On the Propagation of Surface Waves across a Uniform Wake

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Summary

To get better understanding of the interference between surface waves and a frictional wake of a ship, a simple mathematical model is introduced. The wake is assumed as a current of uniform width and uniform velocity in the surrounding still water. Two infinite vortex sheets are employed in constructing the uniform current. Numerical studies show that the incident waves are considerably attenuated through the process of reflection and transmission across the uniform current. Experimental studies on a ship model of roughened surface show an encouraging result to verify the present theoretical approach.

1. Introduction

The linearization of the equation of motion together with the boundary condition has been usually employed so far to analyze the ship waves. The waves in the linearized field follow a law of propagation in a uniform stream. In the steady motion, the phase velocity of the plane wave in the direction of an angle α relative to the uniform flow is $U \cos \alpha$, so that the wave length is $\lambda = 2\pi U^2 \cos^2 \alpha / g$, where g is the acceleration of gravity. However, observations of waves behind a ship show a discrepancy between the measured wave length and its prediction by the theory. According to the measurements by Inui et al.¹⁾ concerning the wave pattern behind a ship model, the measured wave length is shorter than the theoretical one. Gadd²⁾ observed that waves generated by ship models are subject to a remarkable distortion when they trespass into a wake belt behind a thin plate. These phenomena are attributed to the change of the base flow velocity due to the viscous wake. Though the structure of the wake is a complicated turbulent flow field, the average characteristic can be regarded as a sort of shear flow. The ship waves propagate on such a non-uniform flow. The law of propagation is different from that in a uniform flow field. The refraction of surface waves by non-uniform currents has been discussed by several researchers, such as Johnson³⁾, Longuet-Higgins and Stewart.⁴⁾ The analytical treatment by the hydrodynamic theory is rather difficult and approximate methods are

unavoidable. The deformation of incident waves when penetrating into non-uniform wake was discussed by Savitsky⁵⁾, who utilized an analogy with the kinematical theory of refraction of light in a heterogeneous medium.

Though the structure of the wake is very complex, a simple model may suffice to a qualitative description of the phenomena. The simple model which is employed in order to enable a hydrodynamic analysis is such as a current of uniform width and uniform velocity in the surrounding still water upon which a train of waves propagates. The analysis is based on the assumption that the fluid is inviscid and the motion is irrotational except on the boundary of the wake or current where simple vortex sheets are assumed. This problem was first discussed by Maruo and Hayasaki⁶⁾ but an error was found in a part of boundary conditions to be satisfied at the wake frontier. A revised calculation was carried out by Evans⁷⁾ for plane waves across a single vortex sheet. In formulating the problem Evans used a method which involves the use of complete eigenfunction expansions. The method is equivalent to the Green's function approach of Maruo and Hayasaki, who attempted a full numerical solution to the resulting integral equations which determine the strength of the vortex sheets.

In the present paper the error of the equations of the boundary condition at the wake frontier is corrected and the integral equations are solved numerically as in the previous paper by Maruo and Hayasaki. Calculations are carried out first for a simple plane wave across a uniform wake belt associated with two vortex sheets. Then the deformation of Kelvin waves by a uniform wake is calculated based on the concept of elementary waves.

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Finally, wave pattern analyses of ship models both of smooth and roughened surfaces are carried out in order to examine whether the present theoretical model describes even qualitatively the effect of viscous wake on the ship waves.

2. A Wake Model and Boundary Value Problems

As shown in Fig. 1 the flow field is divided into three parts I, II and III by an infinite strip of uniform current of width h . In order to deal with a steady motion, a uniform flow in the x direction with velocity U_1 is assumed in the domains I and III. In the domain II, i.e. inside the strip, there is a uniform current of velocity U_2 ($< U_1$). The boundary of the strip is expressed by parallel lines $y=0$ and $y=h$, at which the tangential velocity is discontinuous.

Now an incident wave is assumed in the domain I or III, that propagates in the negative x -direction with velocity U_1 . Then it is expressed by a time independent velocity potential ϕ_0 . The discontinuous surface at $y=0$ will originate another disturbance potential ϕ_1 in the domain I and a disturbance potential ϕ_2 in the domain II. The former may be called the reflected wave and the latter the transmitted wave. If the incident wave is regarded as a wave element which composes the resultant ship's wave, the fluid motion in the domain III is symmetric with that in I in reference to the vertical plane $y=h/2$, and the function ϕ_2 is symmetric with respect to the same plane. No singularities are assumed in the lower half space except on the boundary surfaces, so that ϕ_0 and ϕ_1 are harmonic in I, i.e.

$$\nabla^2 \phi_0 = 0, \quad \nabla^2 \phi_1 = 0 \quad (1)$$

and ϕ_2 is harmonic in II

$$\nabla^2 \phi_2 = 0 \quad (2)$$

The boundary conditions satisfied by these potentials are as follows. The vortex sheet is a free boundary and the fluid velocity is tangential to it. If we write the equation of the vortex sheet as $y=\eta$, the linearized kinematical conditions are

$$U_1 \frac{\partial \eta}{\partial x} - \frac{\partial}{\partial y}(\phi_0 + \phi_1) = 0, \quad U_2 \frac{\partial \eta}{\partial x} - \frac{\partial \phi_2}{\partial y} = 0 \quad (3)$$

on the plane $y=0$ or $y=h$. Eliminating η between these equations, we have

$$U_2 \frac{\partial}{\partial y}(\phi_0 + \phi_1) = U_1 \frac{\partial \phi_2}{\partial y}. \quad (4)$$

In the previous paper by Maruo and Hayasaki⁽⁶⁾, the normal components of the velocities at $y=0$ and $y=h$ are assumed continuous. Then the following boundary condition was derived:

$$\frac{\partial}{\partial y}(\phi_0 + \phi_1) = \frac{\partial \phi_2}{\partial y} \quad (5)$$

However, it is rational that the boundary surfaces are regarded as free surfaces which are deformed by the incident waves. Thus the equation (4) is used in the present paper. Since the boundary surface is a free surface, the pressure is identical on both sides. Since the disturbance velocities due to ϕ_1 and ϕ_2 are small in comparison with U_1 and U_2 , there is a relation for the linearized pressure as follows

$$U_1 \frac{\partial}{\partial x}(\phi_0 + \phi_1) = U_2 \frac{\partial \phi_2}{\partial x} \quad (6)$$

on the same plane. Integrating with respect to x , one obtains

$$U_1(\phi_0 + \phi_1) = U_2 \phi_2 \quad \text{at } y=0 \text{ and } y=h. \quad (7)$$

Since the motion is steady, the free surface condition in I is

$$\frac{\partial^2}{\partial x^2}(\phi_0 + \phi_1) + K_1 \frac{\partial}{\partial z}(\phi_0 + \phi_1) = 0 \quad (8)$$

where $K_1 = g/U_1^2$. If ϕ_0 is assumed to satisfy the free surface condition, one can write

$$\frac{\partial^2 \phi_1}{\partial x^2} + K_1 \frac{\partial \phi_1}{\partial z} = 0 \quad \text{at } z=0 \quad (9)$$

Inside the wake, on the other hand, the corresponding condition in II becomes

$$\frac{\partial^2 \phi_2}{\partial x^2} + K_2 \frac{\partial \phi_2}{\partial z} = 0 \quad (10)$$

where $K_2 = g/U_2^2$. In the domain III, there is a fluid motion which is symmetric with $\phi_0 + \phi_1$ in reference to the vertical plane $y=h/2$ because of the aforementioned assumption. The unknown function ϕ_1 and ϕ_2 are expressed by means of appropriate Green functions relating to the boundary condition

$$-\frac{\partial^2 G^{(m)}}{\partial x^2} + K_m \frac{\partial G^{(m)}}{\partial z} = 0 \quad \text{at } z=0 \quad (m=1, 2) \quad (11)$$

These Green functions must satisfy the radiation condition

$$\lim_{x \rightarrow -\infty} \sqrt{|x|} G^{(m)} = 0 \quad (12)$$

Employing the wave-making double sheet potential, Maruo and Hayasaki⁽⁶⁾ derived the following formulas for ϕ_1 and ϕ_2 :

$$\phi_1 = -\frac{1}{2\pi} \iint \Phi_1(x', z') G_y^{(1)}(x, y, z; x', 0, z') dx' dz', \quad (13)$$

$$\begin{aligned} \phi_2 = & \frac{1}{2\pi} \iint \mu(x', z') G_y^{(2)}(x, y, z; x', 0, z') dx' dz' \\ & - \frac{1}{2\pi} \iint \mu(x', z') G_y^{(2)}(x, y, z; x', h, z') dx' dz' \end{aligned} \quad (14)$$

where $G^{(m)}(x, y, z; x', y', z')$ is the wave making source, and $\Phi_1(x, z)$ is the value of ϕ_1 on the plane $y=0$. The integral equation to determine the unknown function $\mu(x, z)$ is derived from (4) and (7) as follows

$$\begin{aligned}
v_0 + \frac{1}{2\pi} \iint \Phi_0(x', z') G_{yy}^{(1)}(x, 0, z; x', 0, z') dx' dz' \\
= \frac{1}{2\pi} \iint \mu(x', z') \left\{ \frac{U_2}{U_1} G_{yy}^{(1)}(x, 0, z; x', 0, z') \right. \\
+ \frac{U_1}{U_2} G_{yy}^{(2)}(x, 0, z; x', 0, z') \\
- \frac{U_1}{U_2} G_{yy}^{(2)}(x, 0, z; x', h, z') \left. \right\} dx' dz' \\
- \frac{1}{4\pi^2} \frac{U_2}{U_1} \iint \mu(x', z') dx' dz' \\
\times \iint G_{yy}^{(2)}(x'', 0, z''; x', h, z') \\
\times G_{yy}^{(1)}(x, 0, z; x'', 0, z'') dx'' dz'' \quad (15)
\end{aligned}$$

where Φ_0 and v_0 are the known values of ϕ_0 and $\partial\phi_0/\partial y$ on the plane $y=0$ respectively. If ϕ_0 gives a regular wave in the domain I, the function v_0 and Φ_0 are periodic functions of x such as

$$v_0 = a(z) \cos Kx + b(z) \sin Kx \quad (16)$$

$$\Phi_0 = A(z) \cos Kx + B(z) \sin Kx \quad (17)$$

Then the unknown function $\mu(x, z)$ is also a periodic function in this case, so that

$$\mu(x, z) = p(z) \cos Kx + q(z) \sin Kx \quad (18)$$

Therefore the integral equation (15) is rewritten in a form of its Fourier transforms:

$$\begin{aligned}
a(z) + \int_{-\infty}^0 \{A(z') C_{yy}^{(1)}(0, z; 0, z') \\
- B(z') S_{yy}^{(1)}(0, z; 0, z')\} dz' \\
= \int_{-\infty}^0 p(z') M(z, z') dz' \\
- \int_{-\infty}^0 q(z') N(z, z') dz' \quad (19)
\end{aligned}$$

$$\begin{aligned}
b(z) + \int_{-\infty}^0 \{B(z') C_{yy}^{(1)}(0, z; 0, z') \\
+ A(z') S_{yy}^{(1)}(0, z; 0, z')\} dz' \\
= \int_{-\infty}^0 q(z') M(z, z') dz' \\
+ \int_{-\infty}^0 p(z') N(z, z') dz' \quad (20)
\end{aligned}$$

where $C^{(m)}$, $S^{(m)}$ are the Fourier transforms of the Green function $G(x, y, z; x' y', z')$ as follows

$$\begin{aligned}
C^{(m)}(y, z; y', z') &= \frac{1}{\pi} [K_0 \{K \sqrt{(y-y')^2 + (z-z')^2} \\
&+ K_0 \{K \sqrt{(y-y')^2 + (z+z')^2}\}] \\
&+ \frac{K}{\pi} \text{P.V.} \int_{-\infty}^{\infty} e^{K(z+z') \cosh u} \cos \{K(y-y') \sinh u\} \\
&\times \frac{du}{K_m \cosh u - K} \quad (21) \\
S^{(m)}(y, z; y', z') &= \frac{2e^{K(z+z')K^2/K_m}}{\sqrt{1-(K_m/K)^2}} \\
&\times \cos \{K \sqrt{(K/K_m)^2 - 1} (y-y')\} \quad (K > K_m) \\
&= 0 \quad (K < K_m) \quad (22)
\end{aligned}$$

$M(z, z')$ and $N(z, z')$ are the kernel functions of the integral equations:

$$\begin{aligned}
M(z, z') &= \frac{U_2}{U_1} C_{yy}^{(1)}(0, z; 0, z') + \frac{U_1}{U_2} C_{yy}^{(2)}(0, z; 0, z') \\
&- \frac{U_1}{U_2} C_{yy}^{(2)}(0, z; h, z') \\
&- \frac{U_2}{U_1} \int_{-\infty}^0 \{C_{yy}^{(2)}(0, z''; h, z') C_{yy}^{(1)}(0, z; 0, z'') \\
&- S_{yy}^{(2)}(0, z''; h, z') S_{yy}^{(1)}(0, z; 0, z'')\} dz'' \quad (23)
\end{aligned}$$

$$\begin{aligned}
N(z, z') &= \frac{U_2}{U_1} S_{yy}^{(1)}(0, z; 0, z') + \frac{U_1}{U_2} S_{yy}^{(2)}(0, z; 0, z') \\
&- \frac{U_1}{U_2} S_{yy}^{(2)}(0, z; h, z') \\
&- \frac{U_2}{U_1} \int_{-\infty}^0 \{C_{yy}^{(2)}(0, z''; h, z') S_{yy}^{(1)}(0, z; 0, z'') \\
&+ S_{yy}^{(2)}(0, z''; h, z') C_{yy}^{(1)}(0, z; 0, z'')\} dz'' \quad (24)
\end{aligned}$$

If ϕ_0 gives a regular waves in the domain II, i.e. in the wake belt, the boundary conditions on the planes $y=0$ and $y=h$ are as follows:

$$U_1 \frac{\partial}{\partial y} (\phi_0 + \phi_2) = U_2 \frac{\partial \phi_1}{\partial y} \quad (25)$$

$$U_2 \frac{\partial}{\partial x} (\phi_0 + \phi_2) = U_1 \frac{\partial \phi_1}{\partial x} \quad (26)$$

The integral equations for $p(z)$ and $q(z)$ are derived in the similar manner as for the regular wave in the domain I:

$$\begin{aligned}
a(z) + \left(\frac{U_2}{U_1}\right)^2 \int_{-\infty}^0 \{A(z') C_{yy}^{(1)}(0, z; 0, z') \\
- B(z') S_{yy}^{(1)}(0, z; 0, z')\} dz' \\
= \int_{-\infty}^0 p(z') M(z, z') dz' - \int_{-\infty}^0 q(z') N(z, z') dz' \quad (27)
\end{aligned}$$

$$\begin{aligned}
b(z) + \left(\frac{U_2}{U_1}\right)^2 \int_{-\infty}^0 \{B(z') C_{yy}^{(1)}(0, z; 0, z') \\
+ A(z') S_{yy}^{(1)}(0, z; 0, z')\} dz' \\
= \int_{-\infty}^0 q(z') M(z, z') dz' + \int_{-\infty}^0 p(z') N(z, z') dz' \quad (28)
\end{aligned}$$

where

$$\begin{aligned}
M(z, z') &= -\left(\frac{U_2}{U_1}\right)^2 C_{yy}^{(1)}(0, z; 0, z') - C_{yy}^{(2)}(0, z; 0, z') \\
&+ C_{yy}^{(2)}(0, z; h, z') \\
&+ \left(\frac{U_2}{U_1}\right)^2 \int_{-\infty}^0 \{C_{yy}^{(2)}(0, z''; h, z') \\
&\times C_{yy}^{(1)}(0, z; 0, z'') - S_{yy}^{(2)}(0, z''; h, z') \\
&\times S_{yy}^{(1)}(0, z; 0, z'')\} dz'' \quad (29) \\
N(z, z') &= -\left(\frac{U_2}{U_1}\right)^2 S_{yy}^{(1)}(0, z; 0, z') - S_{yy}^{(2)}(0, z; 0, z') \\
&+ S_{yy}^{(2)}(0, z; h, z')
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{U_2}{U_1} \right)^2 \int_{-\infty}^0 \{ C_{y''}^{(2)}(0, z''; h, z') \\
& \times S_{y''}^{(1)}(0, z; 0, z'') + S_{y''}^{(2)}(0, z''; h, z') \\
& \times C_{y''}^{(1)}(0, z; 0, z'') \} dz'' \quad (30)
\end{aligned}$$

3. Distortion of a Plane Wave across the Uniform Wake

Let us assume a plane wave as the initial disturbance ϕ_0 in the half space $y \leq h/2$ of the domain I or II.

$$\begin{aligned}
\phi_0 = C_0 e^{K_m z \sec^2 \alpha_m} \cos \{ K_m \sec^2 \alpha_m (x \cos \alpha_m \\
+ y \sin \alpha_m) \} \quad (31)
\end{aligned}$$

where $m=1$ for ϕ_0 in domain I and $m=2$ for ϕ_0 in domain II. The direction of the wave propagation makes an angle $\pi + \alpha_m$ with respect to the x axis. In the other half space $y \geq h/2$ the direction of the wave propagation makes an angle $\pi - \alpha_m$ with respect to the x axis because of the symmetry condition. The integral equations are solved numerically by a method explained in the Appendix. The reflected wave ϕ_1 and the transmitted wave ϕ_2 are determined as other plane waves accompanied by some local terms which are appreciable only near the discontinuous surfaces. The elevation of the water surface is given as follows.

When ϕ_0 is assumed in the domain I,

$$\zeta = -\frac{U_1}{g} \left(\frac{\partial \phi_0}{\partial x} + \frac{\partial \phi_1}{\partial x} \right)_{z=0} = \zeta_0 + \zeta_1 \quad \text{in I} \quad (32)$$

$$\zeta = -\frac{U_2}{g} \left(\frac{\partial \phi_2}{\partial x} \right)_{z=0} = \zeta_2 \quad \text{in II} \quad (33)$$

When ϕ_0 is assumed in the domain II,

$$\zeta = -\frac{U_1}{g} \left(\frac{\partial \phi_1}{\partial x} \right)_{z=0} = \zeta_1 \quad \text{in I} \quad (34)$$

$$\zeta = -\frac{U_2}{g} \left(\frac{\partial \phi_0}{\partial x} + \frac{\partial \phi_2}{\partial x} \right)_{z=0} = \zeta_0 + \zeta_2 \quad \text{in II} \quad (35)$$

For the convenience sake, the above mentioned wave elevations which include the local disturbances are rewritten in a following form which is normalized by the amplitude of the incident wave ζ_0 :

$$\bar{\zeta} = \bar{Z}(y) \sin \{ Kx + \nu(y) \} \quad (36)$$

where $\bar{Z}(y)$ and $\nu(y)$ are functions of y . Further, let us call $\bar{Z}$ the resultant wave amplitude and ν the phase lag. Fig. 2 shows a comparison of the resultant wave amplitudes defined above for two cases: one is the resultant wave amplitude obtained by the use of the correct boundary condition (4) and the other is the resultant wave amplitude based on the equation (5) which was used by Maruo and Hayasaki in the previous paper⁶⁾. In this computation the wake breadth is assumed to be infinity and ϕ_0 is in the domain I. From Fig. 2 it turns out that the difference of the boundary conditions (4) and (5) produces only a small effect on the resultant wave amplitude both in the domain I and II for various

phase angles of the incident wave. Further, the differences of phase lags $\nu(y)$ between the boundary conditions (4) and (5) are negligibly small. The resultant wave elevation and the phase lag based on the correct boundary condition are computed for a finite wake breadth ($K_1 h = 2.0$, $U_2/U_1 = 0.9$) as shown in Fig. 3 and Fig. 4 where the incident wave ϕ_0 is in the domain I and II respectively. From these figures it is found that the attenuation of resultant wave amplitudes in the domain I is remarkable. In the domain II there are interferences between the waves originated from the vortex sheets at $y=0$ and $y=h$. For a certain angle of the incident wave, the resultant wave amplitude exceeds the amplitude of the incident wave.

Next, in order to know the effect of wake breadth on the wave deformation the asymptotic wave systems far from the vortex sheets are calculated where the local wave terms are disappeared. Let us suppose an incident plane wave system:

$$\begin{aligned}
\zeta_0 = C_0 \sin \{ K_m \sec^2 \alpha_m (x \cos \alpha_m + y \sin \alpha_m) \}; \\
(m=1, 2) \quad (37)
\end{aligned}$$

The asymptotic plane wave system due to the induced potentials ϕ_1 and ϕ_2 are written as follows.

$$\phi_1: \zeta_1 = C_1 \sin \{ K_1 \sec^2 \alpha_1 (x \cos \alpha_1 + y \sin \alpha_1) + \gamma_1 \} \quad (38)$$

$$\begin{aligned}
\phi_2: \zeta_2 = \zeta_2^0 + \zeta_2^h \\
= C_2^0 \sin \{ K_2 \sec^2 \alpha_2 (x \cos \alpha_2 - y \sin \alpha_2) + \gamma_2^0 \} \\
+ C_2^h \sin \{ K_2 \sec^2 \alpha_2 (x \cos \alpha_2 + y \sin \alpha_2) + \gamma_2^h \} \quad (39)
\end{aligned}$$

where γ_1 , γ_2^0 and γ_2^h are the phase differences from the incident wave. For instance, in the domain I, the resultant wave system is written in the following form

$$\begin{aligned}
\zeta = \zeta_0 + \zeta_1 \\
= C \sin \{ K_1 \sec^2 \alpha_1 (x \cos \alpha_1 + y \sin \alpha_1) + \gamma \} \quad (40)
\end{aligned}$$

where C is the resultant wave amplitude and γ the phase difference from the incident wave. Fig. 5 and Fig. 6 show the values of C/C_0 and γ for the variation of wake breadths versus incident wave angles when $U_2/U_1 = 0.9$. A remarkable change is observed between wave systems below and above the critical angle α_{cr} which is defined by $\alpha_{cr} = \cos^{-1}(K_1/K_2) = \cos^{-1}(U_2^2/U_1^2)$. When ϕ_0 is in the domain I (Fig. 5), the incident wave is completely reflected on the vortex sheet at $y=0$ in the case $\alpha_1 < \alpha_{cr}$, i.e. there is no transmitted wave. Therefore, there is very little contribution from the vortex sheet at $y=h$. In other words, the difference of wake breadth has very small effect upon the reflected wave amplitudes. Further, the phase differences γ from the incident wave are negligibly small when $\alpha_1 < \alpha_{cr}$ as shown in Fig. 5. When ϕ_0 is in the domain II, on the other hand, the transmitted waves in the domain I appear only when $\alpha_1 > \alpha_{cr}$ as shown Fig. 6. From

Fig. 1 Coordinate system

Fig. 3 $\bar{Z}$ and ν for incident wave in I Fig. 5 C and γ in I for incident wave in I Fig. 2 $\bar{Z}$ and ν for correct boundary condition (4) Fig. 4 $\bar{Z}$ and ν for incident wave in II Fig. 6 C and γ in I for incident wave in II

Fig. 7 Comparison of wave profiles with and without wake for incident wave in the domain I

Fig. 8 Comparison of wave profiles of Wigley's model ($L=5m$, $F_n=0.29$)

the figure it is also indicated that the differences of wake breadths have no influence on the maximum and minimum values of C/C_0 and γ .

4. Application

The plane waves treated in the previous section can be regarded as wave elements which construct a wave system such as the Kelvin wave. By using the method developed in the previous section, the effect of wake belt on the Kelvin wave system is studied. Let us suppose first an asymptotic incident wave system in the domain I and far from the wake belt:

$$\zeta_0 = \int_0^{\pi/2} A_0(\alpha_1) \cos\{K_1 \sec^2 \alpha_1 (x \cos \alpha_1 + y - h/2 \sin \alpha_1)\} d\alpha_1, \quad (41)$$

where $A_0(\alpha_1)$ is the amplitude function. In the present study $A_0(\alpha_1) = \sec \alpha_1$ is chosen. The resultant wave amplitudes $A(\alpha_1)$ and the phase differences $\gamma(\alpha_1)$ are calculated against a number of α_1 -values by the use of Fig. 5. Then the following asymptotic wave system is constructed in the region where the local disturbances by the vortex sheet are negligibly small:

$$\zeta = \int_0^{\pi/2} A(\alpha_1) \cos\{K_1 \sec^2 \alpha_1 (x \cos \alpha_1 + y - h/2 \sin \alpha_1) + \gamma(\alpha_1)\} d\alpha_1 \quad (42)$$

Fig. 7 shows a comparison of wave profiles calculated by the use of formulas (41) and (42). As suspected from Fig. 5 a remarkable attenuation of the wave heights with very little phase difference is observed.

In order to know whether the present theoretical predictions are even qualitatively in agreement with the phenomenon observed in the ship's wave system,

measurements of the wave heights of Wigley's parabolic models with smooth and roughened surface were conducted in the Nagasaki Experimental Tank of Mitsubishi Heavy Industries, Ltd. By roughening the surface of a ship model it is expected that the viscous wake is strengthened and its effect then appears as the change of the wave system. The hull surface of the Wigley's parabolic model of 5m

Fig. 9 Comparison of wave spectra of Wigley's parabolic model

Fig. 10 Comparison of resistance tests of Wigley's model

long was roughened artificially by scraping the paraffine surface with a 0.8mm deep and 1.25mm pitch saw-toothed steel plate along the frame lines. Fig. 8 shows a comparison of the longitudinal wave profiles measured at 40% L distance from the model center line. As the result of roughening the surface an attenuation of the wave height is observed as predicted by the present theory. However, the magnitude of the attenuation is not so large as the theoretical calculation shown in Fig. 7. A comparison of wave spectra obtained by the wave pattern analyses at Froude number $U/\sqrt{gL} = 0.29$ is shown in Fig. 9 together with the wave spectrum calculated by Michell's theory. An attenuation of the wave spectrum by the roughening the surface is clearly noticed. Fig. 10 shows the resistance coefficients of wave pattern analyses of both roughened and smooth surface models together with the results of towing tests and the theoretical calculation by Michell's theory. It is noticed that there is a noticeable attenuation of wave pattern resistance by the roughening of the model surface. Although these experimental studies are very indirect ones to examine the present theory, the attenuation of the wave system by strengthening the viscous wake may indicate the validity of the present theoretical approach to the problem of wave and wake interaction.

5. Concluding Remarks

In the present studies the interaction between the regular plane waves and two vortex sheets which are distributed in the infinite range ($-\infty < x < \infty$) is considered. Therefore, this mathematical model of a wake may be applied to the flow region far

behind a ship. In order to calculate theoretically the wave making resistance of a ship with such mathematical wake model, semi-infinite vortex sheets which start from the after part of the ship should be considered as studied by Beck⁹⁾. In this case the vortex sheets themselves induce wave systems from the starting points. Therefore, there are very complex interactions among wave systems, such as ship waves, waves originated from the vortex sheets and wave systems induced by the reflection and the transmission of those waves across the vortex sheets. A more sophisticated wave and wake model should be developed in the future.

The studies described in the present paper may be useful for a study of such complicated interaction problems.

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Appendix Notes on the Numerical Calculation

The Cauchy principal value integral which is contained in the function $C^{(m)}(y, z; y', z')$, the second term of (21), was evaluated numerically based on the method proposed by Monacella¹⁰⁾, where the range of integration is subdivided first and the singularity is then chosen to be both at the origin and at the midpoint of one of the subdivided ranges of the integration. By doing so the singularity is ignored.

In solving the integral equations (19) and (20), or (27) and (28), the integral range with respect to z' is chosen to be equal to the one wave length of the incident wave, i.e. $0 \geq z' \geq -\lambda$, where $\lambda = 2\pi U_1^2/g$ when ϕ_0 is in the domain I, and $\lambda = 2\pi U_2^2/g$ when ϕ_0 is in the domain II. The integral range is then divided into 25 segments of equal interval $\varepsilon (\varepsilon = \lambda/25)$. The values $p(z')$ and $q(z')$ for each abscissa are assumed unknown. Since the kernel functions contains the singularity of a type $1/(z-z')^2$ when $z=z' \neq 0$, the integral segment which includes this singularity is specially evaluated. Choosing a small positive value $\delta (\delta \ll \varepsilon)$, the integral segment is evaluated by taking the so called Hadamard's finite part of a divergent integral¹¹⁾:

$$I_1(z) = \int_a^b \frac{p(z')}{(z-z')^2} dz'$$

$$= \lim_{\delta \rightarrow 0} \left\{ \int_a^{z-\delta} \frac{p(z')}{(z-z')^2} dz' + \int_{z+\delta}^b \frac{p(z')}{(z-z')^2} dz' - p(z) \frac{2}{\delta} \right\}$$

where $b-a=\varepsilon$. Fig. 11 and Fig. 12 show the solutions $p(z)$ and $q(z)$ for various δ values. It was found that the solutions become stable when $\delta < 0.01$ where the coordinate system is normalized by the

Fig. 11 Vortex strength $p(z)$ for various δ

Fig. 12 Vortex strength $q(z)$ for various δ

wave length λ . In the present study $\delta=0.01$ is chosen. Next, when $z=z'=0$, i.e. on the free surface, the kernel function has an additional infinite integral:

$$\text{P.V.} \int_0^\infty f(z, z', u) du$$

$$= \text{P.V.} \int_0^\infty \frac{e^{K(z+z') \cosh u} \sinh^2 u}{K_m \cosh u - K} du,$$

which comes from the second term of (21). In this case the finite part is also taken in the similar manner to Hadamard's method:

$$I_2(z)|_{z=0} = \int_{-\lambda}^0 p(z') \left\{ \text{P.V.} \int_0^\infty f(z, z', u) du \right\}_{z=0} dz'$$

$$= \lim_{\delta \rightarrow 0} \left[\int_{-\lambda}^{-\delta} p(z') \left\{ \text{P.V.} \int_0^\infty f(0, z', u) du \right\} dz' - p(0) \text{P.V.} \int_0^\infty \frac{f(0, -\delta, u)}{K \cosh u} du \right].$$