(昭和 48 年 5 月日本造船学会春季講演会において講演)

Fatigue Crack Propagation under Controlled Stress Intensity Factor-(I)

Hiroshi Itagaki*, *Member*

Tetsuo Ogawa*, *Member*

Yoshito Mitani**, *Member*

Summary

This paper shows how the information about the fatigue crack propagation can be obtained by the test under controlled stress intensity factor and discusses the significance of such data. A new testing method was proposed to control the testing machine during the propagation of the crack based on the approximate evaluation of the stress intensity factor. The dependence of the growth rate on stress intensity factor was confirmed.

1. Introduction

The primary purpose of this report is to determine the fatigue crack growth law of a mild steel and the involved material constants as precisely as possible by controlling the stress intensity factor, this factor will be called K -value hereinafter. To do so, the authors propose a new testing method which can be carried out rather easily at any laboratory where there is an electro-hydraulic servo system.

In order to accentuate the significance of the accumulation of the fatigue test results under controlled K -value the reliability of a fatigue sensitive structure will be discussed briefly. A structure in which the propagation or growth of the fatigue crack is a serious problem must be a fail-safe structure in the sense that the initiation of fatigue cracks or the existence of small cracks are allowable or unavoidable. The catastrophic failure of such structure due to fatigue may be defined by

(event 1) a fatigue crack becomes a through-thickness crack,

(event 2) the length of a fatigue crack becomes long enough to satisfy the condition of the unstable (rapid) fracture, or

(event 3) the crack growth rate due to cyclic load becomes extremely high.

All of these may be taken into account for some structures and one or two for the others.

Event 3 occurs when the fatigue crack length is large or extremely high stress acts upon the structure. When the former is the case, the crack is possibly detected and necessary steps must be taken before the fatal event happens and, so as to the latter, the probability of the occurrence of the other failures such as collapse as a whole is as high as that of event 3. The significance of event 3 as the fatigue failure is, therefore much less than that of event 1 or 2. This must be taken into account on determining experimentally the resistance of the material against fatigue.

Now let T_1 and T_2 be the time to the occurrence of event 1 and 2, respectively, and the reliability of a structure with respect to the fatigue failure can be defined by

$$Pr(\text{life} \geq t) = Pr(T_1 \geq t \text{ and } T_2 \geq t), \quad (1)$$

if event 1 is fatal and

* Faculty of Engineering, Yokohama National University

** Graduate student, the University of Tokyo

$$Pr(\text{life} \geq t) = Pr(T_2 \geq t), \quad (2)$$

if event 1 is not fatal,

where $Pr(A)$ = probability of event A .

If $Pr(T_2 \geq t) \ll Pr(T_1 \geq t) \doteq 1$,

$$Pr(\text{life} \geq t) = Pr(T_2 \geq t) \quad (1')$$

and if events 1 and 2 are mutually independent,

$$Pr(\text{life} \geq t) = Pr(T_1 \geq t)Pr(T_2 \geq t). \quad (1'')$$

The factors which will appear in eqs. (1) and (2) may be different from each other but these two cases are similar from the view point of the random process and can be treated as the level crossing problem. If the (time dependent) fatigue crack length is considered as a random process, event 1 is reduced to the crossing of the constant level, plate thickness, and event 2 to that of the random level which is a function of random variables such as the applied stress, temperature, crack length and material constants*1. From this point of view the research of the crack growth law and the statistics of the material constants are said very important. It is mainly because of this fact that there are a lot of experimental studies on the fatigue crack propagation. Results of the experimental studies as well as the theoretical ones show that the crack growth rate is governed by K -value^{(1), (2), (3), (4), (5)}. Experimental studies, except very few, were carried out under controlled load, usually constant load, and by measuring crack growth rate at a certain crack length the relation between K -value and the rate were obtained. The conclusions from these tests may not be very positive because of the inherent ambiguity due to the varying K -value. Swanson, et al.⁽⁶⁾ and Mukherjee, et al.⁽⁷⁾ carried out K -value controlled fatigue test (KCF test) with stepdown load on Al-clad stainless steel and Polymethylmethacrylate, respectively. They showed that the fatigue crack growth rate was constant if the maximum and minimum K -values were kept constant. The results of the load controlled fatigue test (LCF test) and those of KCF test are not different from each other because both data scatter in the same zone. However, this does not necessarily mean that LCF test is good enough to investigate the characteristics of the fatigue crack growth. It is seen from the above mentioned results that a fatigue crack propagated through the material more than 100 mm under nearly constant K -value and the total number of the cycles was order of 10^5 . The obtained growth rate, therefore, is the mean value of the material properties over 100 mm, or mean from 10^5 samples—if we assume that a striation is formed during one load cycle, the mean growth rate is equivalent to the mean striation space. Generally speaking the mean from samples of this size shows very small dispersion and can be the mean property of the material as a whole. And yet the growth rate for a given K -value were different from specimen to specimen, e.g. 0.65×10^{-3} mm/cycle to 0.28×10^{-3} mm/cycle for $K = 23.5 \text{ kg/mm}^{3/2}$. Suppose a LCF test is done with this material and the crack length is measured every ΔN cycles. The growth rate thus obtained is an average over the length of $\Delta N \times (\text{growth rate})$ under increasing K -value. It is obvious that the data must show wider dispersion. The dispersion may decrease if ΔN increases but the change of K -value during ΔN cycles will become too large to correlate the growth rate to the applied K -value. The wider the dispersion is, the more ambiguous the fatigue crack law becomes. To reduce the dispersion, growth rate is sometimes evaluated from a faired curve of the crack length versus cycle and plotted against the corresponding K -value, from which the parameters such as the proportional constant or exponent in the expression of the growth law are determined. This procedure is somewhat absurd since a growth law is constructed from growth laws arbitrarily determined for each specimen—fairing of crack length-cycle curves is equivalent to assuming a growth law. A rela-

*1 Material constant such as the critical COD for the initiation of brittle fracture should be regarded as a random variable.

tion between the growth rate and K -value is often assumed a priori with the aid of the theoretical analysis and parameters are determined to be consistent with the experimental results. Without very detailed and exact analysis of the fatigue mechanism this method may involve too much arbitrariness.

Practically speaking, the question is if it is permissible to make use of the data obtained by LCF test for the reliability analysis relevant to event 1 or 2. LCF test will produce useful data if it is arranged to simulate very closely the actual phenomenon, because the life of the structure may be similar to that of the specimen or model. To the contrary, if the test is very simple and intended to provide basic data for the estimation of the life, LCF test is not competent because it is very difficult to get information about the characteristics of the random (fatigue) process. It has been pointed out⁽⁸⁾ that the statistical properties of the crack propagation factor (in eq. (3)), especially its space correlation, have great influence upon the probability distribution of the fatigue life. Such space correlation or gradual change of the crack growth rate may not be determined by LCF test. The above discussion will be unnecessary when the actual load amplitude is almost constant and the crack length is extremely short or, on the contrary, the fatigue crack grows extremely long. The crack propagation factor can be assumed constant during propagation—a sample from the parent population for the short crack and the expected value for the long one. On the other hand the real load history is often of random nature and therefore the behavior of the fatigue crack becomes more complicated than that under constant load.

Considering the fatigue crack propagation under a narrow band stationary random load with the assumptions that the crack growth law is

$$\frac{dC}{dN} = kh(C)\sigma^m \quad \text{for LCF} \quad (3)$$

or

$$\frac{dC}{dN} = k\Delta K^m \quad \text{for KCF} \quad (4)$$

and the crack propagation factor, k , is not influenced by the load sequence, we have

$$C_n = \sum_{i=1}^n k_i h_{i-1} \sigma_i^m \quad \text{for LCRF} \quad (5)$$

or

$$C_n = \sum_{i=1}^n k_i \Delta K_i^m \quad \text{for KCRF} \quad (6)$$

where σ_i = stress amplitude of the i -th cycle,
 C_{i-1} = crack length after the $(i-1)$ -th cycle,
 $h_{i-1} = h(C_{i-1})$ = a function of C_{i-1} ,
 k_i = crack propagation factor at the i -th cycle,
 ΔK_i = amplitude of K -value.

Assuming that C_i depends on σ_j ($j=1, 2, \dots, i-1$) but σ_i and σ_j ($1 \leq j \ll i$) are independent each other, the expectations of eqs. (5) and (6) are

$$E(C_n) \doteq E(\sigma^m) \sum_{i=1}^n E(k_i h_{i-1}) \quad \text{for LCRF} \quad (5')$$

$$E(C_n) \doteq n E(k) E(\Delta K^m) \quad \text{for KCRF}. \quad (6')$$

Since random variables k_i and C_{i-1} are mutually dependent, i. e., $E(k_i h_{i-1}) \neq E(k_i) E(h_{i-1})$, eq. (5') is more complicated than eq. (6'). The actual phenomenon, however, corresponds to eq. (5) rather than eq. (6).

The assumption that the history of the load does not affect the crack propagation factor is not realistic. Experimental studies show that the difference of the stress amplitudes, $\sigma_i - \sigma_j$ ($i, j=1, 2, \dots$),

consequently that of K -value, $K_i - K_j$, increases or decreases the growth rate⁵⁾. If so, eqs. (5') and (6') do not hold, for $E(k_i h_{i-1} \sigma_i^m) \neq E(\sigma^m) E(k_i h_{i-1})$ and $E(k_i \Delta K_i^m) \neq E(k) E(\Delta K_i^m)$. We have to return to eqs. (5) and (6). Eqs. (5), (5'), (6) and (6') are useful only if the information on the effects of the stress history on k_i 's is known. Such information may be available through a lot of tests under deterministic load sequences, though not practical.

If, however, an equivalent crack propagation factor, k_r , under random load is assumed, the crack length may be expressed as

$$C_n = \sum_{i=1}^n k_{ri} h_{i-1} \sigma_i^m \quad \text{for LCRF} \quad (7)$$

$$C_n = \sum_{i=1}^n k_{ri} \Delta K_i^m \quad \text{for KCRF}, \quad (8)$$

the expectations

$$E(C_n) = E(\sigma^m) \sum_{i=1}^n E(k_{ri} h_{i-1}) \quad (7')$$

$$E(C_n) = n E(k_r) E(\Delta K^m). \quad (8')$$

It is seen that the assumption of the equivalent crack propagation factor can be easily tried by KCF test but not by LCF test^{*2}. From these facts it is said that the data needed for the reliability analysis will be obtained much easily by carrying out the fatigue crack propagation tests under controlled K -value and therefore, the development of a handy KCF test method is of significance.

2. Calculation of Stress Intensity Factor

Analytical solutions of the stress intensity factors are expressed in terms of the applied stress, crack length and other constants,

$$K(\sigma, C) = \sigma g(C) \quad (9)$$

where $g(C)$ = a function of the crack length.

It is, however, not necessary to measure the length of the crack to calculate K -value under the given stress, if a certain quantity which is a function of the crack length and the applied stress is known. One of the most convenient and adequate quantities is the displacement measured at an appropriate point on the crack surface, or at any point. The outline of the procedure is as follows. The displacement along the crack surface is

$$v(x, 0) = \sigma f(x, C) \quad (10)$$

or

$$C = f^{-1}(x, v/\sigma) \quad (10')$$

where f = a function of crack length and position x ,

f^{-1} = inverse function of f with respect to C ,

and $o-xy$ coordinate system is defined in Fig. 1.

Substituting eq. (10') into (9)

$$K = \sigma g(f^{-1}(x, v/\sigma)). \quad (11)$$

If the analytical or numerical solutions for $f(\cdot)$ and $g(\cdot)$ are given, K -value can be calculated from the applied stress and the displacement.

Since on-line control of the testing machine to realize the assigned time history of K -value requires expensive electronic devices such as a digital computer, AD/DA converters, etc., the direct application of eq. (11) is not practical. One of the alternatives is to approximate eq. (11) by other simple function so that an analogue operational circuit can be used.

Suppose a crack in an infinite plate is subjected to the uniform stress normal to the crack surface

^{*2} Nearly constant growth rate was obtained under random K -value⁶⁾.

(Fig. 1). K -value and COD at the center of the crack are

$$K = \sigma \sqrt{\pi C} \quad (12)$$

$$2v_0 = 4(1-\nu^2)\sigma C/E \quad \text{for plane strain} \quad (13)$$

respectively, where $v_0 = v(0, 0)$, E = Young's modulus and ν = Poisson's ratio.

Substituting eq. (13) into eq. (12)

$$K^2 = 2v_0 \sigma \pi E / 4(1-\nu^2). \quad (14)$$

It is seen that K -value will be controlled if the product of σ and v_0 is controlled.

Analogous to eq. (14), K -value for a finite width plate is approximately

$$\begin{aligned} K^2 &= 2v_0 \sigma \pi E \tan(\pi C/2b) / 4(1-\nu^2) \log(\sec(\pi C/2b) + \tan(\pi C/2b)) \\ &= 2v_0 \sigma \pi E f_1(C/b) / 4(1-\nu^2) \end{aligned} \quad (15)$$

where f_1 = a function of C/b and b = plate width.

K -value, again, can be controlled by the product of σ and v_0 for a certain range of C/b , say $C/b \leq 0.4$, where f_1 is essentially unity. The values of f_1 are given in Table 1.

Table 1 Correction factor for C/b

C/b	0.001	0.010	0.030	0.050	0.100	0.200	0.333	0.400	0.500	0.667	0.800
$(f_1)^{1/2}$	1.000	1.000	1.001	1.001	1.000	1.009	1.025	1.038	1.065	1.153	1.292

Though there is no theoretical evidence, one of the approximation of K -value for most cases*3 is possibly

$$K = \text{const} \sqrt{\sigma v_0}. \quad (16)$$

The experimental examination of the above approximation, eq. (16), was done with the three point bending test where K -value is given by⁹⁾

$$K t b^2 / 6 M C^{1/2} = A_0 + A_1(C/b) + A_2(C/b)^2 + A_3(C/b)^3 + A_4(C/b)^4 \quad (17)$$

where t = plate thickness, M = moment = $P \cdot s/2$, s = span and A 's are constant. K -value computed from the measured displacement and stress differed from eq. (17) by less than 3% for $C/b = 0.4$.

3. Experiment

In this study the beforementioned three point bending test was adopted because of its stability and easiness to observe the fatigue crack. The shape and dimensions of the specimen are shown in Fig. 2. Measurement of COD is done with a LVDT attached to the specimen as shown in Fig. 2-b. The shape of the specimen was so decided that the displacement at the edge of

Fig. 1 Crack subjected to stress

Fig. 2 (a) Shape and dimensions of specimen
(b) C.O.D. measuring method

*3 K -value due to a wedge force can not be approximated in this fashion for K -value decreases with C while v_0 increases.

Table 3 Test results

Specimen No.	C_1^{*4} (mm)	C_2^{*4} (mm)	$\bar{\Delta K}$ (kg/mm ^{3/2})	$\delta_{\Delta K}$	$\delta_{\Delta K}/\bar{\Delta K}$	dC/dN (mm/cycle)	Range (mm/cycle)	
CK-10	12.00	30.51	48	0.91	0.019	2.90×10^{-5}	2.85×10^{-5}	2.99×10^{-5}
CK- 7	12.18	19.04	51	0.85	0.017	3.68×10^{-5}	3.63×10^{-5}	3.73×10^{-5}
CK-14	12.73	34.67	64	0.96	0.015	9.72×10^{-5}	9.63×10^{-5}	9.80×10^{-5}
CK-11	12.01	34.92	76	1.30	0.017	1.33×10^{-4}	1.31×10^{-4}	1.34×10^{-4}
CK- 8	11.89	38.84	97	1.26	0.013	2.83×10^{-4}	2.79×10^{-4}	2.86×10^{-4}
CK-13	12.00	25.89	120	1.58	0.013	3.49×10^{-4}	3.47×10^{-4}	3.50×10^{-4}
CK- 9	11.82	39.87	159	1.65	0.010	7.82×10^{-4}	7.79×10^{-4}	7.85×10^{-4}
CK-17	14.63	35.47	201	3.12	0.016	2.01×10^{-3}	1.97×10^{-3}	2.06×10^{-3}

*4; Crack growth rate was analyzed between C_1 and C_2 .

the crack length and number of cycles is linear except for the very beginning of the propagation of the fatigue cracks initiated from fine a saw-cut.

The variation of the K -values seen in Figs. 4 and 5 are somewhat exaggerated because the applied load was read with a high speed 2 digits AD converter—the accuracy is $\frac{1}{2}$ LSD. Apart from the accuracy of the AD converter, there is another possibility of error in K -value measurement. The fatigue

Fig. 4 C-N diagram

behavior is, strictly speaking, not elastic and therefore the relation between COD and the applied load is not necessarily linear. A certain amount of permanent opening must accumulate during propagation process, even if the applied load is very low and the plastic deformation is localized. If we assume that COD-load relation is linear during unloading, we have

$$v_0 = aP + v_r \quad \text{for unloading} \quad (18)$$

where a =proportional constant depending of the crack length,

P =load and v_r =residual COD.

Thus, instead of eq. (16)

$$K = \text{const} \sqrt{P(v_0 - v_r)} \quad (19)$$

should be used.

Fig. 5 C-N diagram

When the applied K -value was relatively high, say $K=160 \text{ kg/mm}^{3/2}$, zero adjustment of the displacement transducer was necessary for a few times to reduce the error due to v_r . The average K -values and coefficients of variation are listed in Table 3.

The average growth rates were computed from the C-N diagrams by the least square method and listed in Table 3 together with the ranges for 95% confidence level. The general trend of the crack growth observed on the both sides of the specimen are similar (see Figs. 4 and 5, the solid circles are data from the records of CD gages). Consequently the wavy aspect of the C-N diagrams can be interpreted as the phenomenon relevant to the gradual variation of the growth rate. This variation does not necessarily coincide with the apparent fluctuation of K -value.

The growth rate calculated for small increment of the fatigue crack length are plotted against crack length in Fig. 6. The instantaneous growth rate disperses considerably. Taking the accuracy of the measurement of the crack length into account (order of .01 mm), it can be said that the variation of the growth rate (both the wavy gradual change and the dispersion of short range growth rate) reflects the change of the local material properties. Note that the ranges listed in Table 3 should be much smaller than this variation since the values in Table 3 correspond to the range of estimates from a large number of samples.

The average growth rate is certainly dependent of K -value. The size of the plastically deformed zone along the crack path is uniform as long as K -value is kept constant (Photo 2). This fact may confirm that the growth rate is governed by the plastic deformation around the crack tip and K -value is a good measure of this plastic deformation.

Crack Growth Law

To check the linearity between the logarithm of K -value and that of the growth rate, the least

square method was applied though the size of the data is rather small.

The results are

$$E\left(\frac{dC}{dN}\right) = k(\Delta K)^m \quad (20)$$

$$= 0.73 \times 10^{-9} (\Delta K)^{2.8} \quad (20-1)$$

$$\text{or} \quad = 1.07 \times 10^{-9} (\Delta K)^{2.7}; \text{ excluding CK-17.} \quad (20-2)$$

The ranges of the estimated values, k and m , with 95% level are

$$3.39 \times 10^{-9} \sim 1.58 \times 10^{-10}, \quad 2.44 \sim 3.11$$

$$\text{and } 8.34 \times 10^{-9} \sim 1.41 \times 10^{-10}, \quad 2.22 \sim 3.15$$

for eq. (20-1) and eq. (20-2), respectively.

Eq. (20-1) is drawn in Fig. 7 with the experimental results. The broken lines show the range of $\pm 2S_E$, where S_E = standard error. From this figure it seems reasonable to express the crack growth law in the form of eq. (20). If, however, the data are replotted in arithmetic scale as Fig. 8, inadequacy of eq. (20) is clarified. The solid and broken lines correspond to those in Fig. 7.

Bearing in mind that the obtained growth rates are the mean values taken over a large number of samples, the range shown in Fig. 8 is unreasonably wide, in other words, it is meaningless. As long as one adheres to eq. (20) and logarithmic scale, this weakness of the unreasonably wide range of estimation is unavoidable, for it is somewhat inherent to the method. By the same token, the observed arbitrariness of the exponent, m , may be explained. In passing, it is strange that there is very few discussion about the properties of the exponent from the stand point of experimental study, while the theoretical studies gave some definite values to it. If the exponent

is one of the material constants, it is no wonder that it lies in a certain range. However, if it is a quantity that should be determined by the analysis indifferent to the materials and be a universal constant as treated in theoretical studies, the wide range of the observed values of the exponent is nothing but the experimental error. This point will be discussed later.

It is apparent that the so called 4-th power law does not apply to the present data, though " $m=4$ " is one of the possible values which coincides with Rice's analytical prediction.

A curve fitting, therefore, was tried for a quadratic equation,

$$E\left(\frac{dC}{dN}\right) = B_0 + B_1 \Delta K + B_2 \Delta K^2. \quad (21)$$

(a) CK-17

(b) CK-13

(c) CK-10

Fig. 6 Short range growth rate

Photo 2 Plastic deformation around fatigue crack

Fig. 7 Growth rate vs. K -value in logarithmic scale Fig. 8 Growth rate vs. K -value in arithmetic scale

The results is as follows.

$$B_0 = 2.34 \times 10^{-5}, \quad B_1 = -1.61 \times 10^{-6}$$

$$B_2 = 3.98 \times 10^{-8}, \quad S_E = \text{standard error} = 3.0 \times 10^{-5}$$

In the above curve fitting the data of CK-17 ($\Delta K = 200$) was omitted because if it is included the standard error becomes bigger than 10^{-4} . This fact may imply that the mechanism of the fatigue crack growth under high level of ΔK is different from that under low K -values, or that the size of the specimen was not large enough for high K -values because of widely spreaded yielding (see Photo 2). Eq. (21) is also drawn in Fig. 8 with the confidence limits of 95% level ($\pm 2S_E$). As far as the estimation of the expected growth rate for a given K -value is concerned, eq. (21) is much more useful than eq. (20).

The authors are not ready to state positively which of the two is true, that is, whether the exponent is a material constant or a universal. However, the authors would like to point out the possibility that the relation between the growth rate and K -value is expressed in the form of eq. (21).

If the size effect is assumed in the sense that the smaller the highly stressed zone, the less defects are contained in the zone, a critical K -value may exist. The weakest link within the distance r from the tip of the crack is separated by the first several cycles which weakens the remaining part and after a certain number of cycles a striation of size r is formed. In other words, K -value is partially

effective. Taking into consideration the fact that the striation space is proportional to the square of the K -value for wide range, the growth rate may be written as

$$\frac{dC}{dN} = k(\Delta K - \Delta K^*)^2 \quad (22)$$

where ΔK^* is the critical value (a random variable). Suppose two random variables, k and ΔK^* , are independent of each other and their probability density functions are f_k and f_* , respectively. The expectation of the growth rate is

$$\begin{aligned} E\left(\frac{dC}{dN}\right) &= \int_0^\infty dx \int_0^{\Delta K} dy (x - y)^2 f_k(x) f_*(y) \\ &= E(k)(\Delta K^2 F(\Delta K) - 2\Delta K M_1(\Delta K) + M_2(\Delta K)) \end{aligned} \quad (23)$$

where

$$F(y) = \int_0^y f_*(y) dy = \text{distribution of } \Delta K^* \quad (24)$$

and

$$M_i(y) = \int_0^y y^i f_*(y) dy. \quad (25)$$

If ΔK is large enough, say $\Delta K \geq E(\Delta K^*) + \sigma^*$, where $E(\Delta K^*)$ and σ^* are the expectation and the standard deviation of ΔK^* , respectively, eq. (23) becomes

$$E\left(\frac{dC}{dN}\right) \doteq E(k)((\Delta K - E(\Delta K^*))^2 + (\sigma^*)^2). \quad (26)$$

Comparing coefficients in eq. (21) and eq. (26), we have

$$\begin{aligned} E(k) &= B_2 = 3.98 \times 10^{-8} \\ E(\Delta K^*) &= -B_1/2B_2 = 20.2 \end{aligned} \quad (27)$$

and

$$\sigma^* = \sqrt{B_0/B_2 - (B_1/2B_2)^2} = 13.4.$$

These figures may be used for the estimation of the growth rate under very low K -value. For example, assuming extremal distribution for f_* , eqs. (24) and (25) can be evaluated and the expected growth rate will be obtained.

Eq. (21) does not hold for large K -values as seen from the experimental results. However, it is good enough for the design purpose. Note that $\Delta K = 200$ is not realistic for fatigue, considering the fact that such high K -values can be attained only under the extremal conditions, for example, a crack of length 100 mm subjected to the applied stress of 20 kg/mm², which may fall under the previously defined category of the event 3. Moreover it is not practical to discuss the growth rate in the order of μ/cycle when there already exists such a long crack.

Comparison with LCF Test

Load controlled fatigue tests were carried out on two specimens:

CP-15 for $\Delta K = 60 \sim 170$

CP-16 for $\Delta K = 110 \sim 210$

C - N curves are shown in Figs. 9 and 10. The circles are experimental data and the solid lines are power functions of N obtained by the least square method,

$$C = a_0 + a_1 N + a_2 N^2 + a_3 N^3 + a_4 N^4 \quad (28)$$

where

$$\left. \begin{aligned} a_0 &= 7.79 \\ a_1 &= -3.36 \times 10^{-5} \\ a_2 &= 3.02 \times 10^{-9} \\ a_3 &= -3.41 \times 10^{-14} \\ a_4 &= 1.77 \times 10^{-19} \\ S_E &= 0.23 \end{aligned} \right\} \text{(CP-15)} \quad \left. \begin{aligned} &8.64 \\ &5.09 \times 10^{-5} \\ &8.79 \times 10^{-8} \\ &-6.92 \times 10^{-12} \\ &2.07 \times 10^{-16} \\ &0.13 \text{ (mm)} \end{aligned} \right\} \text{(CP-16)}.$$

Fig. 9 N - C curve under constant load amplitude, CP-15

Fig. 10 N - C curve under constant load amplitude, CP-16

(a) CP-15

(b) CP-16

Fig. 11 Growth rate vs. K -value

Judging from these the above curve fitting seems successful. The crack growth rates are plotted in Fig. 11: a, b; $\circ$ are measured, $\bullet$ computed from the derivative of eq. (28).

The measured growth rate shows wide scatter. The general trend is similar to that shown in Fig. 6 as expected. The computed growth rate shows wavy tendency which is not seen in Fig. 6. This is due to the special form of eq. (28). There is, however, no reason to reject eq. (28) (S_E 's are small enough).

Since there is no way to treat these data to analyze the relation between the growth rate and K -value, except for using logarithmic scale to draw a straight line, eq. (20) was applied to obtain the following results.

CP-15	CP-16	CP-15 & 16
$k=2.9\sim 4.1\times 10^{-11}$	$2.1\sim 2.4\times 10^{-10}$	$1.8\sim 2.2\times 10^{-10}$
$m=2.8$	$2.7\sim 3.3$	$2.8\sim 3.3$
number of samples=27	24	51

The growth rates calculated with these combinations of k and m may not show much difference, at most 50% upto $\Delta K=150$, but the crack length estimated with $m=3$ and $k=3\times 10^{-11}$ is quite different from that with $m=3$ and $k=2\times 10^{-10}$. At least the data of CP-15 and 16 must be handled as a group of samples taken from the same source, so $m=3$ and $k=2.0\times 10^{-10}$ will be taken.

Now, eq. (20) as well as eq. (21) is supposed to give the expected crack growth rate as a matter of course. To see if this is true, the estimated and the measured growth rates are plotted in Fig. 12 (○; eq. (20-1), ●; eq. (21)). The shadowed area shows the bounds of the estimation with 95% confidence. These are not the limits for a random variable, dC/dN , to fall in between with the probability .95, but, roughly speaking, for the average value of a number of samples^{*5}. Both the central values and the bounds obtained from LCF test show not only considerable deviation from the measured but also a definite trend. This is due to, as mentioned before, the inaccuracy of determination of the growth rate and the use of logarithmic scale which is inevitable in LCF test. It may be said that LCF test results are valid for rather small K -value, though intension is usually for higher K -value. In order to get results comparable to KCF test, more specimens and more sophisticated technique to handle data will be necessary. Experiment will become tedious unless automatic measurement of crack length is accomplished. Note that very few measurements of the crack length are necessary to determine the average growth rate with KCF test.

Fig. 12 Comparison of LCF and KCF test results

Conclusion

The proposed method was successfully applied to the fatigue crack propagation test under controlled stress intensity factor and proved to be a good method to obtain the information on the fatigue crack propagation. The results obtained by this study are summarized as follows.

- 1) Average crack growth rate is constant if maximum and minimum values of the stress intensity factor are kept constant.
- 2) Obvious fluctuation of the growth rate was observed.

^{*5} This band width can be reduced by taking more data into analysis. Since the data used for the least square method were taken from eq. (28) at appropriate N , the total number of the points can be increased indefinitely. Considering that dC/dN is a continuous function of N , the so called standard error will remain within a certain limit, but the band width shown in Fig. 12 will be reduced to zero. This means that we have to determine a proper sample size to get a proper band width and yet there is no way to demonstrate it.

- 3) Even the average growth rate taken over an incremental crack length of a few mm reflects the local properties of the material.
- 4) The logarithmic growth rate is seemingly a non-linear function of the logarithm of the stress intensity factor.
- 5) Giving certain values to the exponent, m , and proportional constant, k , is merely expediential and may lead to erroneous estimation of the fatigue crack growth rate.

References

- 1) Y. Minami and H. Itagaki: An Investigation on the Low-Cycle Fatigue Fracture (2nd report), J. Soc. Naval Architects of Japan, Vol. 116, (1964).
- 2) J. R. Rice: Mechanics of Crack Tip Deformation and Extension by Fatigue, ASTM STP15, (1967).
- 3) P. C. Paris and F. Erdogan: A Critical Analysis of Crack Propagation Laws, Trans. ASME, (Dec., 1963).
- 4) K. Nagai: Propagation Characteristics of Fatigue Crack, Bulletin J. Soc. Naval Arch. Japan, No. 479, (1969).
- 5) G. A. Miller: The Dependence of Fatigue Crack Growth Rate on the Stress Intensity Factor and the Mechanical Properties of Some High Tensile Strength Steels, Trans. ASM. Vol. 61, (1968).
- 6) S. R. Swanson, F. Cicci and W. Hoppe: Crack Propagation in Clad 7079-T6 Aluminum Alloy Sheet Under Constant and Random Amplitude Fatigue Loading, ASTM STP 415, (1967).
- 7) B. Mukherjee and D. J. Burns: Fatigue-crack Growth in Polymethylmethacrylate, Experimental Mech. (Oct., 1971).
- 8) H. Itagaki and M. Shinozuka: Application of the Monte Carlo Technique to Fatigue-Failure Analysis under Random Loading, ASTM STP 511, (1972).
- 9) "Fracture" IV, Academic Press, (1969), pp. 63-64.