \quad K_1 = 0 \cdot 128 \text{ sec}^{-1}, \quad (16)$$

respectively.

Comparing (16) with (12) it will be seen that the dynamic damping coefficient in pitching motion is much larger than that in rolling motion. It is obvious that the pitching motion of an actual ship in motion-ahead in a calm sea is generally very quickly damped,—a fact agreeing well with our present theory. The condition that the pitching of an actual ship is rather marked in motion-ahead in rough weather merely results from the excitation due to large waves, but not from the smallness of the dynamic damping.

7. Damping Resistance in the Vibration of a Ship in Motion-ahead.

This problem was dealt with in the previous paper and also in Section 2 of the present paper. But, for ascertaining its nature more clearly, we shall now show the treatment of the problem and exemplify it by using the data of the "Revenge."

If we assume that the ship is in the form of a uniform bar, the equation of the flexural vibratory motion is

$$\rho A \frac{\partial^2 y}{\partial t^2} + \{ |k_0| + |k_1| V \} b \frac{\partial y}{\partial t} + EI \frac{\partial^4 y}{\partial x^4} = 0. \quad (17)$$

Putting $y = YT$, the equation transforms to the following pair of expressions

$$\rho A \frac{d^2 T}{dt^2} + \{ |k_0| + |k_1| V \} b \frac{dT}{dt} + n^2 T = 0, \quad (3')$$

$$EI \frac{d^4 Y}{dx^4} = n^2 Y. \quad (18)$$

From (3') the damping coefficient due to dynamic damping alone assumes the form

$$K_1 = \frac{b}{2\rho A} |k_1| V, \quad (19)$$

where b is the breadth of the bar and ρA the mass per unit length of the same bar. If, on the other hand, M be the mass of the ship, the above damping coefficient may be replaced by

$$K_1 = \frac{b_1 L_1}{2M} |k_1| V,$$

where L_1 , b_1 are the effective length and breadth, along both of which the dynamic damping force of water is imparted.

Let us assume that

$$L_1 = 200' (= 6,060 \text{ cm}), \quad b_1 = 70' (= 2,120 \text{ cm}),$$

$$M = 14,600 \text{ tons} = 1.46 \cdot 10^{10} \text{ gr.mass}, \quad |k_1| = 0.358 \text{ gr.mass/cm}^3,$$

then

$$K_1 = \frac{1.57}{10^4} V \text{ sec}^{-1}, \quad (V \text{ in cm/sec}) \quad (20)$$

from which it is possible to guess the damping coefficient of vibration due to dynamic damping alone, some examples being shown below

$V(\text{kts}) = 1.94$	9.7	10	12	19.4
$K_1(\text{sec}^{-1}) = 0.0157$	0.0786	0.0810	0.0973	0.157

As already shown, the damping coefficient K_0 of the ship's vibration in her stand-still condition is nearly equal to that due to dynamic damping at her speed of 6 or 7 knots. It should be added that in this case, too, the actual damping coefficient of the vibration is $K_0 + K_1$.

Since our present result shows the damping force only qualitatively, the value $|k_1| = 0.358 \text{ gr.mass/cm}^3$ therefore represents its coefficient in the order of magnitude.

Since vibrational forces due to the engines or propellers usually increase as the square of the shaft revolution, even if the dynamic damping be augmented with the ship's speed, the amplitudes of the ship's vibrations should become larger and larger with increase in speed. Apart from the dynamic damping, there is, besides, evidence that, beyond the resonance condition, the vibration amplitude in forced oscillation tends to decrease as the result of energy conditions, whence, even should the vibrational force increase as the square of the ship's speed, the vibration amplitude at any higher frequency is likely to remain constant apart from the effect of damping resistance, the vibrational acceleration, however, increasing as the square of the ship's speed. It follows then that when the vibration is resisted by the dynamic damping force, the amplitude of the same vibration tends to decrease with

increase in ship speed, the acceleration, on the other hand, quickly increasing with the same increase.

8. Applicability of the Results of Model Experiments to Problems connected with the Dynamic Damping of Ship Vibrations, as Confirmed by the Resistance to Ship's Rolling.

In the previous paper, from the results of model experiments in a circulation water tank, the dynamic damping of the vibrations of a ship in her motion-ahead was predicted. Although, from the nature of things, it is likely that the experimental results could be availed of in the case of the vibration damping of an actual vessel, since the vibration amplitudes of any ship apparently increase with ship speed, some doubt has remained as to whether or not the damping conditions in a full-sized ship would obey the law that we have formulated. Although the condition that the amplitudes generally increase with speed, can be explained by the vibrational forces increasing with shaft revolution, the statement that the order of magnitude of the damping resistance can be applied to any other case, has not yet been confirmed.

On the other hand, the results of rolling experiments with the "Revenge" that were conducted about fifty years ago, have supplied us with a good deal of data for confirming the applicability of the damping coefficient that we have deduced.

Notwithstanding that the vibrational condition in our experiments of a model plate in a circulation water tank differed entirely from the conditions that attended the rolling experiments of the "Revenge" in oscillatory frequency as well as in displacement velocity, the predicted damping coefficient of rolling of the "Revenge" fairly agrees with the same coefficient that had been actually observed in the same ship. It is then possible for us to conclude that the data of our model experiments immediately apply to the vibrations of an actual ship whose frequencies and amplitudes lie within the range of the vibrating model plate in our experiments. In extending our investigation to the case of the pitching motion of a ship, the reason that the pitching of a ship in motion-ahead condition in a calm sea is damped more quickly than in stand-still condition, has been ascertained.

9. Concluding Remarks.

In the present paper the treatment of the problem is entirely based on the results of model experiments, partly shown in our previous paper and partly in the

present one, the validity of the problem being confirmed by the results of rolling experiments with H.M.S. "Revenge" that were made about fifty years ago. Since, as a matter of fact, the problem is merely qualitative, it is impossible for us to discuss at present the precise values of the damping coefficient in question. It is to be hoped that some investigator having access to any of the usual experimental water tanks will conduct experiments relating to the present problem.

In the present investigation, the author received much kind advice from Professor Y. Hiraga, President of the Tokyo Imperial University, to whom he wishes to express his sincerest thank. He also wish to express his warmest thank to the Council of the Kondô Memorial Marine Foundation for aid given to a series of investigations, of which this study is a part.

(本論文に對して講演會の後日に次の written discussion が提出されたので之を掲載する)

妹澤博士の論文に就ての私見

正 員 渡 邊 惠 弘

平板が流水中で振動する際の水より働く hydrodynamic damping に關する妹澤博士の研究は、我々に取つては非常に興味のある有益なもので、非常に面白く拜見しました。私は當日同博士の講演を聞き落したのですが、後で聞きますと、何等の討論も無がつたとの事なので、同博士竝にその研究に敬意を表する意味で少しく私見を述べさせて頂き度いと思ひます。唯私の此處で述べるのは動搖に關する範圍内に於てであります。

妹澤博士は其の實驗結果を船の横及縦動搖に適用して bilge keel による減衰力を計算して居られます。此中横動搖に對する前進速度の影響に就ては、實船及模型を以てする色々の實驗がありますが、縦動搖に關しては見當りません。それで横動搖に關して、少しく感想を述べ度いと思ひます。

從來の横動搖と前進速度との關係の實船による實驗は妹澤博士の言及して居る "Revenge" の外に Bertin が 9 ton ばかりの小さい曳船 "Navette" にて實驗して居り、最近では "Conte di Savoia" (Sperry の Gyro を裝置し B. K. はなし) に就て試験され、又極く最近の報告に依れば和蘭で、特殊の B. K. をつけた驅逐艦で試験して居り、我國の元良式 fin を附けた陸丸の試験も此中に數へられませう。之等の結果によれば何れも前進速度のために、減衰力は著しく増加して居ります。然るに此處に面白い事は "Conte di Savoia" は B.K. が無いのにも拘らず前進速度があると減衰力が非常に大となつて居る事で、之はその模型試験に於ても現れて居ます。(T.S.N.A.M.E. 1936)。此の事に就ては R. E. Froude が或る驅逐艦の實驗の報告に於て、B.K. の無い船が 17kt の前進速度を有する

際の extinction は、同じ船が B.K. をつけて前進速度を持たぬ時の夫れの約2倍になると言つて居ると云ふ事を Payne が述べて居ります (T.I.N.A. 1924)。Payne は此論文の中で前進速度の影響に関する模型試験の結果を報告して居るのですが、矢張り B.K. を附けない船が 20 kt の速度では、速度なしの時の 4 倍の extinction があると言つて居ます。之と関係ある事柄として私の處で行つた實驗で、船に横動揺を與へながら前進速度を與へると、非常に大き週期的な yawing moment が現れ、此 moment は速度が大きいく程大となる事が確められました。speed length ratio が 0.80 附近の速度で 5° 位の振幅の横動揺の際に現れる此 yawing moment の最大値は舵角 5° 位で初めて balance される程度のものであります。是は明に横動揺と速度との爲めに起る船側に働く hydrodynamical pressure によるもので、横動揺を止める方向に働きます。之等の事から、船體それ自身が前身速度に依つて、横動揺を damp する可なり大きい力を有する事が明かである様に思はれます。

B.K. を有する船が前進速度を有する場合に就ては前記の Payne と R. Legendre (B.A.T.M.A. 1932) が實驗して居ります。Payne は其の實驗より、B.K. がある場合は前進速度は小さい動揺角の時以外に於ては damping には影響は無いと結論して居ますが、實驗の曲線を見ると、此時の damping が大きいので速度の影響がはつきりして居らない丈で、速度に依つて damping の増加する傾向のある事は看取されます。Legendre の實驗では 4.9 米の模型で 1 米/秒 の速度迄は、或る時は速度に依つて却つて damping が減る事もあるが、1 米/秒 以上では増加すると云ふ結果が現れて居ますが、此實驗で速度の速い時は實驗の error が入つて居るのではないかと云ふ疑が持たれる故（側壁よりの反射波の爲め）絶對の信頼は置き難い様です。

果して然らば前進速度の爲めに B.K. の作用がどれだけ増加するかと云ふ事は、以上の結果だけでは判明しません。妹澤博士の實驗にも現れて居る様に、板が流水中で振動すれば、此板は飛行機の翼

の様な作用をなして之に働く力は水の速度と共に増加する事は明かで、故淺羽功教授も元良式 fin の理論を此考へから研究して居ります。然しながら普通の船に取付けられて居る B.K. の様に、高さに比して長さの方向に長い即ち aspect ratio の非常に小さい板に對して、aspect ratio の小さい時と同様に考へ得るか否かは疑問であると思はれます。或る直角に置かれた板に働く normal force coef. C_N (=normal force/ $\frac{1}{2} \rho F U^2$) と迎角 α との関係は aspect ratio が大きい時は略々 1 次の関係が成立しますが、a.r. が小さい時は漸次異つて來る事は實驗及理論 (Bollay: Z.A.M.M. Feb. 1939) からも示され大體圖のようになります。

板が振動しない時の此結果を、振動板に適用するには或る

correction が必要とは思ひますが、a.r. の大きい時に此補正なしで近似的に apply される様に a.r. の小さい場合に就いても同様の近似を許す事にしませう。上圖で面白い事は a.r. が小さい時は 45° 附近迄 stall せず、而も C_N が相當大きい事で、即ち普通の B.K. は相當遅い速度で大きい動揺の際も stall せずに有効に働く事が出来ます。

今 B.K. の aspect ratio を極端に小さく取り 0 とすると $C_N = 2\sin^2\alpha$ で r を B.K. 廻轉軸よりの距離、 $\dot{\theta}$ を動揺角速度とすれば $\sin\alpha = \frac{r\dot{\theta}}{U}$ ($U = \sqrt{V^2 + r^2\dot{\theta}^2}$) なる故 B.K. に働く normal force は $\rho r^2 \dot{\theta}^2 F$ (F は B.K. の面積) となり、是は前進速度がない時に B.K. に働く力と同じとなり (此時も $C\rho r^2 \dot{\theta}^2 F$ で $C=1$ である) 而も速度に無関係と云ふ結果になります。此考へ方が果して正しいかどうかは實驗して見なければわかりませんが、是丈けから見れば普通の proportion の B.K. は前進速度に依つて豫期される程 (a.r. の小さい有限値の時は V に比例する項が現れるが小さい故) 減衰力の増加を來さない様に思はれます。此外 Bryan の指摘した如き B.K. と船體との干涉による damping effect が前進速度によつてどう變るか云ふ様な事もありますが、是ははつきり分らないが左様増加するとは思はれません。

勿論妹澤博士の實驗の様な aspect ratio の比較的大きい板を B.K. に使へば元良式 fin の如く前進速度の影響は大きく現れる事は明かで、殊に速度が大なる程影響が大となる事は同博士の analysis から明かであります。最近和蘭で實驗されたものの如く (Jr. W. Rosingh: Shipb. & Ship. R. Oct. 27, 1938) B.K. を長さの方向に短く切つて並べた a.r. の大きい板では前進速度の影響が強く現れる事は上の事よりわかります。前述の Legendre が矢張り同じ様に B.K. を短く切つて實驗しましたが、是では尚その strip の長さが高さの 13 倍 (a.r. $\approx 1/6$) で、而も其の切口は平板でなく根本の厚さが高さの $1/4$ もある楔形のものを使つてある爲め、普通の B.K. に比較して充分有効の働きを示さなかつたのであります。

以上妹澤博士の論文に對する感想を羅列致しました。要するに、從來の實驗結果だけでは未だ十分に明確な結論を得る事は出来ない様に思はれますので、此點將來の研究に期待し度いと思ふのであります。

渡邊博士の御意見に對する御禮

正 員 妹 澤 克 惟

船體の前進中に於ける振動の減衰性を説明する爲に同じ状態に於ける動揺の減衰性を引き出したところが、渡邊博士から種々御親切なる御教示を戴き恐縮に存じます。“Revenge”の外に同じ研究が相當あるらしい事は充分想像して居りましたが、前書に述べたやうに私には正確に調査する時間と根氣がないので割愛して置いた次第であります。ここに種々の場合を列挙して戴いて幸に存じま

す。

B.K. のみを問題になさつたやうですが、B.K. がなくても船底、船側が適当な形であれば、同様な作用のある事は本文中に述べた通りであります。“Revenge”の横揺には其の船底の形から B.K. のみを取りました。ただ B.K. や船底の aspect ratio が小さくなると板面に働く垂直力が減少する事は勿論であり、そのために私の研究では飛行機の翼などよりも相当小なる a.r. の板の實驗を致しました。之でも船の場合よりは可なり隔つて居りますが、本文に述べたやうに、“Revenge”の結果に數分の一と云ふ factor を掛けると、結果が相當よく合ひました。従つて之等の實驗結果を否定する譯には行かぬと思ひます。

尙餘計な事です、振動の分析に於て垂直力 $= (\rho/2) C_N V^2 F$ と書き而も a.r. = 0 のときの Bollay の式 $C_N = 2 \sin^2 \alpha$ の中の $\sin \alpha$ の値として直ちに $\sin \alpha = r\dot{\theta} / \sqrt{V^2 + r^2 \dot{\theta}^2}$ としてしまふのは如何なものでせうか。私をして書かしめれば、振動に於ける垂直力の減衰性の部分は當然 $(\rho/2) (\partial C_N / \partial \alpha) \alpha_1 V^2 F$ (但し $\alpha_1 = r\dot{\theta}/V$)、即ち $\rho \sin 2\alpha r\dot{\theta} V F$ と致します。之は勿論小振動型の式であり、且つ流體力學的にも第1近似の形式であります。この場合に $\alpha = 0$ の位置では減衰力が零となりますが、それ以外の迎角附近の振動では V に比例する減衰力が御座います。更に實船に於て船底や船側の a.r. は必しも零に近いと云はれませんし、B.K. でもその有效部分は a.r. $\rightarrow 0$ と置く譯には行かぬ故、 $\alpha = 0$ の附近でも $\partial C_N / \partial \alpha \neq 0$ であると考へられます。

兎も角も私の振動減衰性の問題には“Revenge”の結果が可なり役立ちましたつもりですけれども、一般の動揺の問題については、終始動揺の御研究を御やりになつてゐる、渡邊博士かどなたかに模型實驗でもよいから上述の減衰性の有無を是非實地によく確めて戴き度いと思ひます。以上大變御親切なる御意見への御禮の序に一二の點を附加へた次第であります。