Considerations on the Critical-Stress for the Maintenance of Propagation of Brittle Fracture in Mild Steel

By Masao Yoshiki, Member*
Takeshi Kanazawa, Member*
and Hiroshi Itagaki, Member**

Abstract

By expressing the surface plastic work done with the aid of Forscher’s conception of relaxation centre\(^1\), the authors examined the significance of the critical stress obtained by the earlier proposed double tension test with flat temperature gradient\(^2\) and explained its temperature dependence. Further the authors studied theoretically and experimentally the correlation between the temperature dependence of the critical stress and that of yielding stress.

1. Introduction

As compared with the stress for initiation of brittle fracture or the stress for starting the propagation of the brittle fracture from the initiated brittle crack (the critical stress obtained from double tension test with uniform temperature\(^3\)), the stress required for the brittle fracture to keep propagating is very small, and subject to temperature influence. The authors in the present article attempt to explain these facts as follows: the brittle fracture of ductile materials such as mild steel is not a perfect brittle fracture; the plastic deformation develops to a certain depth from the fractured surface which depends on the temperature, crack velocity and mode of fracture. Brittle fracture might be regarded a microscopically discontinuous fracture, but according to Boyd\(^4\), a fast-propagating brittle fracture is discontinuous macroscopically as well. Namely, it is an intermittent mode of fracture, which propagates while developing an internal crack ahead of the main crack. In materials such as mild steel this emerges as increased constraint and affects the surface plastic work done, accordingly giving as strong influence on the propagation of brittle fracture as temperature and crack velocity.

2. "n", an Expression of Surface Plastic Work Done

The brittle fracture surface of mild steel reveals an intense disorder of lattice, the degree of which may be taken as an index to the surface plastic work done. When there is a higher degree of such disorder, the surface plastic work done will amount to more, and when there is a lower degree of such disorder, it will amount to less. The degree of disorder of lattice may be represented using the conception of relaxation centre as introduced by Forscher.

Suppose the brittle fracture surface of mild steel develops \(n\) number of relaxation centres and the necessary work to be done for the development of one relaxation centre depends on a given material. Then the surface plastic work done \(S\) will be expressed by

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Suppose the brittle fracture surface of mild steel develops \(n\) number of relaxation centres and the necessary work to be done for the development of one relaxation centre depends on a given material. Then the surface plastic work done \(S\) will be expressed by
where \( S_0 \) = material constant, \( n \) = number of relaxation centres. The number of relaxation centres \( n \) is, according to Forscher, given by

\[
\frac{n}{n} = \frac{1 - n_0}{T_0} e^{\frac{Q}{R T}}
\]

where \( T_0, \sigma_0, a \) = material constants, \( Q \) = activation energy, \( R \) = gas constant.

\[ T = \text{absolute temperature}, \quad \sigma = \text{effective stress} = \left[ \frac{1}{2} \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 + \left( \sigma_3 - \sigma_1 \right)^2 \right]^{1/2}
\]

\( \sigma_1, \sigma_2, \sigma_3 \) = principal stresses, and \( t \) = time at \( t = t_0 \) \( \sigma = \sigma_0 \), at \( t = t \) \( \sigma = \sigma \).

Thus from Eqs. (1) and (2)

\[
S = S_0 \frac{1 - n_0}{T_0} e^{\frac{Q}{R T}}
\]

Using Eq (3), we can easily bring out the influences of temperature, loading rate and triaxiality on the surface plastic work done, as shown in Figs. 1, 2 and 3.

3. Surface Plastic Work Done of Propagating Crack

As stated in the foregoing, the surface plastic work done \( S \) changes with the temperature, loading rate and triaxiality. It would be difficult to analyse these factors simultaneously with the actual propagating crack, in which the changes due to these factors are superposed one on another. To solve this difficulty, the following assumptions are made:

i) Brittle fracture is regarded macroscopically continuous while it is in the first stage (which presents a crystalline appearance), in which brittle fracture is accelerating. The triaxiality in this first stage is put \( d_c \).

ii) In the first stage, the value of fracture stress \( \sigma_f \) will be affected by the crack velocity \( \dot{c} \).

iii) Brittle fracture, after attaining a certain velocity of propagation, assumes to turn into the intermittent mode. The triaxiality in this stage is put \( d_i \).

iv) In the second stage, if the stress \( \sigma \) at infinity in the direction normal to that of crack propagation is constant, the crack velocity \( \dot{c} \) is constant and so will be the fracture stress \( \sigma_f \).

v) The stress \( \sigma_1 \) normal to the direction of crack propagation is as indicated in Fig. 4, where the fat-line may be represented by E. Yoffè's solution of moving Griffith crack, namely

\[
\sigma_1 = \frac{\alpha x - \dot{c} \dot{c}}{\sqrt{x^2 - \alpha_0^2}}
\]

where \( x' = x - \dot{c} \), \( \alpha_0 \) = crack length = constant,

\( \dot{c} \) = crack velocity.

Besides, when fracture may be regarded continuous, the distance \( \varepsilon \), from the tip of crack to the point where \( \sigma \), equals

Fig. 1

Fig. 2

Fig. 3 (\( d \)=fracture stress/effective stress)
to \(\sigma_f\), is constant while \(\sigma_f\) varies with the velocity; when it may be regarded discontinuous, \(\varepsilon\) changes while \(\sigma_f\) remains constant, even if the crack velocity will be increased under the higher stress, because \(\sigma_f\) will be nearly equal to the ceiling value in this stage.

### a) Fracture assumed continuous

The stress rate at the tip of crack \(\dot{\sigma}\), may be derived from Eq. (4) as \(\dot{\sigma} \propto \dot{\varepsilon} \sigma\).

Then, the surface plastic work done \(S\) under the constant temperature may be approximated by

\[
S = S_0 n \approx \text{const.} \cdot \frac{1}{\dot{\varepsilon}} e^{-\frac{a}{\dot{\varepsilon}}} \sigma_f
\]

In the first acceleration stage the stress condition is affected by the external energy such as energies due to impact, sub-tension of double tension test, etc. (simply to be called initiation energies in further reference) but the surface plastic work done might be expressed by Eq. (5).

Differentiating Eq. (5) with respect to \(c\) (crack length), we get

\[
\frac{\partial S}{\partial c} = \text{const.} \cdot \frac{1}{\dot{\varepsilon}} e^{-\frac{a}{\dot{\varepsilon}}} \sigma_f \left\{ -\frac{1}{\dot{\varepsilon}} + \frac{a}{\dot{\varepsilon}} \frac{\partial \varepsilon}{\partial c} \right\}
\]

When \(\dot{\varepsilon}\) is very small, the second term within the braces on the right hand side of Eq. (6) is considered relatively small as compared with the first term; when \(\dot{\varepsilon}\) is large, it is considered that \(\frac{\partial \varepsilon}{\partial c} \approx 0\). Thereupon, Eq. 5 will be represented diagrammatically by the full line curve in Fig. 5. Namely, as the crack goes on propagating, the surface plastic work done will diminish.

### b) Fracture assumed intermittent

Under this assumption we get, from Eq. (4), \(\dot{\sigma} \propto \frac{\dot{\varepsilon}}{\sigma_f}\).

Due to the rise of triaxiality from \(d_c\) to \(d_i\), the surface plastic work done may be reduced sharply as indicated in Fig. 6.

If the influence of initiation energy is non-existent and the stress at infinity \(\sigma\) is constant, the crack velocity also becomes constant, making \(S\) free from dependence on \(c\). However, when it is attempted to propagate at faster rate by giving higher \(\sigma\), the value of \(S\) differs.

To bring \(\sigma_i\) under the influence of velocity in E. Yoffes formula, we put

\[
\sigma = \frac{H}{1 + \mu} \sigma_e
\]

where \(\sigma_e = \text{constant}\), \(H = \lambda - (\lambda + 2\mu)\gamma + 4\mu \beta^2 \frac{\varepsilon^2}{1 + \beta^2}\), \(\gamma^2 = 1 - \frac{\varepsilon^2}{\dot{\varepsilon}^2}\), \(\dot{\varepsilon}^2 = \frac{\lambda + 2\mu}{\rho}\)

\[
\beta^2 = 1 - \frac{\varepsilon^2}{\dot{\varepsilon}^2}, \quad \dot{\varepsilon}^2 = \frac{\mu}{\rho}, \quad \lambda, \mu = \text{Lamé's elastic constants.}
\]
when \( \dot{\varepsilon} \leq \sqrt{\frac{P}{p}} \), \( H=\left(\lambda+\mu\right)\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_t}\right)^2 \), then Eq. (7) is written
\[ S=\text{const.} \cdot \dot{\varepsilon}^3 \quad (\text{or const.} \cdot \sigma^3) \quad (8) \]

In the case of intermittent fracture, \( S \) will become larger in the field of higher stress through which the crack propagates. Fig. 6 represents the relation between the surface plastic work done \( S \) and crack velocity \( \dot{\varepsilon} \) under two different temperature. As seen in Fig. 6 the surface plastic work done \( S \) assumes to be reduced sharply from point \( A_1 \) to \( B_1 \) (or from \( A_2 \) to \( B_2 \)); this is caused by the rise of triaxiality from \( d_1 \) to \( d_1' \) (or \( d_2' \)). As a matter of fact, the triaxiality is considered to change continuously. Then, \( S \) will change following the dotted line in the Figure. As stated before, the value of \( S \) increases with the temperature rise, moreover the transition from continuous to intermittent fracture will become difficult to take place. Under this consideraton the \( S-\dot{\varepsilon} \) curve in Fig. 6 drawn so as to extend upward right as the temperature rises.

As is apparent from Fig. 6, the velocity is determined at which the change of mode occurse under a certain temperature; this velocity is hereafter to be designated critical velocity \( \dot{\varepsilon}_{cr} \).

4. Energy Condition and Crack Velocity

Under a certain temperature the surface plastic work done \( S \) has a minimum value at a certain velocity. Now suppose, under the influence of initiation energy, a crack proceeds to \( \dot{\varepsilon}=\dot{\varepsilon}_0 \) with the velocity \( \dot{\varepsilon} = \dot{\varepsilon}_0 = \dot{\varepsilon}_{cr} \), and consider the case when the crack is hereafter propagates through the critical stress field.

When the propagating crack satisfies the Mott’s energy condition\(^6\), we have
\[
\frac{\partial W_E}{\partial \dot{\varepsilon}} + \frac{\partial W_S}{\partial \dot{\varepsilon}} + \frac{\partial W_k}{\partial \dot{\varepsilon}} = -\frac{\pi c}{(\lambda+2\mu)}\sigma \dot{\varepsilon}^2 + 2S_{\min} + \frac{k\rho \sigma_\text{cr}^2}{2(\lambda+2\mu)^2} \sigma + \frac{k\rho \sigma_\text{cr}^2}{2(\lambda+2\mu)^2} \sigma \dot{\varepsilon}^2 = 0 \quad (9)
\]
where \( \frac{\partial W_E}{\partial \dot{\varepsilon}} = \text{elastic energy release rate} \), \( \frac{\partial W_S}{\partial \dot{\varepsilon}} = \text{surface plastic work rate} \), \( \frac{\partial W_k}{\partial \dot{\varepsilon}} = \text{kinetic energy increase rate} \), \( \dot{\varepsilon} = \text{crack velocity} \), \( S_{\min} = \text{minimum surface plastic work done under a certain temperature} \), \( \kappa = \text{constant} \), \( \rho = \text{density} \).

Seeking the crack velocity from Eq. (9) with \( S_{\min} = \text{const.} \) we obtain
\[
\dot{\varepsilon}^5 = \dot{\varepsilon}_0^5 \{ \frac{c_0 + c_0^2(\alpha-1)}{c^2} - \frac{l}{e} + 1 \} \quad (10)
\]
where \( \alpha = \frac{\dot{\varepsilon}_0^5}{\dot{\varepsilon}_t^5} \), \( \dot{\varepsilon}_t = \text{crack velocity when} \ c \ \text{tends to infinity} \), \( l = \text{material constant} \).

Then, from Eq. (9) and (10), the energy condition becomes
\[
\frac{\pi(\lambda+2\mu)}{2} \sigma \dot{\varepsilon}^2 - S_{\min} = 0 \quad (11)
\]

The crack velocity will vary with the increase in the crack length as shown in Fig. 7. Namely, when the crack propagates into the critical stress field at velocity \( \dot{\varepsilon}_{cr} \), it will continue to do so at nearly constant velocity, accordingly with \( S= S_{\min} = \text{const.} \). When the stress is less than the critical value, the crack velocity will diminish with the increase in crack length. In this case, as explained in the preceding section, the condition \( S= S_{\min} = \text{const.} \) does not hold and \( S \) begins to increase with the decrease in crack velocity. Consequently the crack is arrested as indicated by the dotted line in Fig. 7.
5. Critical Stress obtained by Double Tension Test with Flat Temperature Gradient

Let us consider the condition in which a crack propagates through a specimen having such a temperature distribution as shown in Fig. 8.

The value of $S_{\text{crit}}$, corresponding to the temperature distribution is plotted as the dotted line in Fig. 9. If the stress value is such as can give critical velocity for $T = T_1$, $S$ for $T < T_1$ will be larger than $S_{\text{crit}}$ corresponding to the temperature $T < T_1$ as shown in Fig. 9, because the crack velocity for $T < T_1$ may be larger than the critical value.

Representing the surface plastic work rate and the elastic energy release rate as $\frac{\partial W_s}{\partial C}$ and $\frac{\partial W_e}{\partial C}$ respectively, we obtain the relation between these rates and the crack length as shown in Fig. 9.

Then the difference between the elastic energy release rate and the kinetic energy increase rate will be, from Eq. (11),

$$I_{\sigma_{cr}}^2 = S_{\text{crit}}(T_1).$$

Next, if the stress value is less than such as can give the critical velocity for $T = T_1$, the fracture of intermittent mode turn into the continuous mode; accordingly the triaxiality changes from $d_i$ to $d_c$, sharply elevating the value of $S$. At the instant the surface plastic work rate varies with $C$ as illustrated by Fig. 10, namely $S_t$ at $C = c_t$ denotes such a large value of $S$ as cannot be deemed to represent brittle fracture surface. Therefore, crystalline appearance of the fractured surface will be ended at the crack length $C = c_t$. In the other word brittle fracture will be arrested at $C = c_t$.

In the analysis so far made, we did not consider the difference of the value of the surface plastic work done in the direction of plate thickness, it will be shown as in Fig. 11. Accordingly, $\eta$-values differ from plate surface to plate core.

Let the mean of the surface plastic work done, i.e. $S_{\text{mean}}$, be given by

$$S_{\text{mean}} = S_{\text{crit}} \eta_{\text{mean}}$$

where $\eta = \text{constant independent on material}$; this value may be smaller as the plate thickness is larger.

Then replacing $S_{\text{crit}}$ of Eq. (11) with $S_{\text{mean}}$, we get

$$\sigma_{cr}^2 = \frac{\pi(\lambda + 2\mu)}{I} S_{\text{mean}} \eta_{\text{mean}}$$

and from Eq. (8) we derive

$$\eta_{\text{mean}} = \text{const.} \sigma_{cr}^{3/2} e^{-\frac{q}{RT}}$$
Therefore
\[ \ln \sigma_{cr} = K_0 - \frac{Q}{RT} \]

where \( p' = \frac{2p}{4-3p} \) = constant independent on material.

Thus Eq. (14) gives the temperature dependence of \( \sigma_{cr} \).

Diagrammatical representation of Eq. (14) is Fig. 12. The critical velocity necessary to change fracture from continuous mode to intermittent one becomes larger and larger as the testing temperature rises, and it will approach the maximum velocity that a brittle fracture can attain, rendering the propagation of intermittent fracture impossible. This is the so-called upper arresting temperature. Moreover, Eq (14) will lose validity in the extremely low temperature zone and the high temperature zone near the upper arresting temperature, because the value of \( p' \) in Eq. (14) may be altered in these temperature zones, and then the curve may turn out as the dotted line. Fig. 13 & 14 exemplifies the experimental results by double tension test with flat temperature gradient.

As stated before the surface plastic work done at the time of initiation of propagation of brittle fracture is extremely large as compared with that while a brittle fracture is propagating. So it is understood that it needs a little energy for the propagation to be maintained, whereas it needs much energy for a brittle crack to be accelerated to a certain velocity and turned into intermittent mode of fracture. This is the reason why the critical stress for maintenance of propagation is small.

In the double tension test with flat temperature gradient we have the temperature distribution such as shown by Fig. 8; when the applied stress is larger than the critical one, the brittle fracture of intermittent mode propagates through the uniform temperature region. Therefore the critical stress obtained from the double tension test with flat temperature gradient is the stress necessary for maintenance of propagation. On the contrary the critical stress obtained from the double tension test with uniform temperature is the stress for acceleration of propagation, which has, generally, a higher value than the critical stress for maintenance of propagation and may depend on the initiation energy.

### 6. Relation between the Temperature Dependence of Yielding Stress and that of Critical Stress

According to Forscher the temperature dependence of yielding stress in mild steel is expressed by
\[ \ln \sigma_y = K_0 - \frac{Q}{RT} \left( \frac{1}{1 + a(\sigma_y - \sigma_c)} \right) \]  

When the stress rate is exceedingly small, we can obtain the following equation from Eq. (15)
\[ \ln \sigma_y = \frac{Q}{RT} - \ln a' \]  

If the values of \( Q/R \) in Eq. (16) and \( 'pQ/R \) in Eq. (14) are obtained from experimental results of
tension test and double tension test with flat temperature gradient under various temperatures, we can get the value of \( p' \), and accordingly \( p \). The value of \( Q/R \) and \( p'Q/R \) of steels tested (plate thickness being 38~45 mm) are given in Fig. 15. From this figure we can see that the value of \( p \) lies between 1.25 and 1.30.

**Conclusion**

The surface plastic work done during the propagation of brittle fracture in mild steel, characteristics of which is represented by \( n \), is studied for its relation with the mode of fracture and the temperature. Through this study it is confirmed that the brittle fracture of mild steel is sensitive to the temperature; the stress for maintenance of propagation is very small compared with the stress for acceleration of a brittle crack. Meanwhile the conclusion is reached that the critical stress obtained from the double tension test with flat temperature gradient is the stress for maintenance of propagation of brittle fracture. Further it is deduced and experimentally checked that the temperature dependence of this critical stress may be expressed by Eq. (14).

**Acknowledgement**

It should be noted with sincere gratitude that some of the experimental data cited in the present paper have been furnished from the results of the joint research carried out by the 37th research committee of the Shipbuilding Research Association of Japan.

**References**