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On the Effectiveness of Panting Stringers and Web Frames of a Ship (2nd Report)

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Abstract

In the first report, the author studied the "Effectiveness of Panting Stringers and Web Frames of A Ship", and showed that the stringers with web frames whose sizes were determined by the Rules of Classification Societies are not strong enough to support side frames.

Recently, however, strong stringers with web frames are often adopted, which is known as "Web and Stringer System". This paper gives the result of an analysis made to the structure of panting zone of a 8,750 G. T. cargo ship which has been strengthened in such a way as stated above and shows that the effectiveness of "Web and Stringer System" is practically sufficient in this case.

Introduction

In the first report,** the author studied the "Effectiveness of Panting Stringers and Web Frames of A Ship," and showed that the stringers with web frames whose sizes were determined by the Rules of Classification Societies are not strong enough to support side frames. Recently, however, strong stringers with web frames are often adopted, which is known as "Web and Stringer System". This paper gives the result of an analysis made to the structure of panting zone of a 8,750 G. T. cargo ship which has been strengthened in such a way as stated above.

Beside a close analysis on the above problem, the author has undertaken a simpler kind of calculation on the preceding problem. As the result of the comparison made to both the methods, i.e., a precise analytical method and a convenient calculation, the latter has been found to be appropriate as far as it is concerned with "the Web and Stringer System".

Assumptions and Procedures of Analysis

In this report, the uniform pressure of 1 kg/cm^2 is assumed to be applicable to the whole surface of shell plating in panting zone as the severest loading conditions.***

The structure between two transverse bulkheads in panting zone is consisted of two panting stringers and three web frames with side frames and shell plating as shown in Fig. 1. (Fig. 1 is omitted.) or Fig. 3. For simplicity, upper and lower ends of frames and web frames and connections of stringers to the bulkheads are assumed to be all clamped.

As the first stage, an analysis is given to a slightly different structure which has symmetrical arrangement of web frames and stringers in longitudinal direction and side frames of equal size, as shown in Fig. 2.

Fig. 2

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*** The reason for adopting this condition is given in the 1st Report.

At this stage, the thickness of shell plating was assumed to be constant at all parts and equal as well to 13.5mm, the average value of real plating. However, as the size of the webs of upper and lower stringers are varied respectively, moments of inertia were treated separately and the values which had been obtained by calculated moment of inertia on each span separately for upper and lower stringers were applied.

In the 2nd stage, the result of the above calculation was compared to what was obtained by evaluating the moments of inertia of both upper and lower stringers on the basis of an assumption that both of them is equivalent to the mean of its real value.

And in the 3rd stage, calculation was made by assuming that the value of the moments of inertia I of both upper and lower stringers is equal to that which is given by

$$(1/I) = (1/2) \cdot \{(1/I_u) + (1/I_l)\} \quad (1)$$

where I_u and I_l are the actual values of the moments of inertia of upper and lower stringers respectively, as stated in the first step calculation.

Therefore, in the fourth step, the same assumptions as adopted in the third step are used only on the treatment of the moments of inertia of stringers, but actual arrangement of web frames

Fig. 3

and stringers are analyzed as shown in Fig. 3, where the thickness of shell plating is assumed to be taken 13mm and 14mm for the fore and aft part of Fr. No.155 respectively, and moments of inertia of side frames are separately calculated for every web frame spacing.

In the last stage, a simple calculation was undertaken on the basis of an assumption

that web frames subject the concentrated load which is equivalent to the total sum of the pressure, even and 1kg/cm^2 in its strength upon the part indicated with hatched lines in Fig.4 (i.e., the part encircled by the straight lines that pass through the mid-points of adjoining spans of web frames and those of stringers) at each crossing point of every web frame and stringer, i.e., the crossing point P of a stringer and a web frame in Fig. 4 for instance.

In that case, the deflections of the web frames w , at the crossing points of the web frames and the stringers can be easily obtained by the elementary calculations.

The base lines of deflection curves of stringers are obtained by connecting rectilinearly the deflected crossing points of web frames and the stringers.

And superposing the deflection curves of stringers clamped at both ends of each web frame span of stringers, approximate deflection curves for stringers may be obtained, where the stringers are assumed to be applied to the

uniform external pressure 1kg/cm^2 between the part divided by straight lines through mid-points between the stringers and adjoining stringers of consecutive spans of web frames.

Fig. 4

Throughout all stages of analysis, the calculations of effective breadth of shell plating which are applied to the evaluation of the moments of inertia of all the stiffening members are made on the basis of C. H. A. Schade's theory* for the case where all the ends of side frames are clamped. And all the points at which stringers are connected to bulkheads have been assumed to be clamped, too. By using accurate or approximate deflection curves for stringers obtained by the above steps, it is possible to estimate the supporting forces of stringers to any side frame at their connecting points.

Comparing the bending moments in side frames due to external pressure applied to one frame spacing and supporting forces from stringers with the bending moments due to external pressure only, the effectiveness of panting stringers and web frames to any side frame is made clear.

Analysis for the Structure with Symmetrically Arranged Stringers in Longitudinal Direction

For the structure shown in Fig. 2, the analysis is performed by using the following data :
Stringers :

	Location of Span	(0,1) and (2,3)	(1,2)
	Length of Span	$l_{01}=l_{23}=476$ cm	$l_{12}=544$ cm
Mts of Inertia	Upper Stringer	$I_{u01}=11.690 \times 10^4$ cm ⁴	$I_{u12}=12.098 \times 10^4$ cm ⁴
	Lower Stringer	$I_{l01}=14.281 \times 10^4$ cm ⁴	$I_{l12}=14.834 \times 10^4$ cm ⁴
ratio of mts. of inertia		$r_{01}=I_{l01}/I_{u01}=1.222$	$r_{12}=I_{l12}/I_{u12}=1.230$

ratio of spans $\rho=l_{12}/l_{01}=1.143$

Web Frames :

Full length, $l_{web}=205 \times 3=615$ cm

Spans of web frames, see Fig. 2 $a=b=205$ m

Moments of inertia, $I_{web}=33.276 \times 10^4$ cm⁴

Torsional rigidities of web frames are neglected, as they affect slightly the stringers.

Side Frames :

Full length, $l_{fr}=615$ cm, Frame spacing. $S=68.0$ cm

Spans of side frames, see Fig. 2. $a=b=205$ cm

Moments of inertia. $I_{fr}=0.5666 \times 10^4$ cm⁴

General Formulae Derived from Separate Treatment of the Moments of Inertia of Upper and Lower Stringers. (1st Step of Analysis)

Equations of stringers at any span

Taking ξ axis along stringer and from each left end of span, the equations of flexure of stringers are

$$\left. \begin{aligned} \lambda_{uu}(d^4 y_u/d\xi^4) + \lambda_{ul}(d^4 y_l/d\xi^4) + y_u &= \eta_u \\ \lambda_{lu}(d^4 y_u/d\xi^4) + \lambda_{ll}(d^4 y_l/d\xi^4) + y_l &= \eta_l \end{aligned} \right\} \quad (2)$$

where y_u and y_l are respectively the deflections of upper and lower stringers at any section of stringers, η_u and η_l are also the deflections of side frames at the crossing points of the stringers due to external pressure in the case where stiffening effects due to stringer are neglected. $\xi=0$ and $\xi=1$ correspond to the left and right ends of stringers at any span, suffix

* C. H. A. Schade : Trans. Soc. Nav. Arch and Mar. Eng. Vol. 59 (1951) and Vol. 61 (1953)

01 or 12 is omitted so far as there is no fear of causing confusion, and in the same way, the following coefficients λ_{uu} etc. are given by

$$\left. \begin{aligned} \lambda_{uu} &= EI_u \mu_{uu} S / l^4, & \lambda_{ul} &= EI_l \mu_{ul} S / l^4 \\ \lambda_{lu} &= EI_u \mu_{lu} S / l^4, & \lambda_{ll} &= EI_l \mu_{ll} S / l^4 \end{aligned} \right\} \quad (3)$$

where, l is the span of stringer, and μ_{ul} is the deflection at the crossing point of a given frame and the upper stringer when unit concentrated load is subjected at the crossing point of the frame and the lower stringer, and μ_{uu} , μ_{lu} and μ_{ll} are defined similarly. Then the reciprocal theorem gives us the relation $\mu_{ul} = \mu_{lu}$.

Parameters determined by relative flexural rigidities between side frames and stringers are defined as follows,

$$\left. \begin{aligned} \alpha_I &= \{(\lambda_{uu} + \lambda_{ll} + \sqrt{(\lambda_{uu} - \lambda_{ll})^2 + 4\lambda_{ul}\lambda_{lu}}) / 8(\lambda_{uu}\lambda_{ll} - \lambda_{ul}\lambda_{lu})\}^{1/4} \\ \alpha_{II} &= \{(\lambda_{uu} + \lambda_{ll} - \sqrt{(\lambda_{uu} - \lambda_{ll})^2 + 4\lambda_{ul}\lambda_{lu}}) / 8(\lambda_{uu}\lambda_{ll} - \lambda_{ul}\lambda_{lu})\}^{1/4} \end{aligned} \right\} \quad (4)$$

In this case $a=b$ is put and we define following constants. $K = (I_{u12} S l_f r^4) / (81 I_f l_f r l_{12}^4)$, $r \equiv r_{12} = I_{12} / I_{u12}$, then we have $\lambda_{uu} = 8K/27$, $\lambda_{ll} = 8rK/27$, $\lambda_{ul} = 11rK/54$, $\lambda_{lu} = 11K/54$,

$$\left. \begin{aligned} \alpha_I &= [\{8(1+r) + (64-7r+64r^2)^{1/2}\} / (10rK)]^{1/4} \\ \alpha_{II} &= [\{8(1+r) - (64-7r+64r^2)^{1/2}\} / (10rK)]^{1/4} \end{aligned} \right\} \quad (5)$$

Denoting

$$\begin{aligned} \beta_I &= (1 - 4\alpha_I^4 \lambda_{uu}) / (4\alpha_I^4 \lambda_{ul}) = (27 - 32\alpha_I^4 K) / (22r\alpha_I^4 K) \\ \beta_{II} &= (1 - 4\alpha_{II}^4 \lambda_{uu}) / (4\alpha_{II}^4 \lambda_{ul}) = (27 - 32\alpha_{II}^4 K) / (22r\alpha_{II}^4 K) \end{aligned}$$

We obtain the slope deflection coefficients derived from equations (2)

$$\left. \begin{aligned} A_{uu} &= \{\beta_{II} A(\alpha_I) - \beta_I A(\alpha_{II})\} / (\beta_{II} - \beta_I), & A_{ul} &= -\{A(\alpha_I) - A(\alpha_{II})\} / (\beta_{II} - \beta_I), \\ A_{lu} &= r\beta_I \beta_{II} \{A(\alpha_I) - A(\alpha_{II})\} / (\beta_{II} - \beta_I), & A_{ll} &= -r\{\beta_I A(\alpha_I) - \beta_{II} A(\alpha_{II})\} / (\beta_{II} - \beta_I), \text{ etc.} \end{aligned} \right\} \quad (6)$$

where $A(\alpha_I)$ etc. are the functions of α which are given by (12) in the first report and by substituting the value of parameter α_I or α_{II} obtained from (4) or (5).

Coefficients of flexure for web frames

Deflections of web frames of the connecting points of upper and lower stringers respectively may be

$$\left. \begin{aligned} v_u &= \mu_{web,uu} P_u + \mu_{web,ul} P_l \\ v_l &= \mu_{web,lu} P_u + \mu_{web,ll} P_l \end{aligned} \right\} \quad (7)$$

where $\mu_{web,uu}$ etc. are coefficients of flexure of web frames, then the reciprocal theorem gives us the relation $\mu_{web,ul} = \mu_{web,lu}$.

Solving (7) with respect to P_u and P_l we obtain

$$\left. \begin{aligned} P_u &= (EI_{u12} / l_{12}^3) \cdot (\bar{\mu} v_u + \bar{\mu}' v_l) \\ P_l &= (EI_{l12} / l_{12}^3) \cdot (\bar{\mu}' v_u + \bar{\mu} v_l) \end{aligned} \right\} \quad (8)$$

$$\text{where} \quad \left. \begin{aligned} \bar{\mu} &= (l_{12}^3 / EI_{u12}) \cdot \{\mu_{web,uu} / (\mu_{web,uu}^2 - \mu_{web,ul}^2)\} \\ \bar{\mu}' &= -(l_{12}^3 / EI_{l12}) \cdot \{\mu_{web,ul} / (\mu_{web,uu}^2 - \mu_{web,ul}^2)\} \end{aligned} \right\} \quad (9)$$

When $a=b$, we have

$$\bar{\mu} = (2592 I_{web} l_{12}^3) / (5 I_{u12} l_{web}^3), \quad \bar{\mu}' = -(1782 I_{web} l_{12}^3) / (5 I_{l12} l_{web}^3), \quad (10)$$

Fixing moments and shearing forces of stringers, when they are fixed at every end of spans, are

$$\left. \begin{aligned} \bar{M}_u &= \mp r \eta EI_u \{(\beta_{II} - 1)(D_I - C_I) - (\beta_I - 1)(D_{II} - C_{II})\} / (\beta_{II} - \beta_I) l^2 \\ \bar{M}_l &= \mp r \eta EI_l \{\beta_I (\beta_{II} - 1)(D_I - C_I) - \beta_{II} (\beta_I - 1)(D_{II} - C_{II})\} / (\beta_{II} - \beta_I) l^2 \\ Q_u &= \pm r \eta EI_u \{(\beta_{II} - 1)(H_I - G_I) - (\beta_I - 1)(H_{II} - G_{II})\} / (\beta_{II} - \beta_I) l^3 \\ Q_l &= \pm r \eta EI_l \{\beta_I (\beta_{II} - 1)(H_I - G_I) - \beta_{II} (\beta_I - 1)(H_{II} - G_{II})\} / (\beta_{II} - \beta_I) l^3 \end{aligned} \right\} \quad (11)$$

where double sign in front of right side of (11) represents the described quantities of (1,0) end for upper sign, (1,2) end for lower sign.

Elastic equation of stringers for any two consecutive spans $(\alpha-1, \alpha)$ and $(\alpha, \alpha+1)$.

The equilibrium equation of shearing force for upper stringer on the α -th web frame is

$$\begin{aligned} & (EI_u/l^2)_{\alpha, \alpha-1} \cdot \{(D_{uu})_{\alpha, \alpha-1} \cdot \theta_{u, \alpha-1} + (D_{ul})_{\alpha, \alpha-1} \cdot \theta_{l, \alpha-1}\} + \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (C_{uu})_{\alpha, \alpha-1} \\ & - (EI_u/l^2)_{\alpha, \alpha+1} \cdot (C_{uu})_{\alpha, \alpha+1}\} \cdot \theta_{u, \alpha} + \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (C_{ul})_{\alpha, \alpha-1} - (EI_u/l^2)_{\alpha, \alpha+1} \cdot (C_{ul})_{\alpha, \alpha+1}\} \theta_{l, \alpha} \\ & - (EI_u/l^2)_{\alpha, \alpha+1} \cdot \{(D_{uu})_{\alpha, \alpha+1} \cdot \theta_{u, \alpha+1} + (D_{ul})_{\alpha, \alpha+1} \cdot \theta_{l, \alpha+1}\} - \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (G_{uu})_{\alpha, \alpha-1} \\ & + (EI_u/l^2)_{\alpha, \alpha+1} \cdot (G_{uu})_{\alpha, \alpha+1}\} \cdot v_{u, \alpha} - \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (G_{ul})_{\alpha, \alpha-1} \\ & + (EI_u/l^2)_{\alpha, \alpha+1} \cdot (G_{ul})_{\alpha, \alpha+1}\} \cdot v_{l, \alpha} + (EI_u/l^2)_{\alpha, \alpha-1} \cdot \{(H_{uu})_{\alpha, \alpha-1} \cdot v_{u, \alpha-1} \\ & + (H_{ul})_{\alpha, \alpha-1} \cdot v_{l, \alpha-1}\} - \{(EI_u/l^2)_{\alpha, \alpha+1} \cdot \bar{\mu}_\alpha \cdot v_{u, \alpha} + (EI_u/l^2)_{\alpha, \alpha+1} \cdot \bar{\mu}'_\alpha \cdot v_{l, \alpha} \\ & - \bar{Q}_{u, \alpha, \alpha-1} - \bar{Q}_{u, \alpha, \alpha+1}\} = 0 \end{aligned} \quad (12)$$

Do. for lower stringer is omitted, as it is obtainable by interchanging the suffix u to l in the above equation.

The equilibrium of moments of forces for upper stringer on the α -th web frame is

$$\begin{aligned} & (EI_u/l^2)_{\alpha, \alpha-1} \cdot \{(B_{uu})_{\alpha, \alpha-1} \cdot \theta_{u, \alpha-1} + (B_{ul})_{\alpha, \alpha-1} \cdot \theta_{l, \alpha-1}\} + \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (A_{uu})_{\alpha, \alpha-1} \\ & + (EI_u/l^2)_{\alpha, \alpha+1} \cdot (A_{uu})_{\alpha, \alpha+1}\} \cdot \theta_{u, \alpha} + \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (A_{ul})_{\alpha, \alpha-1} \\ & + (EI_u/l^2)_{\alpha, \alpha+1} \cdot (A_{ul})_{\alpha, \alpha+1}\} \cdot \theta_{l, \alpha} + (EI_u/l^2)_{\alpha, \alpha+1} \cdot \{(B_{uu})_{\alpha, \alpha+1} \cdot \theta_{u, \alpha+1} \\ & + (B_{ul})_{\alpha, \alpha+1} \cdot \theta_{l, \alpha+1}\} + (EI_u/l^2)_{\alpha, \alpha-1} \cdot \{(D_{uu})_{\alpha, \alpha-1} \cdot v_{u, \alpha-1} + (D_{ul})_{\alpha, \alpha-1} \cdot v_{l, \alpha-1}\} \\ & - \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (C_{uu})_{\alpha, \alpha-1} - (EI_u/l^2)_{\alpha, \alpha+1} \cdot (C_{uu})_{\alpha, \alpha+1}\} \cdot v_{u, \alpha} \\ & - \{(EI_u/l^2)_{\alpha, \alpha-1} \cdot (C_{ul})_{\alpha, \alpha-1} - (EI_u/l^2)_{\alpha, \alpha+1} \cdot (C_{ul})_{\alpha, \alpha+1}\} \cdot v_{l, \alpha} \\ & - \{(EI_u/l^2)_{\alpha, \alpha+1} \cdot (D_{uu})_{\alpha, \alpha+1} \cdot v_{u, \alpha+1} + (EI_u/l^2)_{\alpha, \alpha+1} \cdot (D_{ul})_{\alpha, \alpha+1} \cdot v_{l, \alpha+1}\} \\ & + \{\bar{M}_{u, \alpha-1} + \bar{M}_{u, \alpha, \alpha+1}\} = 0 \end{aligned} \quad (13)$$

Do. for lower stringer is omitted by the same reason as stated above. In the above equations coefficients for $\alpha, \alpha+1$ span $(A_{ul})_{\alpha, \alpha+1}$ etc. are given by (6), $\theta_{u, \alpha}$ and $\theta_{l, \alpha}$ are respectively meant by the angles of rotation of the upper and lower stringers on the α -th web frame and $v_{u, \alpha}$ and $v_{l, \alpha}$ are similary meant by the deflections of them at the crossing points of α -th web frame $\bar{\mu}_\alpha$ and $\bar{\mu}'_\alpha$ are given by (10), and $\bar{M}_{u, \alpha, \alpha+1}$, $\bar{Q}_{u, \alpha, \alpha+1}$ etc. are given by (11).

End Conditions for Stringers.

If the ends of stringers connecting to the bulkheads are assumed to be clamped we may write (see Fig. 2)

$$\left. \begin{aligned} v_{u0} = v_{u3} = v_{l0} = v_{l3} = 0, & \quad \theta_{u0} = \theta_{u3} = \theta_{l0} = \theta_{l3} = 0 \\ v_{u1} = v_{u2} = v_{u4}, & \quad v_{l1} = v_{l2} = v_{l4}, \quad \theta_{u1} = -\theta_{u2} = \theta_{u4}, \quad \theta_{l1} = -\theta_{l2} = \theta_{l4} \end{aligned} \right\} \quad (14)$$

Applying the elastic equation of Stringes (12) and (13) etc. with ends conditions (14) to the spans (0,1) and (1,2), we obtain the equations referred to the deflections v_u, v_l and rotations θ_u, θ_l at the crossing points of No.1 web frame and both stringers respectively, we have simultaneous equations with respect to four unknowns θ_u, θ_l, v_u and v_l . Solving this equations and substituting the obtained values of $\theta_u, \theta_l, \varphi_u \equiv v_u/l_{12}$ and $\varphi_l \equiv v_l/l_{12}$ into following slope-deflection equations for stringers, we obtain the end moments at every ends of stringers as follows,

$$\begin{aligned} M_{u10} &= (EI_{u,10}/l_{10}) \cdot \{A_{uu,10} \cdot \theta_u + A_{ul,10} \cdot \theta_l - C_{uu,10} \cdot \varphi_u - C_{ul,10} \cdot \varphi_l\} + \bar{M}_{u,10} \\ M_{l10} &= (EI_{l,10}/l_{10}) \cdot \{A_{lu,10} \cdot \theta_u + A_{ll,10} \cdot \theta_l - C_{lu,10} \cdot \varphi_u - C_{ll,10} \cdot \varphi_l\} + \bar{M}_{l,10} \\ M_{u12} &= (EI_{u,12}/l_{12}) \cdot \{A_{uu,12} - B_{uu,12}\} \cdot \theta_u + \{A_{ul,12} - B_{ul,12}\} \cdot \theta_l + \{C_{uu12} - D_{uu,12}\} \cdot \varphi_u \\ &+ \{C_{ul,12} - D_{ul,12}\} \cdot \varphi_l + \bar{M}_{u,12} \\ \bar{M}_{l,12} &= (EI_{l,12}/l_{12}) \cdot \{A_{lu,12} - B_{lu,12}\} \cdot \theta_u + \{A_{ll,12} - B_{ll,12}\} \cdot \theta_l + \{C_{lu,12} - D_{lu,12}\} \cdot \varphi_u \\ &+ \{C_{ll,12} - D_{ll,12}\} \cdot \varphi_l + \bar{M}_{l,12} \end{aligned}$$

Preliminary functions used in the calculation of deflections of stringers are defined as follows

$$\begin{aligned} h_{1I} &= [h_1(\xi, \alpha)]_{\xi=0.5, \alpha=\alpha_I} = 2[2 \sin \alpha_I \cdot \sinh \alpha_I \cdot \sin(\alpha_I/2) \cdot \sinh(\alpha_I/2) \\ &\quad + (\sin \alpha_I \cdot \cos \alpha_I + \cos \alpha_I \cdot \sinh \alpha_I) \{ \cos(\alpha_I/2) \cdot \sinh(\alpha_I/2) - \sin(\alpha_I/2) \cdot \cosh(\alpha_I/2) \}] / \\ &\quad (\cosh 2 \alpha_I + \cos 2 \alpha_I - 2) \\ h_{2I} &= [h_2(\xi, \alpha)]_{\xi=0.5, \alpha=\alpha_I} = -2[\sin \alpha_I \cdot \sinh \alpha_I \cdot \{ \sin(\alpha_I/2) \cdot \cosh(\alpha_I/2) \\ &\quad - \cos(\alpha_I/2) \cdot \sinh(\alpha_I/2) + (\cos \alpha_I \cdot \sinh \alpha_I - \sin \alpha_I \cdot \cosh \alpha_I) \cdot \sin(\alpha_I/2) \sinh(\alpha_I/2) \} / \\ &\quad \alpha_I (\cosh 2 \alpha_I + \cos 2 \alpha_I - 2), \end{aligned}$$

And similary we put $h_{1II} \equiv [h_1(\xi, \alpha)]_{\xi=0.5, \alpha=\alpha_{II}}$, $h_{2II} = [h_2(\xi, \alpha)]_{\xi=0.5, \alpha=\alpha_{II}}$, where the functions $h_1(\xi, \alpha)$ and $h_2(\xi, \alpha)$ were already introduced in the first report.

Deflection of upper stringer at the mid-point of (0,1) span is given by

$$y_{u,10} = (\beta_{II} - \beta_I)^{-1} \cdot \{ \eta (-\beta_{II} h_{1I} + \beta_I h_{1II} + h_{1I} + h_{1II}) + (v_{uI} - \eta) \cdot (\beta_{II} h_{1I} - \beta_I h_{1II}) - (v_{II} - \eta) \cdot (h_{1I} - h_{1II}) - \theta_{uI} l_{0I} \cdot (\beta_{II} h_{2I} - \beta_I h_{2II}) + \theta_{II} l_{0I} \cdot (h_{2I} - h_{2II}) \} + \eta.$$

Do. of lower stringer is given by

$$\begin{aligned} y_{l,10} &= (\beta_{II} - \beta_I)^{-1} [\eta (-\beta_I \beta_{II} (h_{1I} - h_{1II}) + \beta_I h_{1I} + \beta_{II} h_{1II}) + (v_{uI} - \eta) \beta_I \beta_{II} (h_{1I} - h_{1II}) \\ &\quad - (v_{II} - \eta) (\beta_I h_{1I} - \beta_{II} h_{1II}) - \theta_{uI} l_{12} \beta_I \beta_{II} (h_{2I} - h_{2II}) + \theta_{II} l_{12} \cdot (\beta_I h_{2I} - \beta_{II} h_{2II})] + \eta \end{aligned}$$

Deflection of upper stringer at the mid-point of (1,2) span is given by

$$y_{u,12} = 2(\beta_{II} - \beta_I)^{-1} \cdot \{ (v_{uI} - \eta) \cdot (\beta_{II} h_{1I} - \beta_I h_{1II}) + (v_{II} - \eta) \cdot (h_{1I} - h_{1II}) + \theta_{uI} l_{12} \cdot (\beta_{II} h_{2I} - \beta_I h_{2II}) - \theta_{II} l_{12} (h_{2I} - h_{2II}) \} + \eta$$

Do. of lower stringer is given by

$$\begin{aligned} y_{l,12} &= 2(\beta_{II} - \beta_I)^{-1} \cdot \{ (v_{uI} - \eta) \beta_I \beta_{II} (h_{1I} - h_{1II}) + (v_{uI} - \eta) \cdot (\beta_{II} h_{1I} - \beta_I h_{1II}) + \theta_{uI} l_{12} \beta_I \beta_{II} (h_{2I} - h_{2II}) \\ &\quad - \theta_{II} l_{12} (\beta_I h_{2I} - \beta_{II} h_{2II}) \} + \eta \end{aligned}$$

Equations described below are common for each span of stringer, therefore we omit the suffix (0,1) or (1,2) in following formulae, and suffix u and l mean by upper and lower stringer respectively.

Reduction rates of deflection of side frame due to stringers.

$$\Delta u = \{ (\eta - y_u) / \eta \} \times 100\%, \quad \Delta l = \{ (\eta - y_l) / \eta \} \times 100\%$$

Supporting forces of stringers to side frames when $a=b$

$$R_u = (324 EI_{fr} / 5 l^3_{fr}) \cdot \{ 8(\eta - y_u) - 5.5(\eta - y_l) \}$$

$$R_l = (324 EI_{fr} / 5 l^3_{fr}) \cdot \{ -5.5(\eta - y_u) + 8(\eta - y_l) \}$$

Fixing moments of side frames when $a=b$

$$\text{for upper end, } M_A = (wsl_{fr}^2 / 12) - (2 l_{fr} / 27) (2 R_u + R_l)$$

$$\text{for lower end, } M_B = (wsl_{fr}^2 / 12) - (2 l_{fr} / 27) (R_u + 2 R_l)$$

$$\text{for the mid-point of the span, } M_G = (wsl_{fr}^2 / 24) - \{ l_{fr} (R_u + R_l) / 18 \}$$

$$\text{for the case } R_u = R_l = 0, \quad M_A = M_B = M = (wsl_{fr}^2 / 12)$$

Reduction rates of dittos due to stringers respectively,

$$\Omega_A = \{ 8(2 R_u + R_l) / (9 wsl_{fr}) \} \times 100\%, \quad \Omega_B = \{ 8(R_u + 2 R_l) / (9 wsl_{fr}) \} \times 100\%.$$

Reduction rate of do. at the mid-point of a side frame,

$$\Omega_G = \{ 4(R_u + R_l) / (3 wsl_{fr}) \} \times 100\%$$

Bending stresses of dittos. are respectively given by

$$\sigma_A = M_A / Z_{fr}, \quad \sigma_B = M_B / Z_{fr} \text{ and } \sigma_G = M_G / Z_{fr},$$

where Z_{fr} is the section modulus of the side frame.

Numerical results obtained from above equations are shown in Table II at the end of this paper.

The Result of 2nd, 3rd and 4th Step of Analysis

In the 2nd step of analysis, above results are compared to a more simplified case where the moments of inertia of upper and lower stringers are assumed to be equal to their mean value, and in the third step calculation is made on the structure which is assumed to be of equal value to the moments of inertia of both stringer I given by

$$(1/I) = (1/2) \cdot \{(1/I_u) + (1/I_l)\} \quad (1)$$

Comparing numerical results based on above assumptions, it was made clear that the results derived from the above assumption (1) give the best approximation to that from accurate theory of 1st step.

Table I

Location of Span	(0,1)	(1,2)	(2,3)	(3,4)
$l_{st} \equiv l$ cm	272	476	544	366
s cm	68	68	68	61
I_{fr} cm ⁴	0.5725×10^4	0.5725×10^4	0.5614×10^4	0.5498×10^4
η cm	1.665	1.665	1.648	1.344
I_{st} cm ⁴	10.88×10^4	13.11×10^4	13.19×10^4	11.81×10^4
α	0.926	1.548	1.758	1.243
I_{web} cm ⁴	$I_{web,1} = 38.69 \times 10^4$ $I_{web,2} = 33.28 \times 10^4$ $I_{web,3} = 43.74 \times 10^4$			

Parameter α in above Table is given by $\alpha = (3 l_{st} / l_{fr}) \cdot (I_{fr} l_{fr} / 2 I_{st})^{1/2}$.

Analysis for the Structure with Actual Arrangement of Stringers and Web Frames

In the fourth step of analysis, we deal with the structure shown in Fig. 3 of which scantlings and other data are given in Table II. But using the facts obtained above in preceding paragraph, we assume that the moments of inertia of upper and lower stringers are both equal to I , which is given by equation (1) for each span. Other assumptions referred to scantlings are already stated in the preceding paragraph for each span of stringer.

Analysis for this step are made by using the formulae given in 1st report but we treat six unknowns θ_1 , θ_2 , θ_3 , v_1 , v_2 and v_3 in this case. Numerical results obtained by above mentioned calculations are shown in Table II with the results derived by approximate method already stated in Introduction.

Table II

	Deflections or Stresses and Their Reduction Rates	1 st Step Calculation		2 nd Step Cal.	3 rd Step Cal.	4 th Step Cal.	Approx. Method
		A End (Upper E.)	B End (Lower E.)	$I = \frac{1}{2}(I_u + I_l)$	$\frac{1}{I} = \frac{1}{2}(\frac{1}{I_u} + \frac{1}{I_l})$	$\frac{1}{I} = \frac{1}{2}(\frac{1}{I_u} + \frac{1}{I_l})$	
Deflections of Side Frames at the Mid-points of the Spans of Stringers	y_{c1} cm					0.04250	0.0805
	Δ_{c1} %					97.44	95.2
	y_{12} cm	0.1580	0.1437	0.1501	0.1487	0.3009	0.2746
	Δ_{12} %	90.61	91.46	91.08	91.16	81.93	83.5
	y_{23} cm	0.3996	0.3820	0.3875	0.3946	0.3721	0.3491
	Δ_{23} %	76.25	77.29	76.96	76.54	77.41	78.8
	y_{34} cm	0.1580	0.1437	0.1501	0.1487	0.05203	0.1115
	Δ_{34} %	90.61	91.46	91.08	91.16	96.12	91.7
Longitudinal Stresses of Side Frames at the Mid-points of the Spans of Stringers	σ_{c1} kg/cm ²					1024	1032
	Ω_{c1} %					86.63	86.52
	σ_{12} kg/cm ²	1518	1414	1463	1457	2080	1844
	Ω_{12} %	80.24	81.64	80.96	81.03	72.84	75.92
	σ_{23} kg/cm ²	2504	2376	2427	2455	2560	2348
	Ω_{23} %	67.40	69.08	68.41	68.04	66.80	69.55
	σ_{34} kg/cm ²	1518	1414	1463	1457	1824	1949
	Ω_{34} %	80.24	81.64	80.96	81.03	73.84	72.04
Bending Stresses of Stringers on Web Frames	$\sigma_{st,10}$ kg/cm ²					801	
	$\sigma_{st,12}$ kg/cm ²					750	
	$\sigma_{st,21}$ kg/cm ²	1604	1291	1447	1445	1724	
	$\sigma_{st,23}$ kg/cm ²	1581	1273	1425	1428	1635	
	$\sigma_{st,32}$ kg/cm ²	1581	1273	1425	1428	1207	
	$\sigma_{st,34}$ kg/cm ²	1604	1291	1447	1445	1274	

Conclusions and Acknowledgments

From Table II we obtain following conclusions :

(1) Effectiveness of panting stringers and web frames to side frames of the panting structure designed as "Web and Stringer System" is practically sufficient in this case.

(2) Analysis of them by the proposed conventional method gives fairly good approximation compared with the results derived from accurate theory.

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