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A Study of Vertical Vibration of Ships

(2nd report)

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Summary

Critical frequencies of the vertical vibration were calculated for 2 tankers, 4 high speed cargo liners and an ore-carrier, by means of the digital computation method described in the previous report⁽¹⁾.

The results were compared with the frequencies measured in the sea trials. Although there are some discrepancies between the computed and measured frequencies, the estimation of the frequencies of tankers and bulk carriers can be made with good accuracy by means of the empirical coefficient defined in this report.

Further the authors investigated the effects of the added virtual mass, effective breadth of the plating and the rotational inertia, the results of which showed that the rotational inertia can be neglected up to 6~7 nodal model. For the ships having no longitudinal bulkheads such as bulk carriers and cargo ships, the effective breadth factor seems to affect the frequencies to some degree, but the detailed study on this subject will be investigated in a subsequent report. For the practical computation, Landweber's two-parametre family with Lewis' correction is fairly applicable to the estimation of virtual added mass.

Contents

1. Introduction.
2. Computation method and the numerical accuracy of the computer program.
3. Preparation of data for computation.
4. Comparison of computed with measured results. Empirical coefficient.
5. Discussions on the ship parametres.
 - A. Virtual added mass
 - B. Rotational inertia
 - C. Effective breadth
6. Conclusions and acknowledgement.

1 Introduction

Recently the application of digital computer technique to the ship vibration problem has been undertaken by many authors^{(2)~(7)}. Their results are very useful and expiring to the people who are engaging in this field. The only regret from the designer's point of view is the fact that the ratio of calculated versus measured frequencies is considerably different.

To overcome this difficulty and to establish the practical estimation technique, the programme was

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undertaken in the Mitsubishi Heavy Industries, Ltd., which includes the measurements of hull vibration in the sea trials and the computations of natural frequencies. A part of the results was already published in the 1st report⁽¹⁾ in which 2 bulk carriers and 2 destroyers were dealt with.

This report is a continuation of the previous one and includes the results on 2 tankers, 3 cargo liners and an ore-carrier. Also are included the considerations on the effect of the virtual added mass, rotational inertia and effective breadth upon the natural frequency.

2 Computation method and the numerical accuracy of the computer program.

The details of the method were shown already in the previous report⁽¹⁾ and are omitted in this report. Some remarks on the computer program are as follows.

- (1) Bending rigidity, shear rigidity and rotational inertia are taken into account.
- (2) The ship's length overall is divided into many segments which are not necessarily equal length. The number of segments can be taken up to 70.
- (3) Within each segments the ship parameters such as weight, and rigidity are taken uniform.
- (4) Computation outputs are ; natural frequencies and the correspondind deflections, bending slopes, bending moments and shear forces.

For the purpose of examining the numerical accuracy of the program, the calculations were made for the uniform free-free beam and the results are compared with theoretical one in Table 1. Also are shown in Table 2 the comparisons of per centage errors of the present method with those of the other authors. It is seen from this table that, although the method by De Vries gives the least

Table 1 Comparison of computed natural frequencies with exact solution for uniform free-free beam.

No. of Nodes	Bending Only			Bending+Shear		
	Exact Solution	Computer Result	Error*	Exact Solution	Computer Result	Error*
2	50.82 cpm	50.82 cpm	0 %	46.49 cpm	46.48 cpm	-0.02 %
3	140.05	140.1	0.04	106.40	106.4	0
4	274.67	274.9	0.08	171.63	171.7	0.1
5	453.80	455.4	0.35	236.04	236.6	0.3
6	677.91	680.0	0.31	299.17	300.6	0.5
7	952.33	—	—	360.94	363.8	0.8

$$* \text{ Error} = \frac{\text{Computer Result} - \text{Exact Solution}}{\text{Exact Solution}} \times 100\%$$

Table 2 Comparison of numerical errors of various authors* for the natural frequencies of uniform free-free beam.

Author	Leibowitz				Vreug- Denhil	De Vries	Present Authors	
	Bending Only		Bending+Shear		Bending Only	Bending Only	Bending Only	Bending +Shear
Divisions for Calculation	20 Equal Divs. %	40 Eq. Divs. %	20 Eq. Divs. %	40 Eq. Divs. %	20 Eq. Divs. %	20 Eq. Divs. %	20 Eq. Divs. %	20 Eq. Divs. %
2-Node	0.14	0.04	0.02	0	-2.9	0	0	-0.02
3- "	-0.3	-0.04	-0.6	-0.1	-2.0	-0.004	0.04	0
4- "	-1.2	-0.3	-1.4	-0.4	-4.2	-0.01	0.08	0.1
5- "	-2.5	-0.6	-2.4	-0.6	—	0.04	0.35	0.3
6- "	-4.2	-1.0	-3.5	-0.9	—	0.09	0.31	0.5
7- "	-6.9	-2.1	-4.9	-1.2	—	0.21	—	0.8
8- "	—	—	—	—	—	0.45	—	—

* Definition of error is the same as Table 1.

ror, the present method gives a fairly good results for practical use.

3 Preparation of data for computation.

The ships for which calculations were carried out are shown in Table 3, together with their particulars. Of these ships listed, ships G and H were described already in the previous report⁽¹⁾, and are added here for completeness.

Table 3 List of ships investigated in this report

Ship Name	Type	Lpp×B×D (m)	Full load condition			Test condition		I _V *** (m ⁴)
			Cb	dm*(m)	Δ**(ton)	dm*(m)	Δ**(ton)	
A	Cargo Liner	145×19.5×12.3	0.667	8.82	17045	4.63	8100	****
B	Do.	145×19.5×12.3	0.667	8.82	17045	6.24	11400	
C	Do.	145×19.5×12.3	0.667	8.82	17045	8.35	15900	
D	Do.	150×23×12.8	0.559	9.342	18472	4.952 6.179	8667 11356	52.75
E	Tanker	213×30.5×15.2	0.801	11.485	61270	8.34 11.42	43070 61392	187.1
F	Do.	218×32.9×15.6	0.811	11.557	68933.5	5.223 11.155	29030 66289	212.03
G	Bulk Carrier	192×27.5×16.1	0.791	10.78	46130	5.43	21652	152
H	Bulk Carrier	183×27.4×14.8	0.813	14.8	41680	5.09	19900	115.46
I	Ore Carrier	214×30.8×15.7	0.816	11.637	64163	6.11	32167	198.3

Notes: * Mean draught

** Displacement

*** Moment of inertia of midship section

**** 44.03 up to upperdeck, 57.1 up to bridge deck, 79.52 up to boat deck.

To compute the frequency, the lengthwise distribution of the ship parameters, such as weight (including added virtual mass), bending and shear rigidity are necessary. These data were made according to the method shown in the previous report⁽¹⁾ and the results are shown in Table 7~10 in the following paper. In the tables, the segment number is counted from stern to bow. Location means the coordinates of after and fore ends of the corresponding segments. These coordinates are expressed as the fraction of ship's length overall and measured from the aft end of ship.

Ships A, B and C are sister ships, the only difference being in the weight distributions. Thus the input data for these three are listed together in Table 7.

Ship D, whose input data are shown in Table 8, is a cargo liner with bridge semi-aft, while ship A, B and C are those with bridge amidships. The difference of rigidity distributions between these two types can be seen clearly from Table 7 and 8.

Input data of ships G and H were shown in the previous report⁽¹⁾.

Ship E is a tanker whose rigidity distributions are similar to ship F and her input data are omitted for brevity.

For the cargo liners with bridge amidships or semi-aft, the calculation of bending and shear rigidity has some ambiguity, because the effectiveness of the bridge is not so clear. In this study the calculation of frequencies were made for three cases; i.e. the longitudinal members up to upper deck, bridge deck and boat deck were taken into account for each case respectively (see Table 7 and 8).

4 Comparisons of computed with measured results.

Empirical coefficient.

The computed frequencies, together with the measured ones, are shown in Table 4. For convenience the results are also shown in Fig.1 through Fig.9 From these figures and table following features can be observed.

(1) In the figures where number of nodes is taken as abscissa and the frequency as ordinate, the

Table 4 Comparison of computed with measured

Ship Name	Δ ton	2-node			3-node			4-node		
		measured	computed	meas/comp.	measured	computed	meas/comp.	measured	computed	meas/comp.
A	8,100	107	107.5	0.995	191	193.5	0.987	—	288.3	—
B	11,400	100.5	99.5	1.010	182	180.3	1.010	260	265.5	0.979
C	15,900	86	87.9	0.978	158	161.7	0.977	—	253.4	—
D	8,667	96	93.7	1.025	178	177.5	1.003	—	267.4	—
E	11,356	93	88.0	1.057	—	169.9	—	—	250.0	—
	43,070	—	51.4	—	109	104.5	1.04	(167)	161.2	(1.04)
F	61,392	—	48.5	—	(105)	97.2	(1.08)	155	150.3	1.03
G	29,030	54	49.5	1.09	(118)	104.9	(1.125)	174	165.1	1.05
H	66,289	48.8	43.7	1.12	96	88.1	1.09	148	137.6	1.08
I	21,652	67.5	66.4	1.02	—	133.7	—	192	209.3	0.92
J	19,900	65	63.5	1.02	130	129.9	1.00	181	196.6	0.92
K	32,167	56	50.9	1.10	115	107.4	1.07	—	171.4	—

() means the value with less accuracy.

Fig. 1 Comparison of computed with measured frequencies of ship A (Cargo Liner).

Fig. 2 Comparison of computed with measured frequencies of ship B (Cargo Liner).

Fig. 3 Comparison of computed with measured frequencies of ship C (Cargo Liner).

Fig. 4 Comparison of computed with measured frequencies of ship D (Cargo Liner).

frequencies for ships listed in Table 3

5-node			6-node			Remarks
measured	computed	meas/ comp.	measured	computed	meas/ comp.	
—	389	—	—	494	—	Longitudinal member up to boat deck taken for rigidity calculation D ₀
—	352	—	—	420	—	
—	333	—	—	461	—	
—	360	—	—	432	—	
—	338	—	—	267.9	—	
218	218.8	1.00	—	252.2	(1.05)	
—	202.3	—	(265)	276.1	—	
(230)	216.8	(1.06)	—	233.8	—	
195	188.2	1.04	—	—	—	
—	282.3	—	312	—	—	
222	261.6	0.85	—	—	—	
242	236.4	1.02	—	—	—

Fig. 5 Comparison of computed with measured frequencies of ship E (Tanker).

Fig. 6 Comparison of computed with measured frequencies of ship F (Tanker).

Fig. 7 Comparison of computed with measured frequencies of ship G (Bulk Carrier).

Fig. 8 Comparison of computed with measured frequencies of ship H (Bulk Carrier).

Fig. 9 Comparison of computed with measured frequencies of ship I (Ore Carrier).

For the higher modes, i.e. above 4 node mode, any definite conclusion can not be drawn for the present because the measured data are lacking. From Fig. 2, however, it is supposed that the cargo liner behaves like bulk carrier in the higher mode region, i.e., the actual frequency is lower than the computed value.

In Fig. 10 and 11 are shown the comparison of computed and measured mode curves. Roughly speaking, the agreements between computed and measured mode curves are reasonable. For more minute inspection the detailed measurement is necessary, using many, say more than 20 measuring points along the ship length.

For the frequency estimation in the design stage, the accuracy of estimation is improved by using

Fig. 10 Comparison of computed with measured mode curves of ship F for 66,289 Tons.

computed and the measured values are almost parallel for tanker and ore carrier.

In other words, for tanker and ore carrier, the difference between computed and measured frequencies is almost constant in every modes, calculated frequencies being lower than measured ones.

(2) On the contrary, for bulk carriers, the differences between computed and measured frequencies are very small for the lower mode and becomes larger for the higher mode, the computed values being higher than those measured.

(3) In the case of cargo liner, the agreements of computed and measured frequencies are excellent for the lower modes if the longitudinal members up to boat deck are taken into account for the rigidity calculation.

Fig. 11 Comparison of computed with measured mode curves of ship H.

Table 5 Empirical coefficient for frequency estimation.

Type of Ship	2 node	3 node	4 node	5 node	Notes
Tanker & Ore Carrier	1.10	1.07	1.06	1.02	In the rigidity calculation the longitudinal members up to boat deck are taken into account
Bulk Carrier	1.02	1.00	0.92	0.85	
Cargo Liner	1.02	1.01	0.98	—	
Destroyer	1.01	1.02	1.05	1.00	

$$\text{Empirical Coefficient} = \frac{\text{Measured Frequency}}{\text{Computed Frequency}}$$

empirical coefficient. Here the empirical coefficient is defined as the ratio of measured versus computed frequency. From Table 4 and reference⁽¹⁾ the empirical coefficients as shown in Table 5 have been derived. Judging from the present data, we can estimate the frequency with fair accuracy (say within 3% of errors) by means of these empirical coefficients.

5 Discussions on the ship parametres.

In the previous section the empirical coefficients were derived for frequency estimation by electronic computer. It is believed that this method can serve quite well for the practical use.

Nevertheless, the essential problem regarding the hull vibration has not completely been solved both from the theoretical and practical point of view unless the discrepancies between computed and measured frequencies have vanished.

Since the numerical accuracy of the computer program is sufficient for frequency prediction as mentioned in article 2, the discrepancies between computed and measured frequencies are supposed to have arisen from the fact that the actual configuration of hull vibration is different from that assumed in the theoretical computation.

The difference is supposed to be classified into the following items.

Inaccuracy in the prediction of ship parametres

In preparing the input data for computation, ship parametres such as added virtual mass, rotational inertia, bending and shear rigidity has been derived by some existing theories. If there are some errors in these derivations, the results may be inaccurate.

Roughness in the input data preparation

Even if the above mentioned ship parametres have been derived reasonably, the results of frequency computation again become inaccurate if the numerical roughness of the input data is considerable. As shown in Tables 7~10, ships' length overall has been divided into 40~50 segments in the frequency computation, but this does not mean that rigidity, weight etc. have been predicted exactly for each segments. As shown in the previous report the hull weight distribution has been calculated by means of the modified Biles method. Bending and shear rigidity has been calculated at 10~15 sections along the length and the distribution curve has been drawn smoothly through these points. Such a roughness in the input data preparation may be a part of the cause of discrepancies.

Deviation of ship structure from the beam theory.

Two items above-mentioned are within the category of so-called beam theory. On the contrary, in some cases the ship structure is considered to deviate from the beam theory. Recently the attention has been often paid to such a phenomenon.

Within this category is the coupling of the local vibration.

If the local vibration is coupled with the main hull, it is expected to affect the natural frequency to some extent.

Of three items above-mentioned, the last is a very interesting phenomenon and will be a central

Table 6 Effect of Rotational Inertia on the Frequencies of Ship F.

No. of nodes	Rotational inertia neglected	Rotational inertia taken into account	
		$\beta=0.00246^{**}$	$\beta=0.0246^{**}$
	cpm	cpm	cpm
2	43.7	42.8	38.3
3	88.1	86.9	76.9 19.0*
4	137.6	136.2	122.3
5	188.2	186.4	166.2
6	233.8	231.6	196.8 223.3*
7		273.9	242.1
8		312.7	281.9 332.6*
9		356.8	323.2
10		398.4	366.9
11		443.6	403.9
12		486.8	450.6
13		526.5 555.0*	500.2
14		569.1 594.9*	546.2 583.1*
15		624.7	

* Coupling caused by the effect of rotational inertia

** Actual β of ship F is about 0.00029

Table 7 Input data of ship A, B and C

Segment No.	Location	Bending rigidity up to			Shear rigidity up to			Weight distribution		
		boat deck	bridge deck	upper deck	boat deck	bridge deck	upper deck	ship A	ship B	ship C
	× ship length	× 10 ⁸ ton-m ²			× 10 ⁶ ton			ton/m		
1	0.000~0.026	0.4	0.4	0.4	0.3	0.3	0.3	10	10	10
2	0.026~0.051	0.6	0.6	0.6	1.0	1.0	1.0	30	30	30
3	0.051~0.077	2.4	2.4	2.4	2.1	2.1	2.1	48	52	36
4	0.077~0.103	3.5	3.5	3.5	2.4	2.4	2.4	36	40	82
5	0.103~0.128	3.7	3.7	3.7	2.4	2.4	2.4	40	50	124
6	0.128~0.154	3.8	3.8	3.8	2.3	2.3	2.3	62	70	160
7	0.154~0.180	4.1	4.1	4.1	2.3	2.3	2.3	70	94	166
8	0.180~0.205	4.8	4.8	4.8	2.3	2.3	2.3	90	136	176
9	0.205~0.231	5.7	5.7	5.7	2.6	2.6	2.6	100	160	200
10	0.231~0.256	6.5	6.5	6.5	2.6	2.6	2.6	138	184	220
11	0.256~0.282	7.8	7.4	7.2	2.7	2.6	2.6	148	198	240
12	0.282~0.308	9.3	8.3	7.7	2.8	2.8	2.7	166	228	264
13	0.308~0.333	10.7	9.1	8.2	3.0	2.8	2.7	208	270	286
14	0.333~0.359	12.0	10.0	8.8	3.1	2.8	2.7	218	256	296
15	0.359~0.385	13.5	10.8	9.4	3.2	2.8	2.7	225	286	306
16	0.385~0.411	14.9	11.7	9.9	3.3	2.9	2.7	236	298	318
17	0.411~0.436	16.1	12.3	10.2	3.4	2.9	2.7	276	254	274
18	0.436~0.462	16.4	12.2	9.8	3.4	2.9	2.7	280	258	276
19	0.462~0.487	16.4	12.1	9.3	3.3	2.9	2.6	280	258	276
20	0.487~0.513	16.7	12.0	8.9	3.3	2.9	2.5	280	260	276
21	0.513~0.538	15.6	11.5	8.5	3.4	2.9	2.6	288	282	274
22	0.538~0.564	14.0	10.7	3.0	3.5	3.0	2.7	240	244	344
23	0.564~0.590	12.5	9.8	7.7	3.6	3.0	2.8	232	222	322
24	0.590~0.615	11.3	9.4	7.7	3.4	3.0	2.8	190	206	316
25	0.615~0.641	10.1	8.8	7.7	3.25	3.0	2.8	182	198	308
26	0.641~0.667	8.9	8.3	7.7	3.1	3.0	2.9	170	186	280
27	0.667~0.692	7.8	7.8	7.7	2.9	3.0	2.9	160	226	222
28	0.692~0.718	7.4	7.4	7.4	3.0	3.0	3.0	142	248	102
29	0.718~0.744	7.1	7.1	7.1	3.0	3.0	3.0	124	226	182
30	0.744~0.769	6.7	6.7	6.7	3.0	3.0	3.0	98	196	154
31	0.769~0.795	6.4	6.4	6.4	3.0	3.0	3.0	88	176	154
32	0.795~0.821	6.0	6.0	6.0	3.0	3.0	3.0	66	112	146
33	0.821~0.846	5.7	5.7	5.7	3.0	3.0	3.0	46	64	112
34	0.846~0.872	5.4	5.4	5.4	3.1	3.1	3.1	34	52	80
35	0.872~0.898	5.1	5.1	5.1	3.1	3.1	3.1	26	38	66
36	0.898~0.923	4.7	4.7	4.7	3.1	3.1	3.1	34	36	42
37	0.923~0.949	2.8	2.8	2.8	2.4	2.4	2.4	34	30	26
38	0.949~0.975	1.1	1.1	1.1	1.6	1.6	1.6	18	18	18
39	0.975~1.000	0.3	0.3	0.3	0.7	0.7	0.7	6	6	6

Table 9 Input data of ship F.

Segment No.	Location x ship length	Bending rigidity $\times 10^8$ ton-m ²	Shear rigidity $\times 10^5$ ton	Weight distribution	
				ballast	full load
1	0.000~0.017	1.0	1.27	12.0	8.0
2	0.017~0.035	1.0	1.27	108.0	110.0
3	0.035~0.061	3.8	2.50	120.0	132.5
4	0.061~0.089	1.3	3.30	69.0	91.5
5	0.089~0.091	10.5	3.30	115.0	147.5
6	0.091~0.104	14.8	3.60	147.0	211.0
7	0.104~0.135	19.8	3.75	165.0	217.5
8	0.135~0.163	26.3	3.90	237.0	292.0
9	0.163~0.178	31.0	3.96	283.0	344.0
10	0.178~0.197	34.0	4.20	328.0	387.0
11	0.197~0.215	37.5	4.63	338.0	388.0
12	0.215~0.243	38.2	5.20	407.0	422.0
13	0.243~0.264	38.6	5.80	561.0	920.0
14	0.264~0.286	42.3	6.40	586.0	943.0
15	0.286~0.308	44.0	6.90	604.0	955.0
16	0.308~0.327	45.0	7.30	471.8	1012.0
17	0.327~0.351	45.2	7.60	474.8	1017.0
18	0.351~0.373	45.5	7.60	476.8	1027.0
19	0.373~0.395	45.5	7.95	476.8	1037.0
20	0.395~0.416	45.5	7.95	476.8	1039.0
21	0.416~0.438	45.5	7.95	776.8	1039.0
22	0.438~0.469	45.5	7.95	776.8	762.0
23	0.469~0.508	45.5	7.95	776.8	762.0
24	0.508~0.525	45.5	7.95	736.0	508.0
25	0.525~0.547	45.5	7.95	736.0	508.0
26	0.547~0.568	45.5	7.93	736.0	508.0
27	0.568~0.594	45.5	7.93	1008.0	508.0
28	0.594~0.612	45.5	7.88	1008.0	510.0
29	0.612~0.638	45.5	7.85	1008.0	510.0
30	0.638~0.664	45.5	7.80	476.8	768.0
31	0.664~0.685	45.5	7.75	474.0	768.0
32	0.685~0.703	45.5	7.70	473.2	765.0
33	0.703~0.722	45.5	7.60	469.4	1039.0
34	0.722~0.746	45.5	7.30	466.6	1039.0
35	0.746~0.768	45.0	7.00	463.0	1030.0
36	0.768~0.798	43.8	7.00	452.0	934.0
37	0.798~0.833	38.7	6.20	441.0	946.0
38	0.833~0.855	34.0	5.40	437.0	810.0
39	0.855~0.881	28.5	4.90	388.0	750.0
40	0.881~0.902	23.5	4.50	302.0	678.0
41	0.902~0.933	17.0	2.85	163.0	198.0
42	0.933~0.954	10.5	2.30	97.6	113.6
43	0.954~0.980	4.0	1.38	52.2	68.0
44	0.980~1.000	1.0	1.38	21.0	40.0

Table 8 Input data of ship D ($\Delta=8667$ tons)

Segment No.	Location x ship length	Bending rigidity up to		Shear rigidity up to		Weight distribution ton/m
		boat deck	bridge deck	upper deck	upper deck	
1	0.000~0.025	0.4	0.4	0.25	0.25	19
2	0.025~0.070	0.8	0.8	0.9	0.9	32
3	0.070~0.055	2.6	2.6	2.1	2.1	39
4	0.075~0.099	3.6	3.6	2.1	2.1	28
5	0.099~0.524	4.4	4.4	2.0	2.0	23
6	0.124~0.149	5.2	5.2	2.1	2.1	51
7	0.149~0.171	6.2	6.2	2.15	2.15	58
8	0.174~0.198	6.8	7.8	2.25	2.25	72
9	0.198~0.224	7.4	7.4	2.35	2.35	86
10	0.224~0.218	7.8	7.8	2.45	2.45	111
11	0.248~0.273	8.2	8.2	2.5	2.5	170
12	0.273~0.293	9.2	9.2	2.5	2.5	226
13	0.298~0.323	17.8	14.6	2.7	2.5	238
14	0.323~0.318	19.0	16.0	2.8	2.45	254
15	0.318~0.373	20.4	17.2	2.95	2.45	266
16	0.373~0.338	21.4	18.4	3.05	2.45	273
17	0.398~0.422	20.8	18.0	3.15	2.45	280
18	0.422~0.447	19.0	15.2	3.25	2.45	324
19	0.447~0.472	11.2	11.2	2.5	2.5	255
20	0.472~0.497	11.0	11.0	2.55	2.55	254
21	0.497~0.521	11.0	11.0	2.7	2.7	252
22	0.521~0.547	11.0	11.0	2.75	2.75	252
23	0.547~0.571	10.8	10.8	2.85	2.85	241
24	0.571~0.596	10.4	10.4	2.9	2.9	206
25	0.596~0.621	10.0	10.0	2.95	2.95	186
26	0.621~0.646	9.6	9.6	2.95	2.95	166
27	0.646~0.671	9.2	9.2	3.0	3.0	144
28	0.671~0.696	8.8	8.8	3.0	3.0	116
29	0.696~0.721	8.2	8.2	2.95	2.95	96
30	0.721~0.745	7.6	7.6	2.9	2.9	82
31	0.745~0.770	7.0	7.0	2.85	2.85	67
32	0.770~0.795	6.4	6.4	2.75	2.75	52
33	0.795~0.820	5.8	5.8	2.7	2.7	42
34	0.820~0.845	5.6	5.6	2.9	2.9	75
35	0.845~0.870	5.0	5.0	3.1	3.1	66
36	0.870~0.894	4.6	4.6	3.2	3.2	50
37	0.894~0.919	4.2	4.2	3.05	3.05	42
38	0.919~0.944	3.8	3.8	2.9	2.9	42
39	0.944~0.969	2.6	2.6	2.1	2.1	45
40	0.969~1.000	1.2	1.2	0.75	0.75	19

Table 10 Input data of ship I

Segment No.	Location	Bending rigidity	Shear rigidity	Weight distribution	Segment No.	Location	Bending rigidity	Shear rigidity	Weight distribution
	×ship length	×10 ⁸ ton/m ²	×10 ⁸ ton	ton/m		×ship length	×10 ⁸ ton/m ²	×10 ⁸ ton	ton/m
1	0.000~0.022	3.15	0.5	32	24	0.508~0.530	41.16	8.2	681
2	0.022~0.044	3.78	0.9	64	25	0.530~0.552	40.74	8.1	423
3	0.044~0.066	7.14	1.5	73	26	0.552~0.574	40.32	8.1	423
4	0.066~0.088	11.34	2.1	155	27	0.574~0.596	39.9	8.1	423
5	0.088~0.110	15.12	2.7	135	28	0.596~0.618	39.27	8.1	681
6	0.110~0.132	19.32	3.3	147	29	0.618~0.640	38.85	8.1	681
7	0.132~0.155	22.89	3.9	179	30	0.640~0.662	38.43	7.9	681
8	0.155~0.177	26.04	5.0	221	31	0.662~0.684	38.01	7.9	680
9	0.177~0.199	29.4	6.2	279	32	0.684~0.706	37.59	7.9	680
10	0.199~0.221	31.29	7.2	286	33	0.706~0.728	36.96	7.9	534
11	0.221~0.243	32.76	7.8	334	34	0.728~0.751	36.54	7.9	421
12	0.243~0.265	34.86	7.9	379	35	0.751~0.773	36.12	7.8	421
13	0.265~0.287	36.54	8.0	525	36	0.773~0.795	35.7	7.7	585
14	0.287~0.309	38.22	8.1	545	37	0.795~0.817	34.86	7.5	620
15	0.309~0.331	38.64	8.1	552	38	0.817~0.839	30.24	7.3	595
16	0.331~0.353	39.06	8.1	676	39	0.839~0.861	26.46	7.2	365
17	0.353~0.375	39.48	8.1	676	40	0.861~0.883	22.26	6.9	277
18	0.375~0.397	39.69	8.1	681	41	0.883~0.905	18.48	4.8	217
19	0.397~0.419	40.11	8.2	425	42	0.905~0.927	13.86	3.1	152
20	0.419~0.442	40.32	8.2	425	43	0.927~0.949	9.66	2.0	95
21	0.442~0.464	40.74	8.2	425	44	0.949~0.971	5.04	0.9	60
22	0.464~0.486	41.37	8.2	681	45	0.971~1.000	4.20	1.0	26
23	0.486~0.508	41.37	8.2	681					

topic in the future. Recently Prof. Yamakoshi et al⁽⁸⁾ have published a new theory on the coupled vibration of hull and bottom structure, which is a first step towards the new topic. But the authors leave this subject in the future and it is not treated in this report.

The second item may deserve attention from the practical point of view. In fact the authors have an experience that, in the calculation of the same ship, the computed frequency varies by about 1~3% according to the difference in the data preparation. But the authors have an impression that, although this item can not be neglected, of primary importance is the first item. Hence the first item will be treated exclusively hereafter.

A. Virtual Added Mass.

As mentioned in the previous report⁽¹⁾, the Landweber's two parametre family⁽⁹⁾ with Lewis' three-dimensional factor⁽¹⁰⁾ was used for the estimation of virtual mass for 2-noded vibration. For the higher mode vibrations the virtual mass was taken the same as for 2-noded. For the purpose of examining the validity of the above-mentioned estimation, two aspects must be considered; i.e.

(1) the two-dimensional calculation of added mass, and (2) three-dimensional correction factor for each mode of vibration.

For the two-dimensional calculation (strip theory), various methods have been proposed, such as, Lewis Forms⁽¹⁰⁾, prohaska Forms⁽¹¹⁾, Landweber's two-parametre⁽⁹⁾ and three-parametre family⁽¹²⁾. The main difference among these studies is the difference of the image functions used in the conformal transformation.

Landweber's two-parametre family used in this report has a transform function in the following forms,

$$z = \zeta + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^3} \quad (1)$$

where z : variable in the plane of ship section

ζ : variable in the plane of unit circle.

a_1, a_2 : arbitrary constant.

Since eq (1) has two arbitrary constants, we can express various forms of sections by properly arranging the constants.

In Landwebers method, a_1 and a_2 are determined such as λ and σ of the transformed section are equal to those of the actual ship section, where

$$\left. \begin{array}{l} \lambda : \text{breadth to draft ratio} \\ \sigma : \text{section area ratio.} \end{array} \right\} \quad (\text{see Fig. 12})$$

and
In this method for two sections with different forms but with same λ and σ , the calculated added mass has the same value. Further the transformed forms by eq. (1) are considerably different from the actual forms if λ and σ take excessive values. Thus Landweber defined the domain for λ and σ in which transform (1) is realistic (see Fig. 12). Landweber's three-parametre family is an extension of (1) and takes the form ;

$$z = \zeta + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^3} + \frac{a_3}{\zeta^5} \quad (2)$$

Here an additional constant appears, by which the transform becomes more realistic and the available domain can be extended (see Fig. 12).

It is interesting, from the practical point of view, how the λ and σ of actual ship section distribute. In Fig. 12 are shown the value of λ and σ for two types of ships. Ship F has been described in this report. Model A has lines of fine passenger ships. It is seen in Fig. 12 that the value of λ and σ sometimes go out of the available domain for full ships. For sections whose λ and σ exceeds the limit, shapes of actual and transformed sections are compared in Fig. 13. From the visual impression the discrepancy is not so large. This encourages us to use the two-parametre family.

Fig. 12 Domain for λ and σ within which transforms (1) and (2) are available.

Fig. 13 Comparison of transformed with actual sections whose λ and σ deviate the limits of Fig. 12.

Regarding the three-dimensional correction factor, investigations have been made by Taylor⁽¹³⁾, Lewis⁽¹⁰⁾, Prof. Kumai⁽¹³⁾⁽¹⁴⁾, Burrill et al⁽¹⁵⁾ and Dutton et al⁽⁷⁾.

In spite of these studies, the three-dimensional correction factor is not considered to have been clarified completely, especially for the higher modes. Thus a simple preliminary test has been carried

out in our laboratory.

Vibration experiments were made for model ships in air and on water, and the frequency ratios were derived. On the other hand, the frequency calculation were made with and without added virtual mass, and the frequency ratios were compared with experiments. Two types of model ships were used, model A and model B. Their particulars are shown in Table 11.

Table 11 List of Particulars of Model Ships.

	Model A	Model B
Type	with lines of passenger boat (without Superstructure)	wall sided vessel (without Superstructure)
Material	Plastics	Plastics
L×B×D	280cm×38.6cm×28.6cm	280cm×38.6cm×28.6cm
Draft	11.5cm	11.5cm
Displacement	72kg	115.6kg

Added mass calculations were made according to the method described above. Comparison of measured with computed frequencies is shown in Table 12.

Table 12 Ratio of Natural Frequencies on water to those in air.

Model		2-Node	3-Node	4-Node
A	Experimental	0.798	0.773	0.735* 0.790*
	Computed	0.815	0.758	0.743
B	Experimental	0.648	0.648	0.665
	Computed	0.672	0.670	0.679

* 2 kinds of 4 node vibration appeared in the experiment on water, due to the coupling with local vibration.

From Table 12, the following conclusions are drawn.

- (1) Landweber's 2-parametre family with Lewis' three dimensional correction gives practically good results for 2-noded vibration.
- (2) As Prof. Kumai has stated⁽¹⁴⁾ the assumption that the lengthwise distribution of three-dimensional correction factor is uniform gives fairly good results.
- (3) In the case of wall-sided vessel, the 3-dimensional correction is practically constant in every mode. In the fine ship it decreases gradually in the higher mode. To clarify the details of the tendency, the detailed experiment is necessary. For practical use, the virtual mass can be taken constant in any mode.

B. Rotational Inertia.

In the previous report⁽¹⁾ the authors investigated the effect of rotational inertia on the frequency of destroyers and concluded that the effect of rotational inertia can be neglected.

The effect of rotational inertia is determined by the value of β , where

$$\beta = \frac{I_m}{wl^2}$$

I_m : rotational inertia,

w : weight per unit length

and l : length.

In the case of the above-mentioned destroyer and ship F in full load, $\beta=0.00025$ and $\beta=0.00029$

respectively at midship and are practically of no effect.

Here the hypothetical cases in which $\beta=0.0025$ (10 times the usual value) and $\beta=0.025$ (100 times the actual value) were treated for ship F to examine the effect of rotational inertia. The results are shown in Table 6. From this table it is concluded that rotational inertia effect is small at least under 6~7 node mode. Also from this table the appearance of 2nd spectrum is understood. As an example of mode curve of 2nd spectrum, Fig 14 is presented.

Fig. 14 An example of mode curves for extra modes shown in table 5 (rotational inertia taken about 100 times the actual value).

C. Effective Breadth

In the article 4 the authors have indicated that the frequency tendencies of tankers and ore-carrier are completely different from those of bulk carriers and perhaps cargo liners. This difference depending on the ship type can not be attributed to the virtual mass nor rotational inertia, because the former changes very little according to the ship type and the latter has little effect on the frequency, as has been shown in the preceding discussions.

The most probable cause is the effect of the effective breadth of deck and bottom.

Since tanker and ore-carrier have longitudinal bulkheads, the deck plating between side shell and the bulkhead suffers tension and compression on both sides in the case of bending deflection. Hence the bending of the deck plate in its own plane is small and the effective breadth is practically 100%. On the contrary bulk carrier and cargo liner has no longitudinal bulkhead and the deck plating suffers

Fig. 15 Change of bending rigidity of model B when the effective breadth of deck and bottom is considered according to Schade's theory.

tension and compression on one side only, the bending of the deck plating in its own plane arising easily. Thus the effective breadth is lessened considerably. According to H. A. Schade⁽¹⁶⁾ (17) the former is the effective breadth of double web type and the latter of web free type and both types differ to a great extent.

For the preliminary check of this problem, the calculations were carried out for model B mentioned above, and the results were compared with experiment.

The Bending rigidity distributions with and without effective breadth concept are shown in Fig. 15. It is seen from this figure that the effect of effective breadth on the bending rigidity is considerable especially for the higher mode and in the case of web-free type.

Table 13 Effect of effective breadth on the frequency of model B

No. of Nodes			2	3	4	5
in Air	Experiment		68	142	209	258
	computed	a Effective breadth taken 100%	67	144	227	303
		b Effective breadth of double web type taked	67	139.6	217	290
		<i>Experiment/a</i>	1.014	0.985	0.92	0.851
		<i>Experiment/b</i>	1.014	1.017	0.963	0.890
	Experiment		44	92	139	162
on Water	computed	a Effective breadth taken 100%	45.4	96.4	153.6	204
		b Effective breadth of double web type taken	45.4	93.3	147.0	195.0
		c Effective breadth of web free type taken	—	—	—	165.0
		<i>Experiment/a</i>	0.97	0.955	0.906	0.794
		<i>Experiment/b</i>	0.97	0.986	0.946	0.831
		<i>Experiment/c</i>	—	—	—	0.982

The results of frequency calculation using these rigidities are shown in Table 13. From this table we can conclude that the effective breadth have considerable influence upon the frequency. On this subject, however, much detailed analysis and experiments are necessary to establish the practical method and the detailed discussion is left to the future report.

6 Conclusions and acknowledgement

Frequency calculations were carried out for various type of ships, and the empirical coefficients were derived by the comparison of computed with measured value. Further the effects of ship parameters on the hull vertical frequency were studied and the following conclusions were drawn.

- (1) Frequency prediction of various type of ships can be made with good accuracy, by means of electronic computer and the empirical coefficients shown in Table 5.
- (2) The effect of rotational inertia upon the hull natural frequency is small and can be neglected for the ordinary type of ships.
- (3) For the estimation of added virtual mass of 2-noded vibration, the Landweber's 2-parameter family with Lewis 3-dimensional correction factor gives practically good result. The lengthwise distribution of 3-dimensional correction factor can be taken constant, as Prof. Kumai has pointed out.
- (4) For the higher mode vibration the virtual mass has a tendency to decrease slightly, for practical use, however, it can be taken constant up to 4-node mode. For the minute investigation of virtual mass in the higher mode, more detailed experiment is necessary.
- (5) Effective breadth has considerable effect on the hull natural frequency. To clarify this phe-

nomenon quantitatively, further studies are necessary and open to the future report.

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