A Study on the Bilge Keels


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Summary

The eddy-making resistance due to the bilge keels was investigated in the previous papers. The effect of the ship form on the eddy-making resistance to the rolling of a ship hull is discussed in the present paper from the results of the systematic two-dimensional model experiments. It is found that the eddy-making resistance to the rolling of a ship hull is large in the case of a rectangular section as much as the resistance due to the bilge keels and decreases rapidly as a radius of bilge circle increases, and that the resistance to a parallel middle body of a usual ship becomes negligibly small but its resistance to a fore or after section of a ship becomes a considerable amount.

Next, the eddy-making resistance to the rolling of a ship hull is found to follow approximately the law of comparison by Froude from the results of the similar rhombic model experiment, and a method of approximate estimation of the resistance to the rolling of an actual ship is obtained.

1 Introduction

The eddy-making resistance to the rolling of a ship is divided into two kinds; the resistance to a ship hull and the one due to the bilge keels. The latter was studied in the previous papers and a method of approximate calculation was obtained, and the former is investigated here.

The eddy-making is generally influenced by a separation of flow, therefore, it is not only related to Reynolds' number but also to the form of a body. The coefficient of eddy-making resistance to some forms, such as a circular cylinder or a flat plate, in a uniform flow was obtained experimentally as a function of Reynolds' number. The value of its coefficient is approximately constant at high Reynolds' number. If we limit the subject within the rolling of a ship, an isolated vortex such as is generated behind the bilge keels makes the eddy-making resistance increase conspicuously, and the generation of such the vortex depends mainly on the ship form and may be almost independent of Reynolds' number. Then the eddy-making resistance to the rolling of a ship hull may be supposed to be almost independent of Reynolds' number.

It is both difficult to measure the eddy-making resistance experimentally and to treat it theoretically since the mechanism of generation and the motion of the isolated vortex are yet unknown. In the present paper, first, the rolling resistance to a ship hull is obtained from the systematic two-dimensional model experiment. Subtracting the frictional resistance from the obtained total resistance, the sum of the eddy-making and the wave-making resistance is obtained, and the relation between the ship form and the eddy-making resistance is found supposing that the effect of the ship form on the wave-making resistance is smaller than that on the eddy-making resistance.

If we suppose the eddy-making resistance to the rolling of a ship hull is independent of Reynolds' number, the sum of the eddy-making and the wave-making resistance to the rolling of a ship hull may follow the law of comparison by Froude. This is confirmed by the similar rhombic model experiment and a method of approximate estimation of the rolling resistance to an actual ship is obtained.

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2 Symbols

The meaning of the symbols is as follows:

- $R_{ae}$, $R_{ha}$, $R_{hf}$ = the eddy-making, wave-making and frictional resistance to the rolling of a ship hull,
- $R_{h(e+w)} = R_{ae} + R_{ha}$,
- $C = \text{the coefficient of } R_{h(e+w)}$,
- $\rho = \text{density of water}$,
- $S = \text{the wetted surface area}$,
- $r = \text{the maximum distance from the rolling axis to the bilge circle or KG}$,
- $T = \text{the rolling period}$,
- $T_p = \text{the pitching period}$,
- $\theta_m = \text{the mean rolling amplitude during a swing}$,
- $R = \text{the radius of bilge circle}$,
- $B = \text{the breadth of ship}$,
- $d = \text{the draft of ship}$,
- $\alpha = \text{the angle of inclination of ship side}$,
- $\beta = \text{the angle shown in Fig. 2}$,
- $\delta \theta, \Delta \theta = \text{the decrement of roll per swing for a model (or ship) without, with bilge keels}$,
- $\delta \theta_{fm}, \delta \theta_{fa} = \text{the decrement of roll per swing due to the friction calculated by prof. Kato's formula)}$ for a model without bilge keels, for an actual ship with bilge keels,
- $\delta \theta_{BK} = \text{the decrement of roll due to bilge keels only per swing for an actual ship estimated by the previous paper}$.

3 Three Dimensional Model Experiment

Before investigating the effect of the ship form on $R_{ae}$, here, the distribution of the rolling resistance along the length of the trawler ship model is obtained. Cutting the model in ten parts at its square stations, free damping experiments on the rolling of each above-mentioned part have been made at the same condition as the trawler model in smooth water. A circular cylinder whose length is 50 cm and the radius 15 cm is fitted to the each part for the purpose of the suitable stability. The work done per swing of free roll, $A$, by the trawler model having a rolling axis at $G$ is equal to the sum of the works, $\Delta A$, done per swing by each part about the same axis. Subtract the work done by a circular cylinder during a swing from the work done by a model fitted to the cylinder during a swing and the remainder is the work done by a model $\Delta A$. The obtained experimental $(\Delta A/\sum \Delta A) \delta \theta = \Delta \theta$ is shown in Fig. 1, where $\delta \theta$ is the decrement per swing for a trawler model and $\Delta \theta$ is the component of the decrement of roll per swing along the length of a model, then $\sum \Delta \theta$ makes $\delta \theta$. In this figure, as the trawler model has an initial trim, the rolling resistance is comparatively small to the fore part, but to the after part it is extremely large.

In case of even keel, we can infer that the isolated vortex will be generated at the time of rolling behind a fore or after section of a ship, where $R_{ae}$ will be the same order as the total resistance to the other parts, and that the separation will not develop into a vortex at a parallel middle body of a ship, where $R_{ae}$ will be small because the total rolling resistance is found to be comparatively small in spite of a large amount of the displacement.

4 Two Dimensional Model Experiment

To investigate the effect of a ship form on $R_{ae}$ in detail, the systematic two dimensional model experiments of the rolling have been carried out and also the observations of the stream around the
bilge circle have been made. The models whose length are 50 cm are constructed with wooden pieces combined in various ways as shown in Fig. 2, and two circular cylinders whose length are 88 cm and radius 25 cm are fixed to both sides of a model ship for the purpose of the suitable stability. Subtract the work done by two cylinders during a swing from the total work done by a model fitted to the cylinders and the remainder is the work done by the model δE. The work done δE_{h(e+w)} by both \( R_{h(e)} \) and \( R_{h(w)} \) during a swing is obtained by subtracting from \( \delta E \) the work due to the friction, \( \delta E_{hfr} \), calculated by prof. Kato's formula\(^1\).

If \( R_{h(e+w)} \) is expressed by
\[
R_{h(e+w)} = \frac{1}{2} \rho C S v^2. \tag{1}
\]
the work \( \delta E_{h(e+w)} \) is given by the formula,
\[
\delta E_{h(e+w)} = \frac{8}{3} \rho C \pi^2 S r^3 \theta_m^3 / T^2 \tag{2}
\]
in g, cm, sec, radian.

C value is obtained after analysing the results of two dimensional model experiments and the characters of \( R_{h(e)} \) are brought to light.

(1) Series A. The experiments on the models with wall-side have been made. C value is shown in Fig. 3, as an example, for the models in which the breadth \( B = 50 \) cm, the draft \( d = 17.5 \) cm, \( KG = 20 \) cm and the radius of bilge circle \( R \) varies from 0 to 5 cm in nine ways. C value is extremely large at \( R = 0 \) as shown in this figure and decreases rapidly as \( R \) increases, but C does not diminish so
much when $R$ increases beyond $R=5$ cm. This radical variation of $C$ value by $R$ is mainly caused by $R_{he}$, therefore it is found that $R_{he}$ is very large when $R$ is small and decreases rapidly in accordance with the increase of $R$, and $R_{he}$ will be small when $R \geq 5$ cm.

An interesting sidelight on the motions of water around the above models has been obtained by dropping blue ink into the water so as to make the motion visible in the observation of the rolling of models through the observation window in the tank wall. One of the examples of the stream around the model ($B=50$ cm, $KG=20$ cm, $d=17.5$ cm) is shown in Fig. 4 ; (a) is for $R=5$ cm, (b) for 3 cm and (c) 2 cm respectively. When $R=5$ cm, ink moves along the ship surface and normal flows to the ship surface can not be observed except the free water surface, so $R_{he}$ may be negligibly small there. The vortex can not be observed when $R=3$ cm and something short of a vortex is observed when $R=2$ cm. An isolated vortex such as is generated behind the bilge keels is observed when $R=0$.

In order to examine the scale effect of the generation of the vortex, the observations have been carried out for two models whose breadth are 30 cm and 80 cm ($KG=20$ cm, $d=17.5$ cm in common). Both the normal flows to the ship surface for $R \geq 0.1B$ and a vortex for $R \geq 0.0625B$ can not be observed, but an isolated vortex is observed for $R \leq 0.05B$. The effect of the rolling period and amplitude on the eddy-making is scarcely noticeable, but the eddy-making becomes conspicuous regardless of $R$ as $KG$ or the draft increases.

From the results of the experiments the following empirical formula is obtained for $\theta_m$ is moderately large.

$$C = f_1 \left( \frac{B}{KG} \text{ or } \frac{KG}{B} \right) e^{-a \frac{R}{d}}$$  \hspace{1cm} (3)

The value of $f_1$ is shown in Figs. 5 & 6, and $a$ in Fig. 7. The calculated value of $C$ for the above
example by the formula (3) is shown in Fig. 3. $R_{he}$ occupies the majority of the total resistance when $R$ is small, where the characters of $R_{he}$ may be expressed approximately by the formula (3). $R_{he}$ is comparatively small as seen in Fig. 3 or the observations of the stream when $R \leq 0.1B$, where the calculated $C$ value by (3) may be underestimated.

It was supposed several years ago that $R_{he}$ to an extremely full ship such as a super tanker is a considerably large amount. The calculated value of $C_{(\alpha=15\degree)}$ by the formula (3) is equal to 0.04 at the midship section of 65,000 T super tanker in a full load condition, and therefore $R_{he}$ is considered to be negligibly small to a parallel middle body of a full ship.

(2) Series B...The experiments on the models with the sloping ship side have been made. The results obtained are the following,

$$C = f_1 \left( \frac{B}{KG} \right) f_2(\alpha) e^{-\frac{aR}{d}}$$

where $f_2(\alpha)$ is shown in Fig. 8, $\alpha$ is the angle of inclination of ship side and $B$ is the breadth of model at the water plane.

It is found from Fig. 8 that $C$ decreases as $\alpha$ increases within a certain value of the inclination of ship side, $\alpha_c$, and $C$ conversely increases as $\alpha$ increases beyond $\alpha_c$, and that $\alpha_c$ decreases as $R/d$ increases.

Dr. Watanabe and others investigated experimentally on the effect of the flare using a model having a large radius of bilge circle and found that $R_{he(\alpha, d)}$ increased as the increase of the flare. $R_{he}$ is negligibly small to such the model having a large radius of bilge circle, then $R_{he}$ increases as the flare increases is found from Dr. Watanabe’s experiment. To the contrary, $R_{he}$ decreases as $\alpha$ increases is found from the result that $C$ decreases as $\alpha$ increases, and $R_{he}$ cannot be neglected when both $R$ and $\alpha$ are small.

The $\alpha$ is equal to about 10° at a usual fore section, where $R_{he}$ can not be neglected, and the total resistance to a fore section is about 70% of the one when $\alpha=0$. The value of $KG$ is equal to 20 cm in these experiments, but $R_{he}$ becomes large rapidly as $KG$ or the draft increases as shown below in Series D.

(3) Series C...The relation between $R_{he}$ and a form of bilge is obtained in detail. The results
obtained are shown in Fig. 9, where $\Delta R$ is a virtual increment of $R$ by the angle $\beta$ shown in Fig. 2. The virtual increment $\Delta R$ is defined as follow; $R_{he}$ decreases as $\beta$ increases, then the decrement of $R_{he}$ due to $\beta$ is defined to be equal to the decrement of $R_{he}$ due to the virtual increment $\Delta R$ of $R$.

For an example, $\Delta R$ is about 1 cm when $B=30$ cm, $\beta=20^\circ$ and $R=1$ cm, so $C$ can be calculated by the formula (3) substituting $R=1$ cm $+1$ cm $=2$ cm. $\Delta R/B/2$ increases rapidly as $\beta$ increases and $R_{he}$ is small to the form such as $\beta \geq 20^\circ$, then a chine line as seen in a small ship does not make so much increase the eddy-making resistance to the rolling.

(4) Series D, E···The experiments on the models representing the fore or after section of ships are added. At first, the experiments on the models, with $\alpha$ being equal to $10^\circ$ and the form of the bottom being parabolic as shown in Fig. 2, have been made changing the draft. The form of the bottom is determined to have the first continuous derivative on the whole surface because the discontinuity induces the eddy-making. In order to explain the experimental results by the formula (4) an equivalent $R$ to the above parabolic form is considered. The equivalent $R$, which is shown in Fig. 10, decreases as $KG/B$ increases, and $R_{he}$ is conspicuous and has no relation with the form of the ship bottom when $KG/B \geq 2$. The value of $KG/B$ at a usual fore section is beyond 2, then $R_{he}$ is considerably large there.

Nextly, the experiment has been made on the special formed model representing the square station 1/2. The models and the obtained $C$ value are shown in Fig. 11. $R_{he}$ to this section is extremely large as shown in this figure, where an isolated vortex such as is generated behind the bilge keels is observed. $R_{he}$ to the after section is also found to be comparatively large from the above trawler model experiment.


A method of approximate estimation of the rolling resistance due to the bilge keels was shown in the previous paper\(^2\). Considering $R_{hw}$ and $R_{hr}$, that $R_{hw}$ follows the law of comparison by Froude

<table>
<thead>
<tr>
<th>Model</th>
<th>$L$(cm)</th>
<th>$B$(cm)</th>
<th>$d$(cm)</th>
<th>$W$(kg)</th>
<th>$KG$(cm)</th>
<th>$GM$(cm)</th>
<th>$T$(sec)</th>
<th>$T_f$(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>90</td>
<td>20</td>
<td>7</td>
<td>8.52</td>
<td>6.66</td>
<td>0.97</td>
<td>1.52</td>
<td>0.72</td>
</tr>
<tr>
<td>$A_2$</td>
<td>180</td>
<td>40</td>
<td>14</td>
<td>68.16</td>
<td>13.32</td>
<td>1.94</td>
<td>2.15</td>
<td>1.00</td>
</tr>
<tr>
<td>$A_3$</td>
<td>270</td>
<td>60</td>
<td>21</td>
<td>230.04</td>
<td>19.98</td>
<td>2.91</td>
<td>2.64</td>
<td>1.25</td>
</tr>
</tbody>
</table>
was known and new formula for $R_{hf}$ was obtained by Prof. Kato, and it is necessary to establish the formula for $R_{he}$ in order to extend the results of model experiments to actual ships.

The free rolling experiments on the geometrically similar rhombic three models have been made in similar conditions. The smallest model is shown in Fig. 12, and the dimensions of the others are shown in Table 1, in which the conditions of the experiments are also illustrated. In these conditions $R_{hw}$ is negligibly small from Prof. Hishida's formula. Obtained value of $(\delta \theta - \delta \theta_f)$ is shown in Fig. 13. The calculated value by means of the strip method of $(\delta \theta - \delta \theta_f)$ by the use of the method of estimation of $R_{h(e+w)}$ shown in the previous article is also shown by the dotted line in Fig. 13. $R_{hw}$ is negligibly small and the majority of the experimental $(\delta \theta - \delta \theta_f)$ is due to $R_{he}$ in these conditions, therefore, $R_{he}$ is found from this figure to follow approximately to Froude's law. $(\delta \theta - \delta \theta_f)$ is small compared with the total rolling resistance to a ship with bilge keels, where we may neglect the scale effect of $R_{hf}$.

In the present paper, the following law of comparison for similar ships on the resistance to the rolling of a ship is considered;

$$\Delta \theta_a = \delta \theta_m - \delta \theta_{fm} + \delta \theta_{fa} + \delta \theta_{BK_a}$$  \hfill (5)

![Fig. 12](image1)

![Fig. 13](image2)

![Fig. 14](image3)

![Fig. 15](image4)
where the suffixes \( m \) and \( a \) are referred to a model and an actual ship respectively. In the formula (5), we must obtain \( \Delta \theta_m \) from a model experiment but the other terms are calculated by the method as illustrated in the article 2.

In order to assure the above similarity law, the rolling experiments on two kinds of models and actual ships as shown in Table 2 have been made in similar conditions. The scale of model is 1/25 for \( K \) Ship and 1/35 for \( N \) Ship. The obtained experimental results and the calculated results by the formula (5) of \( \Delta \theta_a \) are shown in Figs. 14 & 15. The calculated values agree fairly, although a little under-estimated, with the experimental results as long as these ships are concerned.

### 6 Conclusion

The characters of the eddy-making resistance to the rolling of a ship hull have been brought to light from the systematic two dimensional model experiments. They are summarized as the following:

1. The eddy-making resistance to the rolling of a ship hull is proportional to \( \exp \left( -\frac{aR}{d} \right) \), where 1 is shown in Fig. 7, \( R \) is the radius of bilge circle and \( d \) the draft, then its resistance is scarcely noticeable to a parallel middle body of a usual ship.

2. In the case of the value of \( KG/B \) is larger than 2, where \( B \) is the breadth of ship at water plane, the eddy-making resistance to the rolling of a fore or after part of a ship becomes a considerably large amount.

3. The angle of inclination of ship side makes the eddy-making resistance to the rolling decrease, and a chine line as seen in a small ship does not make so much increase its resistance.

The eddy-making resistance to the rolling of a ship hull is found to follow the law of comparison by Froude from the analysis of the results of the similar rhombic model experiments, and a method of approximate estimation (the formula (5)) of the rolling resistance to an actual ship with bilge keels is obtained.

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### References

2. N. Tanaka and H. Kitamura; A Study on the Bilge Keels (Part 2) JSNA vol. 103, 1958.