(昭和 45 年 11 月日本造船学会秋季講演会において講演)

Higher Mode Vertical Vibration of Giant Tanker

(2nd Report)

Koji Kagawa*, *Member* Katsuo Ohtaka*, *Member*

Summary

In the higher mode vibration of giant tankers the transverse section of the ship deforms elastically, and the side shell and the longitudinal bulkhead show the individual deflection, as explained in the first report³⁾. In this report, the hydrodynamic inertia has been investigated theoretically for such a case, and the results shows that the hydrodynamic inertia has three components, i.e., added mass of water for longitudinal bulkhead member, that for side shell member and the hydrodynamic coupling term. Theoretical estimation of these three components nearly coincides with the experiment.

The calculation of the natural frequency of the ship using the hydrodynamic inertia derived above agrees with the experiment in a full scale ship with 5% of errors.

The calculation of response to a sinusoidal force has been also carried out, the results of which show that, although the calculation agrees with experiment qualitatively, more study is necessary to acquire the precise result.

1 Introduction

The deformation of the transverse section of the ship should be considered in the study of the higher mode vertical vibration of the giant taker^{1)~4)}. In this report, the followings are studied based upon the results of the 1st report³⁾.

- (a) added weight of water for higher mode vibration
- (b) natural frequency and response of higher mode vibration

The ship hull is simulated by the two parallel beams elastically connected, which is the same that in the 1st report. Each beam represents the longitudinal bulkhead and the side plating respectively and the connecting spring the transverse members. The transfer matrix method developed in the 1st report to calculate the frequency and the response of the ship is used for the numerical calculation in this report. Antisymmetric deformation of the transverse section of the ship is not taken into account.

2 Added virtual weight of water

The ship vibrates approximately as a beam in the lower mode where the transverse section of the ship oscillates vertically without elastic deformation as shown in Fig. 2.1 (a). In this case, an added virtual mass of water of unit length of the ship is represented by $J \cdot m_w$, where J is a three-dimensional correction factor and m_w is the two-dimensional added virtual mass per unit length and is shown as follows :

$$m_w = \frac{1}{2} \rho \pi b^2 C_{V0} \quad (2.1)$$

where

ρ : density of water

* Nagasaki Technical Institute, Technical Headquarters, Mitsubishi Heavy Industries, Ltd.

(a) beam theory

(b) higher mode vibration

Fig. 2.1 Vibration mode of cross section

Fig. 2.2 Cross section of floating body

b : half breadth of section

C_{V0} : hydrodynamic coefficient

In the case of higher mode vibration, however, the transverse section deforms elastically as shown in Fig. 2.1 (b) and, as was pointed out by Kumai⁸⁾ and Matsuura¹⁰⁾, it is not correct to adopt the equation (2.1). In this report, the treatment of hydrodynamic inertia effect of surrounding water for this kind of vibration is presented, under the assumption that the flow is two-dimensional. A section of the two-dimensional floating body is shown in Fig. 2.2. The hydrodynamical boundary condition for the vertical vibration of the section with elastic deformation is drawn as follows.

$$-\frac{\partial \phi}{\partial n} = -\frac{\partial \psi}{\partial s} = V_{(\bar{x}, \bar{y}, t)} \frac{\partial \bar{x}}{\partial s} \quad (2.2)$$

$$\therefore \phi = -\int_0^{\bar{x}} V_{(\bar{x}, \bar{y}, t)} d\bar{x} + \text{const} \quad (2.3)$$

where

ϕ : velocity potential

ψ : stream function

$V_{(\bar{x}, \bar{y}, t)}$: vertical velocity of elastic deflection of boundary shown in Fig. 2.2

$(\bar{x}, \bar{y})$: coordinate of a point on the boundary

It is assumed that the transverse section of the ship in the Z -plane is the conformal transformation of unit circle in ζ -plane through Lewis' formula (so-called the Lewis Forms) as follows.

$$\left. \begin{aligned} z &= b_0 \left(\zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} \right) \\ z &= x + iy, \quad \zeta = r e^{i\theta} \\ b_0 &= \frac{b}{4} \left\{ 3(1+\lambda) - \sqrt{(1+\lambda)^2 + 8\lambda \left(1 - \frac{4\sigma}{\pi} \right)} \right\} \\ a_1 &= \frac{1-\lambda}{2b_0} b \quad a_3 = \frac{1+\lambda}{2b_0} b - 1 \\ \lambda &= \frac{d}{b} \quad \sigma = \frac{S}{2bd} \\ d &: \text{draft} \\ S &: \text{area of section under water surface} \end{aligned} \right\} \quad (2.4)$$

Another assumption is that the vibration mode of ship section in Z -plane is expressed by the following quadratic equation as shown in Fig. 2.3.

$$V(\bar{x}, \bar{y}, t) = \frac{V_1}{e} \{ e + (1-\alpha)(x_1^2 - \bar{x}^2) \} \quad (2.5)$$

$$\text{where } e = b^2 - x_1^2, \quad \alpha = \frac{V_2}{V_1}$$

x_1 : position of the longitudinal bulkhead

V_1 : vertical velocity of the longitudinal bulkhead

V_2 : vertical velocity of the side plating

Fig. 2.3 Elastic deformation of Lewis form

Substituting the equation (2.5) into the equation (2.3), the boundary condition for Lewis form section is given as follows :

$$\left. \begin{aligned} \psi = -\frac{V_1}{e} \left[\{e + (1-\alpha)x_1^2\} \bar{x} - \frac{(1-\alpha)}{3} \bar{x}^3 \right] + \text{const.} \end{aligned} \right\} \quad (2.6)$$

where $\bar{x} = b_0 \{ (1+a_1) \cos \theta + a_3 \cos 3\theta \}$

Using the above-mentioned boundary condition, the velocity potential and the stream function of the half-immersed Lewis form section are derived as follows :

$$\left. \begin{aligned} \phi &= \sum_m \frac{1}{r^m} S_m \sin m\theta, \quad m=1, 3, 5, 7, 9 \\ \psi &= \sum_m \frac{1}{r^m} S_m \cos m\theta, \quad m=1, 3, 5, 7, 9 \end{aligned} \right\} \quad (2.7)$$

where $S_m = f_m V_1 + h_m V_2$

$$\left. \begin{aligned} f_1 &= -\frac{1}{e} \left[\{e + x_1^2\} b_0 (1+a_1) - \frac{1}{3} b_0^3 \left\{ \frac{3}{4} (1+a_1)^3 + \frac{3}{4} (1+a_1)^2 a_3 + \frac{3}{2} (1+a_1) a_3^2 \right\} \right] \\ h_1 &= -\frac{1}{e} \left[-x_1^2 b_0 (1+a_1) + \frac{1}{3} b_0^3 \left\{ \frac{3}{4} (1+a_1)^3 + \frac{3}{4} (1+a_1)^2 a_3 + \frac{3}{2} (1+a_1) a_3^2 \right\} \right] \\ f_3 &= -\frac{1}{e} \left[\{e + x_1^2\} b_0 a_3 - \frac{1}{3} b_0^3 \left\{ \frac{1}{4} (1+a_1)^3 + \frac{3}{2} (1+a_1)^2 a_3 + \frac{3}{4} a_3^3 \right\} \right] \\ h_3 &= -\frac{1}{e} \left[-x_1^2 b_0 a_3 + \frac{1}{3} b_0^3 \left\{ \frac{1}{4} (1+a_1)^3 + \frac{3}{2} (1+a_1)^2 a_3 + \frac{3}{4} a_3^3 \right\} \right] \\ f_5 &= \frac{1}{3e} b_0^3 \left\{ \frac{3}{4} (1+a_1)^2 a_3 + \frac{3}{4} (1+a_1) a_3^2 \right\} \\ h_5 &= -f_5 \\ f_7 &= \frac{1}{4e} b_0^3 \cdot a_3^2 (1+a_1) \\ h_7 &= -f_7 \\ f_9 &= \frac{1}{12e} b_0^3 \cdot a_3^3 \\ h_9 &= -f_9 \end{aligned} \right\} \quad (2.8)$$

The kinetic energy of the fluid is expressed as follows :

$$\left. \begin{aligned} T &= \frac{1}{2} (m_{W1} V_1^2 + m_{W2} V_2^2 + 2 m_{W3} V_1 V_2) \\ \text{where} \quad m_{W1} &= \frac{\rho\pi}{2} \sum m f_m^2 \equiv -\frac{\rho\pi}{2} b^2 C_{V1} \left(\lambda, \sigma, \frac{x_1}{b} \right) \\ m_{W2} &= \frac{\rho\pi}{2} \sum m h_m^2 \equiv -\frac{\rho\pi}{2} b^2 C_{V2} \left(\lambda, \sigma, \frac{x_1}{b} \right) \\ m_{W3} &= \frac{\rho\pi}{2} \sum m f_m h_m \equiv -\frac{\rho\pi}{2} b^2 C_{V3} \left(\lambda, \sigma, \frac{x_1}{b} \right) \end{aligned} \right\} \quad (2.9)$$

Thus, hydrodynamic effect of water for the higher mode vibration of the ship may be expressed by the above-mentioned m_{W1} , m_{W2} and m_{W3} . Neglecting the three dimensional flow of water, the kinetic energy of the water surrounding the ship is expressed by the following equation.

$$T_W = \frac{1}{2} \int_{l_1}^{l_2} \left\{ m_{W1} \left(\frac{\partial y_1}{\partial t} \right)^2 + m_{W2} \left(\frac{\partial y_2}{\partial t} \right)^2 + 2 m_{W3} \left(\frac{\partial y_1}{\partial t} \right) \left(\frac{\partial y_2}{\partial t} \right) \right\} dx \quad (2.10)$$

where y_1 : deflection of longitudinal bulkhead

y_2 : deflection of side plating

On the other hand, the kinetic energy and the strain energy of the ship are as follows :

$$\left. \begin{aligned} T_H &= \frac{1}{2} \int_{l_1}^{l_2} \left\{ m_1 \left(\frac{\partial y_1}{\partial t} \right)^2 + m_2 \left(\frac{\partial y_2}{\partial t} \right)^2 + I_{m1} \left(\frac{\partial \theta_1}{\partial t} \right)^2 + I_{m2} \left(\frac{\partial \theta_2}{\partial t} \right)^2 \right\} dx \\ V &= \frac{1}{2} \int_{l_1}^{l_2} \left\{ EI_1 \left(\frac{\partial \theta_1}{\partial x} \right)^2 + EI_2 \left(\frac{\partial \theta_2}{\partial x} \right)^2 + k' G A_1 \left(\frac{\partial y_1}{\partial x} - \theta_1 \right)^2 \right. \\ &\quad \left. + k' G A_2 \left(\frac{\partial y_2}{\partial x} - \theta_2 \right)^2 + k (y_2 - y_1)^2 \right\} dx \end{aligned} \right\} \quad (2.11)$$

where m_1, m_2 : mass per unit length of longitudinal bulkhead and side plating respectively
 I_{m1}, I_{m2} : mass moment of inertia per unit length of longitudinal bulkhead and side plating respectively
 EI_1, EI_2 : bending rigidity of longitudinal bulkhead and side plating respectively
 $k'GA_1, k'GA_2$: shear rigidity of longitudinal bulkhead and side plating respectively
 θ_1, θ_2 : bending slope of longitudinal bulkhead and side plating respectively
 k : connecting spring constant of transverse members per unit length of ship

The equation of motion is drawn by mean of the Hamilton's principle,

$$\delta \int_{t_1}^{t_2} (T_W + T_H - V) dt = 0 \quad (2.12)$$

Thus, the differential equation for the higher mode vibration is expressed as follows :

$$\left. \begin{aligned} \mathbf{u} &= \{y_1, y_2, \theta_1, \theta_2, V_1, V_2, M_1, M_2\} \\ \frac{d\mathbf{u}}{dx} &= \mathbf{c}\mathbf{u} \\ \mathbf{c} &= \begin{pmatrix} 0 & 0 & 1 & 0 & \frac{1}{k'GA_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{k'GA_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{EI_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{EI_2} \\ k - (m_1 + m_{W1})\omega^2 & -(k + m_{W3})\omega^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(k + m_{W3})\omega^2 & k - (m_2 + m_{W2})\omega^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{m1}\omega^2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{m2}\omega^2 & 0 & 1 & 0 & 0 \end{pmatrix} \end{aligned} \right\} \quad (2.13)$$

where $\mathbf{u}$: column vector

V_1, V_2 : shearing force of longitudinal bulkhead and side plating respectively

M_1, M_2 : bending moment of longitudinal bulkhead and side plating respectively

ω : circular frequency

The equation (2.13) is the same as the equations (2.6), (2.7) and (2.8) in the 1st report³⁾ except elements including m_{W1} , m_{W2} and m_{W3} in the matrix $\mathbf{C}$. From the equation (2.13), the followings are concluded.

(a) m_{W1} and m_{W2} are the added mass of water to the longitudinal bulkhead and the side plating respectively.

(b) m_{W3} concerns to the hydrodynamic coupling.

The added mass of water for the higher mode vibration expressed by the above-mentioned m_{W1} , m_{W2} and m_{W3} has the following relation with the added mass in the beam theory shown by the equation (2.1).

$$C_{V0} = C_{V1} + C_{V2} + 2C_{V3} \quad (2.14)$$

The similar procedure can be applied to the rectangular section whose transforming formula is expressed as follows²⁰⁾ :

$$\left. \begin{aligned} Z &= M \left\{ \zeta + \sum_m \frac{1}{\zeta^m} A_m \right\}, \quad m = 1, 3, 5, 7, 9, 11 \\ \text{where } M &= \frac{d}{\sin \varphi - \sum A_m \sin m\varphi} \\ \lambda &= \frac{\sin \varphi - \sum A_m \sin m\varphi}{\cos \varphi + \sum A_m \cos m\varphi} \\ A_1 &= \cos 2\varphi \end{aligned} \right\} \quad (2.15)$$

$$\begin{aligned}
 A_3 &= -\frac{1}{6} \sin^2 2\varphi \\
 A_5 &= -\frac{1}{10} \cos 2\varphi \sin^2 2\varphi \\
 A_7 &= -\frac{1}{56} (5 \cos^2 2\varphi - 1) \sin^2 2\varphi \\
 A_9 &= -\frac{1}{72} (7 \cos^2 2\varphi - 3) \cos 2\varphi \sin^2 2\varphi \\
 A_{11} &= -\frac{1}{176} (21 \cos^4 2\varphi - 14 \cos^2 2\varphi + 1) \sin^2 2\varphi
 \end{aligned}$$

Fig. 2.5 Acrylite models for added water experiment

Fig. 2.6 Vibration in water of the mass-spring system

Numerical values of C_{V1} , C_{V2} and C_{V3} are shown in Fig. 2.4.

In order to check the above-mentioned results, the model experiment has been carried out. The models are made of acrylite and shown in Fig. 2.5. As the calculation is made for the two-dimensional flow, the model device is also arranged so as to keep the flow two-dimensional. The length of the model is so short compared with the breadth that every section of model undergoes the deformation like that shown in Fig. 2.1 (b). Thus, the model can be approximated for calculation by the mass-spring system of two degrees of freedom shown in Fig. 2.6, where m_1 and m_2 are the mass of the longitudinal bulkhead and the side plating respectively, k_1 is the connecting spring constant of transverse members and k_2 is the spring constant of the suspender. The equation of motion of the system is drawn as follows.

Fig. 2.4 Hydrodynamic coefficient for higher mode vibration

$$\begin{bmatrix} k_1 - \omega^2(m_1 + m_{W1}), & -(k_1 + \omega^2 m_{W3}) \\ -(k_1 + \omega^2 m_{W3}), & (k_1 + k_2) - \omega^2(m_2 + m_{W2}) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0 \quad (2.16)$$

Numerical values of models shown in Table 2.1. The mass of members located inside of the vertical planes through halfbreadth of wing tanks is assigned to the mass of the longitudinal bulkhead (m_1) and the remainders to the mass of the side plating (m_2). Values of added virtual mass of water (m_{W1} , m_{W2} and m_{W3}) are estimated by the theoretical method mentioned above. Measured and calculated results are presented in Tables 2.2 and 2.3. The followings are seen in the tables.

- Calculated results agree with measured ones in qualitative tendency.
- The effect of the hydrodynamic inertia on the frequency is larger in the 1st mode than in the 2nd mode, as was pointed out by Kumai⁶⁾.
- Discrepancy in 1st mode presented in Table 2.3

may be partly caused by the error in estimation of the spring constant of the suspender (k_2), which has the predominant effect on the frequency of the 1st mode and was estimated by the static deflection test, whereas the spring constant of the connecting spring (k_1) has the predominant effect on the frequency of the 2nd mode and was estimated by the vibration test.

Another cause may be in the inadequate rigidity of the model device.

- It can be acceptable for simplification to neglect m_{W3} , though it slightly makes error larger. In the actual ship the flow surrounding a ship is three dimensional and the correction factor should be considered. In the case of vibration based on the beam theory there is a vibration test on a

Table 2.1 Numerical values of models

	Model A			Model B		
$m_1 g$	3.18 kg			1.90 kg		
$m_2 g$	2.34 kg			1.65 kg		
$k_1^{*)}$	12512 kg/cm			1424 kg/cm		
Draft (cm)	$m_{w1} g$ (kg)	$m_{w2} g$ (kg)	$m_{w3} g$ (kg)	$m_{w1} g$ (kg)	$m_{w2} g$ (kg)	$m_{w3} g$ (kg)
1.0	9.88	0.80	1.66			
3.0				10.53	0.92	1.96
4.0				10.80	0.98	2.05
4.9	10.98	1.02	2.15			
10.0	11.90	1.17	2.52			

* k_1 is so determined that the calculated frequency coincides with the measured one in the case A-1 without suspender ($k_2=0$).

Table 2.2 Measured and calculated frequencies of models

			Model A				Model B					
			Measured freq.		Calculated freq.		Measured freq.		Calculated freq.		Calculated freq.**	
Case	Draft (cm)	Suspension	1st* mode (cps)	2nd* mode (cps)	1st mode (cps)	2nd mode (cps)	1st mode (cps)	2nd mode (cps)	1st mode (cps)	2nd mode (cps)	1st mode	2nd mode
In air	A-1	Sponge mat		480		480		200		200		
	A-2	Steel***	75	455	90.1	492.4	75	192	94.4	244.2	do.	
	A-3	Acrylite***	23	472	19.3	480.5	23	195	23.9	201.7		
In water	W-1	1.0	48	393	47.2	415.2						
	W-2	3.0						147		154.3		128.8
	W-3	3.0					...	147	10.3	156.9	11.5	131.6
	W-4	4.0					41	172	36.6	206.5	39.1	180.5
	W-5	4.9		390		397.2						
	W-6	4.9	...	390	9.7	397.9						
	W-7	4.9	47.5	392	44.6	413.9						
	W-8	10.0	50	370	42.9	413.6						

Note * y_1 and y_2 are in phase in 1st mode and out of phase in 2nd mode.

** m_{w3} is neglected in calculation.

*** Spring constant of suspender, evaluated by static deflection test, $k_2=1900$ kg/cm for steel stay, 83 kg/cm for acrylite stay

Table 2.3 Ratio of frequency in water to that in air of models

Suspender	Case	1 st mode			2 nd mode			Model
		Measured	Calculated	Meas./Cal.	Measured	Calculated	Meas./Cal.	
Free	W-5 A-1				0.813	0.827	0.983	Model A
Steel	W-1 A-2	0.640	0.524	1.221	0.864	0.844	1.022	
	W-7 A-2	0.633	0.496	1.276	0.862	0.841	1.024	
	W-8 A-2	0.666	0.477	1.396	0.814	0.841	0.968	
Acrylite	W-6 A-3	...	0.503	...	0.826	0.829	0.997	Model B
Free	W-2 A-1				0.735	0.772 (0.644)	0.952 (1.141)	
Steel	W-4 A-2	0.547	0.388 (0.414)	1.409 (1.320)	0.896	0.836 (0.739)	1.071 (1.212)	
Acrylite	W-3 A-3	...	0.431 (0.481)	...	0.754	0.777 (0.652)	0.971 (1.157)	

Note Numbers in brackets mean the case neglecting m_{w3} in calculation.

Fig. 2.7 Plan and profile of steel model

tugboat by Kumai¹¹⁾ on the three dimensional correction factor. The correction factor, however, has not been investigated for the vibration considered in this report. As it is very complicated to study the correction factor theoretically, it is determined in this report by semi-empirical method using the tank part model of a tanker. The model is made of steel and shown in Fig. 2.7. It has the rectangular cross section and no upper deck. The transverse bulkheads except both ends are made of acrylite. Measured results, together with calculated ones, are presented in Tables 2.4 and 2.5 and Fig. 2.8. The theoretical calculation is made according to the procedure mentioned in the 1st report. The two-dimensional added mass of water is evaluated as described above.

In the calculation the Townsin's three-dimensional correction factor¹²⁾ is used tentatively for the sake of simplicity, although it is not rigorous for the present case*. From the presented results, the followings are concluded.

- (a) Calculated results agree with measured ones in general tendency.

* Townsin's reduction factor is proposed for the beam vibration and neglects the effect of beam-draft ratio for simplification, which is pointed out by Kumai in discussions of reference (12).

Table 2.4 Comparison between measured and calculated frequency of tank part model

No. of nodes		In water (draft=70 mm)			In air		Remarks
		Measured	Calculated		Measured	Calculated	
			N_{w1}	N_{w2}		N_a	
1 st spectrum	2	93 cps	88.1 cps	88.1 cps	138 cps	136.6 cps	Stern vibration
	3	245	242.3	236.2	362	373.7	
	4	467	453.4	435.8	685	728.2	
	5	694	710.1	676.9	1065	1153.3	
	6	928	1013.6	961.2	1658	1655.4	
	7	1178	1354.3	1280.3	...	...	
	8	1323	...	...	...	...	
2 nd spectrum	2	534	423.4	419.5	571	471.3	
	3	...	478.6	474.7	635	526.3	
	4	726	669.0	662.5	...	697.8	
	5	...	995.2	987.2	...	1021.8	
	6	...	...	1402.2	1472	1441.9	
Reduction factor			Townsin's J_n	Townsin's J_2			

- (b) Townsin's reduction factor for 2-noded vibration (J_2) may be applicable to the higher mode vibration for the practical use.

- (c) Since the effect of the hydrodynamic inertia on the 2nd spectrum is slight it is important to study the reduction factor for the 1st spectrum in future works.

3 Natural frequencies of actual ship

Using the added virtual mass studied in the preceding section, the calculation of the natural frequency has been carried out again on the ship A, principal particulars of which were presented in the 1st report. The method is the same that used in the preceding section. The liquid cargo weight is assumed to be perfectly effective for simplification*. Calculated results are presented in Table 3.1 and Figs. 3.1, 3.2 and 3.3 for comparison with measured ones. From these results, the followings are concluded.

- (a) The present method shows good agreement of calculated and measured frequencies in the higher mode vibration of ship A.
- (b) It might be considered that the calculated frequencies in the 1st report became lower because under-estimation of the shear rigidity and over-estimation of the added virtual mass of water.

Table 2.5 Ratio of frequency in water to that in air of tank part model

No. of nodes		Measured	Calculated	
			N_{w1}/N_a	N_{w2}/N_a
1 st spectrum	2	0.674	0.645	0.645
	3	0.677	0.648	0.632
	4	0.682	0.623	0.598
	5	0.652	0.616	0.587
	6	0.560	0.612	0.581
2 nd spectrum	2	0.935	0.898	0.890
	3	...	0.909	0.902
	4	...	0.959	0.950
	5	...	0.974	0.966
	6	...	...	0.972
Reduction factor			Townsin's J_n	Townsin's J_2

* The effectiveness of the liquid cargo was investigated by Kumai⁹⁾.

Fig. 2.8 Comparison between measured and calculated mode curves of tank part model in water

Table 3.1 Comparison between measured and calculated frequency of ship A

No. of nodes	Measured (N_m) (cpm)	Calculated		
		Present method (N_p) (cpm)	Beam theory	
			N_{b1} (cpm)	N_{b2} (cpm)
1st spectrum	2	...	37.4	38.3
	3	...	75.7	79.0
	4	104	116.0	122.1
	5	158	150.8	163.2
	6	182	188.7	205.7
	7	218	222.9	252.2
	8	257	247.7	283.0
	9	281	288.5	325.6
	10	...	308.4	358.0
	11	...	334.7	397.5
	12	...	366.7	436.2
	13	...	393.2	478.5
	14	...	423.8	516.1
	15	...	458.3	...
	16	...	485.5	...
	17	...	511.7	...
	18	...	538.8	...
	19	...	563.4	...
	20	...	594.0	...
2nd spectrum	2	247	276.6	
	3	...	430.1	
	4	...	572.2	
Reduction factor		Townsin's J_2	Townsin's J_2	Townsin's J_n

Fig. 3.1 Ratio of measured and calculated frequency of ship A
(N_p , N_{b1} , N_{b2} ; see Table 3.1)

- (c) The unsatisfactory discrepancy of calculated and measured mode curves at after ship, where the extreme stern vibration is observed, suggests the necessity of future works.

Fig. 3.2 Higher mode vibration of ship A

4 Response of actual ship

The transfer matrix method to calculate the response of the higher mode vibration of a giant tanker was developed in the 1st report where the bending deflection was neglected. The calculated response is shown in Fig. 4.1 where the shear damping only is taken into account for simplification. In the figure the so-called stern vibration is seen in the higher frequency range.

On the other hand, the forced amplitude of the aft end of the ship subjected to a sinusoidal force acting on the end can be expressed by the following analytical equation, provided that the vibratory response is assumed to be the sum of normal functions.

$$y = \sum_{n=2}^{\infty} \frac{F}{\omega_n^2 \sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{\delta_n}{\pi} \cdot \frac{\omega}{\omega_n}\right)^2}} \cdot \int_0^L \frac{w}{g} X_n^2 dx \quad (4.1)$$

where y : amplitude of vibratory response of ship end

F : amplitude of sinusoidal exciting force acting on ship end

ω : circular frequency of exciting force

X_n : n 'th normal mode

ω_n : circular frequency of n 'th normal mode

δ_n : logarithmic decrement of n 'th normal mode

w : weight per unit length of ship including added water

L : ship length

x : coordinate along ship length

g : acceleration of gravity

The reciprocal of the mode factor may be defined to be an effective mass coefficient as follows :¹⁹⁾

$$C_e = \frac{\int_0^L \frac{w}{g} X_n^2 dx}{\int_0^L \frac{w}{g} dx} \quad (4.2)$$

For the higher mode vibration of a giant tanker, the effective mass and the total mass are expressed by the following equations.

Fig. 3.3 Comparison between measured and calculated mode curves of ship A

Fig. 4.1 Absolute amplitude and number of nodes of ship A in shear vibration ($\beta=0.001$ sec)

$$\left. \begin{aligned} \int_0^L \frac{w}{g} X^2 dx &= \int_0^L \{ (m_1 + m_{W1}) y_1^2 + (m_2 + m_{W2}) y_2^2 + 2 m_{W3} y_1 y_2 \} dx \\ \int_0^L \frac{w}{g} dx &= \int_0^L \{ m_1 + m_{W1} + m_2 + m_{W2} + 2 m_{W3} \} dx \end{aligned} \right\} \quad (4.3)$$

The calculated effective mass coefficient of the normal mode of ship A is shown in Fig. 4.2, where "shear mode" means that the bending deflection is neglected. The effective coefficient undergoes irregular fluctuation with the number of nodes. It may be partly caused by the non-uniform distribution of the weight as was pointed out by Kumai¹⁹⁾, and also by the elastic deformation of the section. In the Fig. 4.2 the effective mass coefficient is approximated by simple formula shown as "proposed line" which is used later in this report. The total weight of ship A is 256,690 ton including virtual weight of water. For the resonance in the case of small damping, the equation (4.1) can be reduced to the following equation.

$$\alpha = \frac{\mu}{c_e W/g} \quad (4.4)$$

where α : acceleration of ship end per unit exciting force acting on ship end

μ : dynamic magnification factor

W : total weight including added water

In the shear vibration with shear damping, the dynamic magnification factor and the logarithmic decrement are expressed as follows*.

$$\mu = \frac{1}{\beta \omega}, \quad \delta = \pi \beta \omega \quad (4.5)$$

where β : shear damping coefficient

For comparison of the response calculated by the transfer matrix method and that calculated by using the equations (4.4) and (4.5), numerical results are presented in Table 4.1. In the table, the effective mass coefficient (c_e) is evaluated by using the absolute amplitude of shear vibration and the equation (4.3). These results show good agreement between the transfer matrix method and analytical method. Decreasing of effective mass coefficient due to the effect of damping is presented in Table 4.2. Thus the following equation is derived.

$$\alpha = \frac{\mu}{r c_e W/g} \quad (4.6)$$

where r : reduction factor of c_e due to damping
(ratio of c_e in Table 4.2)

Fig. 4.2 Effective mass coefficient of higher mode vibration of ship A

Fig. 4.3 Ratio of measured and calculated response of ship A and ship B

* It is clarified that the effect of non-uniform distribution of damping has little influence on the response of the ship¹⁹⁾.

Table 4.1 Dynamic response of excitation end (aft end) in shear vibration of ship A

β			0.001 sec					0.0005 sec				
No. of nodes	Item		C_e	μ^*	α_I^{**} (gal/ton)	α_{II}^{***} (gal/ton)	α_I/α_{II}	C_e	μ^*	α_I^{**}	α_{II}^{***}	α_I/α_{II}
	Freq.	(cpm)										
1 st spectrum	2	84.883	0.2660	112.5	1.615	1.653	0.977	0.2403	225.0	2.643	2.768	0.955
	3	114.80	0.2384	83.2	1.332	1.410	0.945		166.4			
	4	149.80	0.1405	63.8	1.732	1.885	0.919		127.5			
	5	174.80	0.2018	54.6	1.034	1.142	0.905		109.3			
	6	211.82	0.1315	45.1	1.309	1.417	0.923		90.2			
	7	243.32	0.1251	39.2	1.198	1.194	1.003	0.1539	78.5	1.948	2.118	0.920
	8	267.84	0.0756	35.7	1.801	1.932	0.932	0.0825	71.3	3.300	3.650	0.904
	9	301.99	0.1265	31.6	0.955	1.083	0.882	0.1882	63.2	1.283	1.634	0.785
	10	326.19	0.0473	29.3	2.364	2.627	0.900	0.0522	58.6	4.281	4.882	0.877
	11	350.09	0.0580	27.3	1.797	1.934	0.929	0.0654	54.6	3.186	3.450	0.923
	12	380.62	0.0492	25.1	1.946	2.100	0.927	0.0568	50.2	3.371	3.697	0.912
	13	405.30	0.0507	23.6	1.773	1.981	0.895	0.0585	47.1	3.074	3.452	0.891
	14	434.44	0.0662	22.0	1.268	1.415	0.897	0.0837	44.0	2.005	2.199	0.911
	15	452.71	0.0809	21.1	0.996	1.338	0.745	0.1190	42.2	1.354	1.985	0.682
	16	497.84	0.0441	19.2	1.660	1.774	0.936	0.0582	38.4	2.516	2.779	0.906
	17	525.49	0.0488	18.2	1.422	1.502	0.947	0.0967	36.3	1.435	1.685	0.852
2 nd spec	2	291.22	0.1547	32.8	0.809	1.800	0.450					
	3	472.77	0.1040	20.2	0.741	0.864	0.858					
Remarks	Shear Vibration											

*) $\mu = \frac{1}{\beta\omega}$ **) $\alpha_I = \frac{\mu}{C_e W/g}$ $W = 256,690$ tons $g = 980$ cm/sec² ***) Calculated by transfer matrix method

Table 4.2 Decreasing of effective mass coefficient and dynamic magnification factor in shear vibration of ship A

Case			I			II			III
β (sec)			0.0005			0.001			0
Item			C_e	Ratio of C_e (I/III)	μ^*	C_e	Ratio of C_e (II/III)	μ^*	C_e
No. of nodes	Freq. (cpm)								
1 st spectrum	2	84.883	0.2403	1.003	174.2	0.2660	0.999	115.2	0.2663
	3	114.80				0.2384	0.995	88.1	0.2396
	4	149.80				0.1405	0.994	69.4	0.1414
	5	174.80				0.2018	0.971	60.4	0.2078
	6	211.82				0.1315	0.922	48.8	0.1426
	7	243.32				0.1539	0.920	85.3	0.1251
	8	267.84	0.0825	0.972	78.9	0.0756	0.890	38.2	0.0849
	9	301.99	0.1882	0.836	80.6	0.1265	0.562	35.9	0.2250
	10	326.19	0.0522	0.974	66.8	0.0473	0.882	32.5	0.0536
	11	350.09	0.0654	0.962	59.1	0.0580	0.852	29.4	0.0680
	12	380.62	0.0568	0.947	55.0	0.0492	0.820	27.1	0.0600
	13	405.30	0.0585	0.974	52.9	0.0507	0.844	26.3	0.0601
	14	434.44	0.0837	0.915	48.2	0.0662	0.723	24.5	0.0915
	15	452.71	0.1190	0.923	61.9	0.0809	0.627	28.3	0.1289
	16	497.84	0.0582	0.891	42.4	0.0441	0.676	20.5	0.0653
	17	525.49	0.0967	0.557	42.7	0.0488	0.281	19.2	0.1735
	2 nd spectrum	2	291.22				0.1547	0.936	72.9
3		472.77				0.1040	0.239	77.2	0.4350
Remarks	Shear vibration								Normal mode

* Deteained so that α_I coincides with α_{II} in Table 4.1

Dynamic response of ship A calculated by the equation (4.6) using values of c_e of natural mode in Fig. 4.2 and μ and r for $\beta=0.001$ sec in Table 4.2 is shown in Fig. 4.3. In the figure, there are response of ship B and another result of ship A, which are calculated also by the equation (4.6) but using the following proposed formula derived from c_e in Fig. 4.2 and μ and r for $\beta=0.001$ sec in Table 4.2.

$$\left. \begin{aligned} c_e &= \frac{0.2}{n+2} + 0.05 \\ \mu &= \frac{400}{n+2} \\ r &= \frac{1}{1+0.1\left(\frac{n}{10}\right)^4} \end{aligned} \right\} \quad (4.7)$$

where n : number of nodes

These proposed formulae are shown in Figs. 4.4, 4.5 and 4.6. From above-mentioned results, followings are concluded.

Fig. 4.5 Dynamic magnification factor (μ)

Fig. 4.4 Effective mass coefficient in higher mode vibration

Fig. 4.6 Reduction factor (r) of effective mass coefficient (c_e) of ship A

- (a) The rough value of response of the higher mode vibration of a giant tanker may be estimated by the equations (4.6) and (4.7) with empirical factor of 0.5.
- (b) It is necessary to study more precisely the response of a giant tanker in future works.

5 Conclusive remarks

A treatment of hydrodynamic inertia for the higher mode vertical vibration of a giant tanker has been studied by theoretical and experimental procedures. The calculation of the frequency and the response of the ship has been made and compared with the measured results. Although the theoretical results agree with the measured ones in general, the followings need future works.

- (a) Three dimensional reduction factor for the effect of hydrodynamic inertia in the higher mode vibration of giant tankers.
- (b) Effect of such local structures on the higher mode vibration of giant tankers as double bottom in engine room, propulsion system and superstructure.
- (c) Damping mechanism and response in higher mode vibration of giant tankers.

The authors express their hearty gratitude to Professor T. Kumai for his kind advice and to Mr. M. Nakano and Mr. T. Hara for their sincere cooperation in model experiments.

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