On the Transverse Strength of Oil Tankers

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Summary

In order to examine exactly the strength of transverse rings of oil tanker with long cargo tanks, it is necessary to take into consideration the effects of deformation of the longitudinal members which support the transverse members. Much investigation has been done in regard to this problem as the shearing deformation of wing tank. It seems to us that the shearing deformation will occur not only in wing tank but also in centre tank. The authors try to make the three dimensional strength calculation for all tank parts and then try to analyze the results of experiment on an actual ship and to make some considerations upon the longitudinal members.

From this study following things are found:
(1) The deformations and the stresses in the transverse ring of a big tanker can be estimated with good accuracy for the practical uses by this method.
(2) The shearing deformations in the cross section increase the stresses in the lower part of the transverse ring.
(3) In way of the tank parts, the shearing force seems to be more shared by the longitudinal bulkhead and to be less shared by the side shell than what is usually considered.

Introduction

In order to examine exactly the strength of transverse rings of oil tanker with long cargo tanks, it is necessary to take into consideration the effects of deformation of the longitudinal members which support the transverse members. This problem was studied as the shearing deformation of wing tank by Dr. T. Okabe and Mr. K. Hori1) about twelve years ago and also by Prof. Mano2) few years ago, both under some special boundary conditions. Prof. Yamakoshi and Mr. K. Kagawa3) studied this problem under more general boundary conditions. They proposed the method of calculation for shearing deformation of wing tank from relative deflection between the side shell and the longitudinal bulkhead due to only the shearing deflection taking the load deviated. They also studied4) the effects of the centre girder upon shearing deformation of wing tank neglecting the relative deflection between the longitudinal bulkhead and the centre girder itself.

In this paper, the authors treat this problem as the shearing deformation of both wing and centre tanks and then try the followings:
(1) to analyze the results of experiment on an actual ship,
(2) to estimate the ratio of stress resultants of each longitudinal member to those of the whole.

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1. Process of Calculation

1.1 General Considerations

(1) Division of Structures

We divide the longitudinal strength members of a ship into four parts, that is, the side shell part, the longitudinal bulkhead part, the bottom centre girder part and the deck centre girder part as shown in Fig. 1-1. And we consider the transverse ring, the watertight bulkhead and the swash bulkhead as the transverse strength members. From the point of view of longitudinal strength, we can treat the longitudinal strength members as a composite parallel beam connected with the transverse strength members. On the contrary, from the point of view of transverse strength, we can also treat the transverse strength members as the rigid frames supported elastically by the longitudinal strength members.

(2) Assumptions for Calculations

In the case of tanker which is constructed in accordance with longitudinal system, the load is transmitted in following sequence; plates → long. frames → tran. strength members → long. strength members. Beyond the tank parts, the member which carries the shearing force is the side shell part only. In this paper, therefore, the three dimensional strength calculation is carried out under the following assumptions:

(i) (a) Cut out whole the tank parts from a tanker floating in upright condition and support the neutral axis of the Hull Girder at both ends (at the engine room front bulkhead and at the fore peak bulkhead) as shown in Fig. 1-2.

(b) Let only the side shell carry the reaction forces at support points. Next share the longitudinal bending moment of the Hull Girder among the longitudinal strength members in proportion to their bending rigidity around the neutral axis of the Hull Girder.

(ii) Treat the longitudinal strength members as a composite parallel beam connected with distributed spring action of transverse rings and of deck and bottom plates and with concentrated spring actions of watertight and swash bulkheads. So it is enough to take the reaction forces at support points of a transverse ring subjected to external loads of one transverse spacing and given the forced displacement at each support point equal to the deflection of corresponding longitudinal member as the vertical loads for the longitudinal strength members, adding suitably the hull weight.

(iii) Consider, at first, the transverse ring supported by longitudinal members, and then calculate the deformation of transverse members subjected to external loads of 1-transverse spacing. Next give
the forced displacement obtained in item (ii) to each support point, and then calculate the deformation of those members. The final deformation of the transverse ring is obtained by superposing above two deformations.

Thus, it is expected that the three-dimensional strength calculation is able to be carried out with sufficient accuracy for practical use if the distribution of external loads coincides to the load curve for the Longitudinal Strength Calculation.

1.2 Method of Calculation of Transverse Strength Members

(1) Transverse Rings

The final state of deformation of transverse ring is obtained by superposing the following states; one is the deformation when is subjected to external loads of 1-transverse spacing with each support point being restrained and others are the deformations when are given the forced displacement equal to the deflection of the corresponding longitudinal member to each support point one by one as shown in fig. 1-3. It is, therefore, enough to calculate the deformations of transverse ring of the following states; one is the state shown in Fig. 1-3 (b) and others are the states shown in Fig. 1-3 (c), (d), (e) and (f) with forced displacement in equal to unity, respectively. Ordinary rigid frame theory is used for this calculation taking into consideration of both bending and shearing deflections and of the effects of existences of variable cross sections. The elongations of members are also considered.

Let us represent the stresses and stress resultants acting on the cross section of the point in the member by general notation of $H_{jk}$, and define the following notations:

- $H_{jk}$: quantity corresponding to the state shown in Fig. 1-3 (a).
- $H_{jk}(0)$: Fig. 1-3 (b).
- $H_{jk}(1)$: Fig. 1-3 (c).
- (d), (e) and (f), where $Y_i$ is equal to unity, respectively.

\[ \text{Fig. 1-3. Deformations of Transverse Ring} \]

(i) Stress Resultants and Stresses

Our objective forces and moments and stresses acting on the cross section of the point in the member can be calculated by the following formulas.
We calculate stresses in the corner according to Dr. Terada’s\textsuperscript{5) theory—an equivalent curved beam theory—with suitable corrections for the distributed loads and for the effect of shapes of T-type corner\textsuperscript{7).}

(ii) Supporting Forces

Supporting forces due to the longitudinal strength members can be represented by the following form.

\[ R_i = R_i(0) + \sum_{i=1}^{4} y_i \cdot R_{ij}(1) \]  \hspace{1cm} (i=1, 2, 3, 4) \hspace{1cm} (1.3)

The reactions of these supporting forces act on as the loads for the longitudinal strength members. The first term denotes an external forces and the second term denotes the connecting forces due to the transverse ring. For the sake of simplicity we assume that these reactions act between transverse rings as uniformly distributed forces. Well, it is possible to represent the connecting forces in the form of linear combination with respect to the relative deflections \((y_j - y_i)\) instead of with respect to the deflections \(y_i\) in above formulas. But the results are the identity.

(2) Transverse Water Tight Bulkheads

(i) Shearing Deformation of Bulkhead Plates

Let us consider the deformation of transverse water tight bulkhead shown in Fig. 1-4 (a). The stress resultants in bulkhead plate will become as shown in Fig. 1-4 (b). Upon considering the simple shearing deformation of the plate between the side shell part and the longitudinal bulkhead part and the mean shearing deformation of the plate among the longitudinal bulkhead and the bottom and deck centre girders, we can obtain the relations between stress resultants and deformation in the following forms.

\[
\begin{align*}
S_{jk} &= S_{jk}(0) + \sum_{i=1}^{4} y_i \cdot S_{jk}(1) \quad \text{(Shearing Force)} \\
N_{jk} &= N_{jk}(0) + \sum_{i=1}^{4} y_i \cdot N_{jk}(1) \quad \text{(Normal Force)} \\
M_{jk} &= M_{jk}(0) + \sum_{i=1}^{4} y_i \cdot M_{jk}(1) \quad \text{(Bending Moment)} \\
\sigma_{jk} &= \sigma_{jk}(0) + \sum_{i=1}^{4} y_i \cdot \sigma_{jk}(1) \quad \text{(Normal Stress)} \\
\tau_{jk} &= \tau_{jk}(0) + \sum_{i=1}^{4} y_i \cdot \tau_{jk}(1) \quad \text{(Shearing Stress)}
\end{align*}
\]  \hspace{0.5cm} (1.2)

\[
J_{S1} = K_{SB1} \cdot (y_2 - y_1), \hspace{1cm} J_{S2} = K_{SB2} \cdot [(y_3 - y_2) + (y_4 - y_2)]. \hspace{1cm} (1.4)
\]

where:

\[
K_{SB1} = k'G_{SB1}b_1 z, \hspace{1cm} K_{SB2} = (k'G_{SB2}/b_2) \times \frac{1}{2}.
\]
(ii) Elongation of C. L. Vertical Webs

We can obtain the forces of $N_3$ and $N_4$ shown in Fig 1-4 (b) from the consideration of elongation of a C. L. vertical web. Suppose that $S_2$ is distributed uniformly along the depth of bulkhead plate, then we can represent $N_3$ and $N_4$ in the following forms.

$$
\begin{align*}
\Delta N_3 &= K_{NV} \left((y_3-y_2)-(y_4-y_3)\right) + K_{SB2} \left((y_3-y_2)+(y_4-y_3)\right), \\
\Delta N_4 &= K_{NV} \left((y_3-y_2)-(y_4-y_3)\right) - K_{SB2} \left((y_3-y_2)+(y_4-y_3)\right).
\end{align*}
$$

where: 
$$
K_{NV} = E(A_{NV}/D) e_{eff}.
$$

(iii) Bending and Shearing Deformation of C. L. Vertical Webs

Let us consider the deformation of C. L. vertical web due to both bending and shearing deflections shown in Fig. 1-5 (a). The stress resultants in the C. L. vertical web are shown in Fig. 1-5 (b). Applying the slope-deflection method to this member, we can represent the relations between stress resultants and deformations in the following forms

$$
\begin{align*}
\Delta M_3 &= \frac{1}{f(h-g)^2} \left\{ h\psi_3 + g\psi_4 - \frac{(g+h)}{l} (u_4-u_3) \right\} + C_{3,4-0} - C_{3,4+0}, \\
\Delta M_4 &= \frac{1}{f(h-g)^2} \left\{ g\psi_3 + f\psi_4 - \frac{(g+f)}{l} (u_4-u_3) \right\} - C_{4,3-0} + C_{4,3+0}, \\
\Delta S_3 &= \frac{1}{l(fh-g^2)} \left\{ -(g+h)\psi_3 - (f+g)\psi_4 + \frac{(f+2g+h)}{l} (u_4-u_3) \right\} + D_{3,4-0} - D_{3,4+0}, \\
\Delta S_4 &= \frac{1}{l(fh-g^2)} \left\{ -(g+h)\psi_3 - (f+g)\psi_4 + \frac{(f+2g+h)}{l} (u_4-u_3) \right\} - D_{4,3-0} + D_{4,3+0}
\end{align*}
$$

where:

$$
\begin{align*}
f &= \frac{1}{EI} \int_0^l (l-Z)^2 dZ + \frac{1}{GI^2} \int_0^l dZ, \\
g &= \frac{1}{EI} \int_0^l \frac{(l-Z)Z}{Iv} dZ - \frac{1}{GI^2} \int_0^l dZ, \\
h &= \frac{1}{GI^2} \int_0^l Z dZ + \frac{1}{GI^2} \int_0^l dZ
\end{align*}
$$

and

$$
A_{SW} : \text{effective cross sectional area against shearing force of a C. L. vertical web}, \\
I_v : \text{effective moment of inertia of cross section of a C. L. vertical web}, \\
C,D : \text{terms of loads regarding to the pressure due to inner fluid}.
$$

Reactions of these above described forces and moments act on the longitudinal members. We should treat these reactions as the concentrated ones acting on the point where the transverse watertight bulkhead is set.
(3) Transverse Swash Bulkheads

We should treat the swash bulkhead according to the same way of thinking in case of an ordinary transverse ring if the swash bulkhead is a type of strong transverse ring. Of course, the reactions should act on as the concentrated ones. And we should treat the swash bulkhead according to the same way of thinking in case of a transverse watertight bulkhead if the swash bulkhead is of a type of perforated plate. In regard to the C. L. vertical web, if fitted, treat as the same method already described.

1.3 Method of Calculation of Longitudinal Strength Members

According to the way of thinking written in chapter 1.1, we carry the strength calculation of the longitudinal strength member as a composite parallel beam as follows.

(1) Equations of Equilibrium

Let us consider the deformation and the stress resultants of each longitudinal strength member as shown in Fig. 1-6 ((a) and (b)). The relations between them are represented in the following forms.

\[ S_i = k' A S_i \frac{dy_i}{dx} (dx - \varphi_i), \]

\[ M_i = -EI \frac{d\varphi_i}{dx}, \]

\[ N_i = E A N_i \frac{d\varphi_i}{dx}, \]

\[(1.7)\]

where: \( k' A S_i \): effective cross sectional area against shearing force of the \( i \)-th member,

\( I_i \): Moment of Inertia of the cross section about the neutral axis of the \( i \)-th member,

\( A N_i \): cross sectional area against normal force of the \( i \)-th member.

The load acting on the longitudinal members are considered as follows.

- **External load**: \( f_i = -R_i(0)/S + \omega_i, \)
- **Vertical connecting force**: \( r_i = -\sum_{j=1}^{4} [R_i(1)/S] \cdot y_j, \)
- **Longitudinal connecting force**: \( q_{ij} = G(t_i \bar{u}/b_j \bar{u}) [(u_i + c_1 \varphi_i) - (u_j + c_1 \varphi_j)], \)

\( q_{ijL} = G(t_i \bar{u}/b_j \bar{u}) [(u_i - c_1 \varphi_i) - (u_j - c_1 \varphi_j)], \)

where: \( S \) is transverse spacing, and \( \omega_i \) is hull weight for each member.

![Fig. 1-6 Displacement and Stress Resultant.](image)

![Fig. 1-7 Shearing Deformations of Deck Plates.](image)
The force \( q_{i,ju} \) and \( q_{i,jL} \) are the longitudinal connecting forces due to the shearing deformation of bottom due deck plate in proportion to the relative longitudinal displacement between the adjacent members as shown in Fig. 1-7.

Considering the equilibrium of all the forces and moments, we obtain the following equations (refer to Fig. 1-8):

\[
\begin{align*}
\frac{dS_i}{dx} &= -f_i, \\
\frac{dM_i}{dx} &= S_i + c_i(q_{i,ju} - q_{i,bu}) - c_iL(q_{i,jL} - q_{i,bL}), \\
\frac{dN_i}{dx} &= -(q_{i,ju} - q_{i,bu}) - (q_{i,jL} - q_{i,bL}),
\end{align*}
\]

\((i=1, 2, 3, 4)\).

Substituting the equations (1.7) and (1.8) into equations (1.9) and translating suitably the independent variables and the unknown functions, we can obtain the fundamental differential equations for this problem in the following form.

\[
\frac{dy}{dx} = V \cdot y + W
\]

where:
- \( y \): functional column vector having 24 elements,
- \( V \): matrix of coefficients, 24 rows by 24 columns,
- \( W \): column vector denoting the loads with 24 elements.

(2) Relation between the Deformation of Hull Girder and that of the Individual Member

Denoting the deformation components and the stress resultants of the Hull Girder by using suffix-0, we can obtain the following equations of equilibrium for elementary part of the Hull Girder:

\[
\begin{align*}
\frac{dS_0}{dx} &= -2f_1 - 2f_2 - f_3 - f_4, \\
\frac{dM_0}{dx} &= S_0, \\
\frac{dN_0}{dx} &= 0.
\end{align*}
\]

From these differential equations, we can solve the unknown deformation components \( y_0, \varphi_0 \) and \( u_0 \), having nothing to do with \( y_i, \varphi_i \) and \( u_i \) if the boundary conditions are defined. On the other hand, it is easy to find the following relations between the stress resultants of the Hull Girder and that of the individual member, that is:

\[
\begin{align*}
S_0 &= 2S_1 + 2S_2 + S_3 + S_4, \\
M_0 &= 2(M_1 + J_0N_1) + 2(M_2 + J_0N_2) \\
&\quad + (M_3 + J_0N_3) + (M_4 + J_0N_4), \\
N_0 &= 2N_1 + 2N_2 + N_3 + N_4.
\end{align*}
\]

Substituting the equations (1.7) into (1.12) and integrating them, we can define the relations between the deformation of the Hull Girder and that of the individual member in the following forms.
\[
2 \alpha s_1 y_1 + 2 \alpha s_2 y_2 + \alpha s_3 y_3 + \alpha s_4 y_4 \\
= \alpha s_0 y_0 - \int [\alpha s_0 \varphi_0 - (2 \alpha s_1 \varphi_1 + 2 \alpha s_2 \varphi_2 + \alpha s_3 \varphi_3 + \alpha s_4 \varphi_4)] dx, \\
2 \gamma_1 \varphi_1 + 2 \gamma_2 \varphi_2 + \gamma_3 \varphi_3 + \gamma_4 \varphi_4 \\
= \gamma_0 \varphi_0 + (2 \beta_1 u_1/\partial e_1 + 2 \beta_2 u_2/\partial e_2 + \beta_3 u_3/\partial e_3 - \partial_1 u_1/\partial e_1), \\
2 \alpha N_1 u_1 + 2 \alpha N_2 u_2 + \alpha N_3 u_3 + \alpha N_4 u_4 = \alpha N_0 u_0 \\
\text{where}: \quad \alpha s_i = A s_i / A S, \\
\gamma_i = I_i / I, \\
\alpha N_i = A N_i / A N, \\
\partial_1 = \partial e^2 A N_i / I \\
\text{and } A S, I, A N \text{ are the normalization factors.}
\]

From above equation (1.13) we can know that the deformations and the stress resultants of the Hull Girder and those of arbitrarily three of four members are linearly independent. On the other hand, we can write the equations (1.11) (which are expressing the equilibrium of the elementary Hull Girder) by the linear combinations with respect to those chosen suitably from the equations (1.7) (which are expressing the equilibrium of individual elementary member). This means that in order to determine the deformation and the stress resultants of each longitudinal member uniquely by solving the fundamental differential equations (1.9) or (1.10), it is necessary to take into consideration the end conditions for the Hull Girder.

(3) Boundary Condition

From the preceding considerations and by the assumptions noted in section 1, 1 paragraph (2), we take the following conditions as the boundary conditions for this problem.

(i) At the Engine Room Front Bulkhead

\[
\begin{align*}
2 \alpha s_1 y_1 + 2 \alpha s_2 y_2 + \alpha s_3 y_3 + \alpha s_4 y_4 &= 0 \\
M_1 &= (I_1/I_0) \cdot M_A - M A_{1-2} + M A_{1+2} \\
2 \alpha N_1 u_1 + 2 \alpha N_2 u_2 + \alpha N_3 u_3 + \alpha N_4 u_4 &= 0 \\
S_1 &= -d S_1 - d S_2 \\
M_2 &= (I_2/I_0) \cdot M_A - d M A_{2-0} + d M A_{2+0} \\
N_2 &= (d e_0 A N_2/I_0) \cdot M_A - d N A_{2-0} + d N A_{2+0} \\
S_2 &= d N A_0 \\
M_3 &= (I_3/I_0) \cdot M_A - d M A_3 \\
N_3 &= (d e_0 A N_3/I_0) \cdot M_A - d N A_3 \\
S_3 &= -d N A_0 \\
M_4 &= (I_4/I_0) \cdot M_A - d M A_4 \\
N_4 &= -d M A_4 \\
S_4 &= -d M A_4 \\
\end{align*}
\]

(ii) At the Eorce Peak Bulkhead

\[
\begin{align*}
2 \alpha s_1 y_1 + 2 \alpha s_2 y_2 + \alpha s_3 y_3 + \alpha s_4 y_4 &= 0, \\
M_1 &= (I_1/I_0) \cdot M_F + d M F_{1-0} - d M F_{1-0}, \\
N_1 &= (d e_1 A N_1/I_0) \cdot M_F + d N F_{1-0} - d N F_{1-0}.
\end{align*}
\]
\[ S_2 = - \Delta S_1 + \Delta S_3, \]
\[ M_2 = (I_2/I_0) \cdot M_F + \Delta M_{2, -0} - \Delta M_{2, +0}, \]
\[ N_2 = (J_{y, 0}(y_2/I_0) \cdot M_F + \Delta N_{2, -0} - \Delta N_{2, +0}, \]
\[ S_3 = - \Delta N_{F, 3}, \]
\[ M_3 = (I_3/I_0) \cdot M_F + \Delta M_{3, 5}, \]
\[ N_3 = (J_{x, 0}(x_3/I_0) \cdot M_F + \Delta S_{F, 3}, \]
\[ S_4 = \Delta N_{F, 4}, \]
\[ M_4 = (I_4/I_0) \cdot M_F + \Delta M_{4, 4}, \]
\[ N_4 = - (J_{x, 0}(x_4/I_0) \cdot M_F - \Delta S_{F, 4}. \]

\( (i = 1, 2, 3, 4) \]
\[ S_{1,0} = S_{2,0} - \Delta S_1, \]
\[ M_{1,0} = M_{1,0} - \Delta M_{1, -0} + \Delta M_{1, +0}, \]
\[ N_{1,0} = N_{1,0} - \Delta N_{1, -0} + \Delta N_{1, +0}, \]
\[ S_{2,0} = S_{1,0} + \Delta S_1 - \Delta S_2, \]
\[ M_{2,0} = M_{2,0} - \Delta M_{2, -0} + \Delta M_{2, +0}, \]
\[ N_{2,0} = N_{2,0} - \Delta N_{2, -0} + \Delta N_{2, +0}, \]
\[ S_{3,0} = S_{2,0} + \Delta S_2, \]
\[ M_{3,0} = M_{3,0} - \Delta M_3, \]
\[ N_{3,0} = N_{3,0} - \Delta S_3, \]
\[ S_{4,0} = S_{3,0} - \Delta N_4, \]
\[ M_{4,0} = M_{4,0} - \Delta M_4, \]
\[ N_{4,0} = N_{4,0} + \Delta S_4, \]

where \( M_A \) and \( M_F \) denote the longi. bending moment at both ends and \( \Delta M_{1, -0} \sim \Delta M_{2, -0}, \Delta N_{1, -0} \sim \Delta N_{2, -0} \) denote the effects of the pressure due to liquid in tanks and suffix \(-0\) and \(+0\) denotes after side and fore side of the point, respectively.

(4) Method of Numerical Calculation

Numerical calculation of the fundamental differential equation is carried out according to the way of thinking which was shown in the paper published before. We have taken care of to keep the accuracy of results of calculation as shown in Appendix.

2. Experiment on an Actual Ship

In order to verify the results of calculation noted in Chapter 1, we carried out the experiment on a big tanker which had been built in our Nagasaki Shipyard and Engine works (S. No. 1631,
120,000 D/W) when in her Tank Test on the sea. We must say that this experiment had been carried out before the same kind of experiment which was committed to our company by the 83rd Division of the Japan Ship Research Association.

2.1 Test Conditions

The principal dimensions and the test conditions of the subject ship are listed in Table 2-1. The load conditions measured and the rough midship section are shown in Fig. 2-1 (a) and (b), respectively. The items and the instruments of experiment are listed in Table 2-2. The experiments had being

<table>
<thead>
<tr>
<th>Table 2-1 Principal Dimensions and Test Conditions of Subject Ship</th>
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<tbody>
<tr>
<td><strong>Items</strong></td>
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Fig. 2-1 (a) Ballast in Testing Cond.  Fig. 2-1 (b) Typical midship Section
carried out from May the 13th to May the 15th, 1966. The measurements were made in successive load conditions till the Tank Test Pressure was attained. Among the 4-conditions listed in Table 2-1 the Test No. 0 was what to determine the zeros and the Test No. 3 was corresponding to the condition of the Tank Test itself.

Table 2-2 Items of Measurements and Instruments

<table>
<thead>
<tr>
<th>Items Measured</th>
<th>Position</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stresses in Trans. Ring</td>
<td>Fr. No. 71</td>
<td>Electric Wire Strain Gauge</td>
</tr>
<tr>
<td>Shearing Deformation of the Cross Section (Deflections on Side Shell, Long. BHD and on Dk Girder).</td>
<td>On the Deck Level</td>
<td>Transitt and Water Tube</td>
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<tr>
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<td>At Fr. No. 69, 71, 73 and 75</td>
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</table>

2.2 Results of the Experiment

The measured values of the shearing deformations in the cross sections at Fr. No. 69, 71, 73 & 74 and of the normal stresses in face plates and the shearing stresses in web plates of the transverse ring at Fr. No. 71 are shown in Fig. 2-2 (a), (b) and (c), in correspondence with the successive conditions of loading, respectively. In these figures the corresponding results of calculation for each condition of loading are shown at the same time. Some explanations about these results of experiment are as follows:

1. The results for each condition of loading are almost in proportion to the weight of ballast water which was loaded.

2. The Shearing Deformation of the Cross Section: The relative deflections of the side shell part with respect to the longitudinal bulkhead part became the amounts of the order about 8 mm. and about 6 mm. upwards at the ordinary transverse ring with Fr. No. 71 and at the water tight bulkhead with Fr. No. 74, respectively. And the relative deflections of the deck centre girder part with respect to the longitudinal bulkhead part became the amounts of the order about 1.5 mm. and about 2 mm. downwards at the ordinary transverse ring with Fr. No. 71 and at the water-tight bulkhead with Fr. No. 74, respectively. These values were all measured under the Tank Test pressure. As to the relative deflections of bottom centre girder

Fig. 2-2(a) Shearing Deformation in Cross Section
part with respect to the longitudinal bulkhead part, the measurements were not made because of the difficulty. The reason why the differences between the relative deflections at Fr. No. 71 and those at Fr. No. 74 were small was perhaps that the length of the tank loaded was not so long.

(3) Normal Stresses in Face Plates: The maximum stress was the compressive stress in the corner of the bottom transverse near the longitudinal bulkhead in wing tank and its magnitude became about 25 kg/mm² when under the Tank Test pressure. This stress occurred in the face plate which was tapered down in its scantlings. In the bilge corner, there occurred somewhat large tensile stress and its magnitude became about 15 kg/mm² when under the Tank Test pressure. The other parts where the large stresses were measured were the corner of the deck transverse near the longitudinal bulkhead and the vertical web attached on the longitudinal bulkhead between the struts. Values of these stresses were much smaller than those previously mentioned.

(4) Shearing Stresses in Web Plates: The large shearing stresses were measured in the upper and the lower R-end of the corner where the lower strut was connected to the vertical web attached on the longitudinal bulkhead and in the lowest vertical web attached on the longitudinal bulkhead where the bracket of the bottom transverse in centre tank started. The values of these stresses was of the order about 7~10 kg/mm² when under the Tank Test pressure. It was especially noted that the shearing stresses in web plates were much affected by the attachment of the collar plates to the slots.

2.3 Some Considerations

(1) Comparisons between the Results of the Calculation and the Experiment

(i) As to the shearing deformation of the section, it can be said that we can well characterize the results of experiment and estimate them with accuracy of about 85~90% by the results of calculation. The results of experiment, however, are somewhat larger than those of calculation in general.
(ii) As to the normal stresses in face plates of a transverse ring, it can be said that we can well characterize and estimate the results of experiment by those of calculation with quite the good accuracy for the parallel parts and the corners. But it can not be said so for the parts where the sections are changing their scantlings from the parallel parts into the corners. This means that it is necessary to study further on the method of treatments for the so called rigid parts and for the variable cross sections in ordinary rigid frame theory used for a transverse ring and on the method of calculation for stresses in corners.

(iii) As to the shearing stresses in web plates of a transverse ring, it should be noted that we measured the shearing stresses on so few points as to make decided discussion. But it seems that we can estimate the results of experiment by those of calculation taking into account of the effects of collar plates attached to the slots.

Putting together the above mentioned things, it seems to be able to say that we can estimate the state of deformation and stresses of the transverse ring of a big tanker under the Tank Test pressure with necessary accuracy for practical use by calculating them according to the way of thinking explained in the Chapter 1. In order to obtain the results of calculation with more accuracy, we think that it is necessary to study further on the items pointed out in the paragraph (ii) of this section.

(2) The Effects of the Shearing Deformation upon the Distribution of the Stresses in a Transverse Ring

The distribution of the shearing deformation of the cross section over all the tank parts under the Tank Test pressure is as shown in Fig. 2-3 (a). The corresponding values of the normal stresses due to the shearing deformation together with the total values in the transverse ring wing with Fr. No. 71 are show in Fig. 2-3 (b). Those about the shearing stresses in the same transverse ring are shown in Fig. 2-4 (c).

From these figures we can see the followings:

(i) The shearing deformation of the cross section reaches its maximums in way of the tank in which the ballast water is loaded and becomes smaller clear of that tank. This means that the shearing deformation of the cross section becomes larger in way of the tank part where the deviation of the load between on the centre tank and on the wing tank is existence. This is what the many researchers have already pointed out.

(ii) The shearing deformation of the cross section induces the stresses so as to increase the

![Fig. 2-3 (a) Distribution of the Shearing Deformation of the Cross Section over all the Tanks.](image-url)
stresses due to the direct load in the lower members of a transverse ring and to decrease them in the upper members in general in case of the Tank Test. The ratios of the induced stresses to the stresses due to the direct load in the lower members of a transverse ring are of the order about 20-30%, both for the normal and shearing stresses. These ratios will become larger in case that the ballast water is loaded in rather a longer tank than in case of this tank test.

3. Miscellaneous

In Fig. 2-2 (b) we can easily find that the normal stress of the corner becomes much larger if the scantling of the face plate is tapered down within the range of the rounded corner. It is, therefore, necessary not to taper down the scantling of the face plate within the range of the rounded corner in order to avoid the occurrence of such a large stress.

3. Some Considerations upon the Longitudinal Strength Members

How it can become the shearing force and bending moment of the hull girder when the longitudinal strength calculation is carried out on a composite parallel beam instead of on a simple beam? The shearing force and the equivalent bending moment which are shared by each the longitudinal strength member, namely, the side shell part, the longitudinal bulkhead part, the bottom centre girder part and the deck centre girder part, are calculated using the equation (3.1) and are shown in Fig. 3-1 (a), (b) and Fig. 3-2 (a), (b). Fig. 3-1 is of those corresponding to the Heavy Ballast Condition of the tanker on which the experiment was made. Also, Fig. 3-2 is of those corresponding to the Full Load Condition of the same tanker.

\[
\begin{align*}
2S1.e &= 2S_1 \\
2S2.e &= 2S_2 \\
S3.e &= S_3 \\
S4.e &= S_4 \\
2M1.e &= (M_1 + Je_1 \cdot N_1) \times 2 \\
2M2.e &= (M_2 + Je_2 \cdot N_2) \times 2 \\
M3.e &= M_3 + Je_3 \cdot N_3 \\
M4.e &= M_4 - Je_4 \cdot N_4 \\
\end{align*}
\]
Fig. 3-1 (a) Distribution of the Shearing Force (in Heavy Ballast Condition)

Fig. 3-1 (b) Distribution of the Bending Moment (in Heavy Ballast Condition)

Fig. 3-2 (a) Distribution of the Shearing Force (in Full Load condition)

Fig. 3-2 (b) Distribution of the Bending Moment (in Full Load condition)
The shearing forces and bending moments calculated by the ordinarily Longitudinal Strength Calculation corresponding to the above load conditions are shared into each the strength member in proportion to the ratios of shearing and bending rigidity of each member using the equation (3.2). These values are shown in the corresponding above figures at the same time.

\[
\begin{align*}
2S1.e &= S_0 \times (2A_{S1} + 2A_{S2} + kA_{S3} + kA_{S4}) \\
2S2.e &= S_0 \times (2A_{S2} + 2A_{S1} + 2A_{S2} + kA_{S3} + kA_{S4}) \\
S3.e &= S_0 \times (kA_{S3} + 2A_{S1} + 2A_{S2} + kA_{S3} + kA_{S4}) \\
S4.e &= S_0 \times (kA_{S4} + 2A_{S1} + 2A_{S2} + kA_{S3} + kA_{S4}) \\
2M1.e &= M_0 \times \{2(I_1 + Je^2 \cdot A_{N1})/I_0\} \\
2M2.e &= M_0 \times \{2(I_2 + Je^2 \cdot A_{N2})/I_0\} \\
M3.e &= M_0 \times \{(I_1 + Je^2 \cdot A_{N1})/I_0\} \\
M4.e &= M_0 \times \{(I_1 + Je^2 \cdot A_{N1})/I_0\}
\end{align*}
\]

\[ (3.2) \]

where; \( k \) is a correction factor for the distribution of the shearing stresses in the web plates of bottom and deck centre girder and in this case the value of \( k \) is taken equal to 2/3.

The maximum values of those calculated by the equation (3.1) are compared to those calculated by the equation (3.2) and are listed in Table 3-1.

The following table shows the comparison between the maximum value of shearing force and bending moment:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Force or D. Moment</th>
<th>Member</th>
<th>Due to this Methods</th>
<th>In Proportion to the Rigidity</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shearing</td>
<td></td>
<td>S. Shell</td>
<td>2600</td>
<td>5000</td>
<td>0.50</td>
</tr>
<tr>
<td>Force</td>
<td></td>
<td>L. BHD</td>
<td>2600</td>
<td>5000</td>
<td>0.50</td>
</tr>
<tr>
<td>(Ton)</td>
<td></td>
<td>B. G.</td>
<td>2600</td>
<td>5000</td>
<td>0.50</td>
</tr>
<tr>
<td>(Fr. No. 74)</td>
<td>SUM</td>
<td>-9636</td>
<td>-9645</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending</td>
<td></td>
<td>S. Shell</td>
<td>-13.25 x 10^4</td>
<td>-12.6 x 10^4</td>
<td>1.04</td>
</tr>
<tr>
<td>Moment</td>
<td></td>
<td>L. BHD</td>
<td>-17.86</td>
<td>-17.9</td>
<td>1.01</td>
</tr>
<tr>
<td>(Ton-M)</td>
<td></td>
<td>B. G.</td>
<td>-4.59</td>
<td>-4.16</td>
<td>1.10</td>
</tr>
<tr>
<td>(Fr. No. 79)</td>
<td>SUM</td>
<td>-38.12 x 10^4</td>
<td>-38.14 x 10^4</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Shearing</td>
<td></td>
<td>S. Shell</td>
<td>+2980</td>
<td>+3780</td>
<td>0.79</td>
</tr>
<tr>
<td>Force</td>
<td></td>
<td>L. BHD</td>
<td>+3420</td>
<td>+3690</td>
<td>1.27</td>
</tr>
<tr>
<td>(Ton)</td>
<td></td>
<td>B. G.</td>
<td>-630</td>
<td>-517</td>
<td>1.23</td>
</tr>
<tr>
<td>(Fr. No. 74)</td>
<td>SUM</td>
<td>+7177</td>
<td>+7181</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending</td>
<td></td>
<td>S. Shell</td>
<td>+6.95 x 10^4</td>
<td>+6.14 x 10^4</td>
<td>1.13</td>
</tr>
<tr>
<td>Moment</td>
<td></td>
<td>L. BHD</td>
<td>+8.31 x 10^4</td>
<td>+8.72</td>
<td>1.01</td>
</tr>
<tr>
<td>(Ton-M)</td>
<td></td>
<td>B. G.</td>
<td>+2.48 x 10^4</td>
<td>+2.05</td>
<td>1.23</td>
</tr>
<tr>
<td>(Fr. No. 84)</td>
<td>SUM</td>
<td>+18.52 x 10^4</td>
<td>+18.56 x 10^4</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

In these figures and the table we can observe the following things.
1. Shearing Forces
   (i) In way of the tank parts, the shearing force seems to be more shared by the longitudinal bulkhead and to be less shared by the side shell than what is usually considered in both the two load conditions.
   (ii) The shearing forces are much deviated from those shared by in proportion to the shearing rigidities of the longitudinal members, especially in the bottom and the deck centre girders.
   (iii) The deviations observed in case of heavy ballast condition are larger than those observed in case of full load condition.

2. Bending Moments
   (i) The bending moments are slightly deviated from those shared by in proportion to the rigidities in the side shell and in the longitudinal bulkhead, but are much deviated in the bottom and the deck centre girders, both in the two load conditions.

3. General Tendency
   (i) It can be said that the more rapidly is the load changing along and across the tank parts, the larger will become these deviations for both the shearing forces and the bending moments. These things can be easily understood and these are in accordance with what Prof. Mano proposed before. It can, therefore, be said about the shearing deformation of a tanker as follows:
      (i) When only the shearing deformation in wing tank is discussed, the shearing deflections of the side shell and of the longitudinal bulkhead due to the load deviated are the most important. The method, therefore, proposed before by Prof. Yamakoshi will give the good results for this case.
      (ii) When the shearing deformations both in the wing and the centre tanks are discussed, it is necessary to take into account of both the bending and shearing deflections and the elongations, especially those of the centreline girders.

4. Conclusions

In this paper the method of a three dimensional strength calculation on the tank part structures of a big tanker was deduced and the results of the experiment on an actual ship were analyzed by this method with some considerations upon the longitudinal strength members. As the results of this study the following things were found:

(1) The deformations and the stresses in the transverse ring of a big tanker can be estimated with necessary accuracy for the practical uses according to the method explained here.

(2) The shearing deformations in the cross section increase the stresses in the lower parts of the transverse ring due to the direct load in the magnitude of about 20~30% under Tank Test Pressure in the case of this study.

(3) In way of the tank parts, the shearing force seems to be more shared by the longitudinal bulkhead and to be less shared by the side shell than what is usually considered.

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References:

1) T. Okabe & K. Hori: "Three Dimensional Strength Analysis of Frame Structure of Tankers by
Hardy Cross’ Method.” Journal of the Society of Naval Architects of West Japan, No. 8 & No. 9.


Appendix

Methods of calculation

Using a well known technique our problem is formulated as a boundary value problem of a first order ordinary linear differential equation (A1) in a 24 dimensional vector space with suitable jump conditions at every bulkhead.

\[ Y = AY + F(x) \]  \hspace{1cm} (A1)

where \( A \) is a constant singular matrix and zero is its eigen value of multiplicity two, and \( F(x) \) is a vector function corresponding to load conditions which are assumed constant in each tank, and \( Y \) is an unknown vector function to be determined.

To solve a problem of this kind, a general method is described in reference\( ^3 \), and the method of calculation employed here is an example of it with some modifications to avoid the decrease of accuracy in final results. Although our computer program written in IBM 7040 FORTRAN IV language can solve up to 15 tanks problem, we restrict ourselves here to a simple case of three tanks problem to clarify the modified method. For the sake of simplicity however we omit detailed description of the actual choice of following various matrices and vectors.

Let \( l \) be half tank length, so that bulkheads lie at \( x=0, 2l, 4l \) and \( 6l \). Taking suitable 12 unknowns at each end, we can express the given boundary conditions in the following form:

\[
\begin{align*}
Y(0) &= BZ_1 + C_+ \\
Y(6l) &= BZ_5 + C_- \\
\end{align*}
\]  \hspace{1cm} (A2)

where \( B \) is a suitable \( 24 \times 12 \) matrix, and \( C_\pm \) is a vector, and \( Z_i = (i=1, 2) \) is an unknown vector with 12 elements to be determined.

Let

\[
\begin{align*}
Z_1 &= Y(2l-0) \\
Z_4 &= Y(4l+0) \\
\end{align*}
\]  \hspace{1cm} (A3)

then with appropriate \( 24 \times 24 \) transfer matrices \( T_+ \) and \( T_- \), and vectors \( C_+ \) and \( C_- \), our jump conditions at the intermediate bulkheads can be formulated as

\[
\begin{align*}
Y(2l+0) &= T_+Z_1 + C_- \\
Y(4l-0) &= T_-Z_4 + C_- \\
\end{align*}
\]  \hspace{1cm} (A4)

Integration procedure is carried out from left and right, and equating the integrated results at \( x=1, 3l \) and \( 5l \), we obtain following three equations.

\[
\begin{align*}
R_+(BZ_1 + C_+) + K_+F_1 &= R_-Z_5 + K_-F_1, \\
R_+(T_+Z_1 + C_-) + K_+F_2 &= R_-Z_4 + K_-F_2, \\
R_+Z_4 + K_+F_3 &= R_-(BZ_1 + C_-) + K_-F_3, \\
\end{align*}
\]  \hspace{1cm} (A5)
where $F(i=1,2,3)$ is a value of the vector function $F(X)$ in $i$-th tank, and

$$
\begin{align*}
R_{\pm} &= \exp(\pm tA) = \sum_{j=0}^{\infty} \frac{A^j(\pm t)^j}{j!}, \\
K_{\pm} &= \sum_{j=0}^{\infty} \frac{A^j(\pm t)^j}{j!},
\end{align*}
$$

Solving this linear simultaneous equation (A5) for 72 unknown elements of four vectors $Z_1, Z_2, Z_3,$ and $Z_4$, it is easy to obtain the complete solution of the problem$^9$.

In actual numerical computation, following two methods we adopted are worth mentioning:

1. We meet with some difficulties to calculate $R_+$ and $K_+$ numerically, so we use, instead of the infinite series definition (A6), well known Sylvester's Theorem of confluent type which ensures the existence of finite base matrices to compute these matrix functions$^{10,11}$.

2. To avoid cancelling in meaningful digits, we use double precision arithmetics where necessary, and many stage elimination technique to solve the simultaneous equation (A5).

**REFERENCES OF APPENDIX**

$^a$ See the References 7).
