Theory for Articulated Double-Flap Wavemakers in Water of Constant Depth

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Abstract

Extending the theoretical work of Biéssel, a preliminary study is made of dynamic characteristics of an articulated double-flap wavemaker. The underlying motivation of utilizing a double-flap waveboard is based on its effectiveness in reducing the local disturbances in the immediate vicinity of the waveboard, which is demonstrated through decreasing inertia pressures. The additional degree of freedom in the double-flap waveboard movements is shown to be capable of encompassing a wider range of wavelength that can be satisfactorily generated in a given tank. The inertia pressure is reduced significantly by use of double-flap wavemakers, especially when short waves are generated. The usual wavemaker performance curves are computed for double-flap wavemakers. Comparison with the results of a single-flap wavemaker characteristics reveals that considerable improvement in performance curves is feasible by selecting a suitable combination of design parameters.

1. Introduction

The classical wavemaker theory of Havelock was further refined by Biéssel¹⁾, chiefly in reference to piston-type and flap-type wavemakers in laboratory use. The Biéssel's analysis may be applied to a variety of wavemakers²⁾⁻⁵⁾, with necessary modification in the contact surface boundary condition pertinent to the wavemaker in question. The Biéssel's analysis is based on the first-order linearized hydrodynamic equations of two-dimensional irrotational motions in prismatic channels of rectangular cross section.

Partial experimental verifications of the first-order theory were reported by Ursell, Dean, and Yu⁶⁾ for the case of a piston-type wavemaker, and by Taniguchi and Shibata⁷⁾ for a flap-type wavemaker.

Aside from the problems involved in factual mechanical fabrication, the greater part of unfavorable features of a wavemaker performance, such as waveform distortions very near the waveboard and excessive hydrodynamic loads on the waveboard, may basically be attributed to the added mass effect of the waveboard motions²⁾. The added mass effect may be explained in terms of the inertia pressure, p_i , which represents the local disturbances in the immediate vicinity of the waveboard. The local disturbances are created by, among other things, the difference between the waveboard

motions and the actual water particle movements as desired waves are generated.

Whereas the wave (or normal) pressure p_w , which is the hydrodynamic pressure far down the channel, is primarily a function of λ/h (the ratio of wavelength λ to the tank water depth h), the inertia pressure p_i strongly depends on the waveboard particulars as well as on λ/h .

The common flap-type wavemaker²⁾³⁾⁵⁾⁷⁾ is normally equipped with a single-flap rigid straight waveboard with its lower end hinged somewhere in the middle of the wall. It was found²⁾ that the flap size f_s/h is a crucial design parameter in predicting a single-flap wavemaker performance. In general, if the flap size is moderate or large (say $f_s/h \geq 0.5$), the relative magnitude of p_i is smaller than p_w when long waves (say $\lambda/h \geq 2.0$) are generated, but in the case of short waves p_i increases to the extent of almost overwhelming p_w [Ref. (2)]. Consequently, if the main purpose of a single-flap wavemaker is to generate short waves (e.g., $\lambda/h \leq 0.5$), then the proper size of the flap should be quite small ($f_s/h \cong 0.25$) with a view towards entailing much reduced inertia pressure. From the standpoint of minimizing the inertia pressure (or the local disturbances), it may be stated that small flaps are preferable when generating short waves, and that flaps of increased sizes should be chosen in order to generate moderate and long waves. In the extreme limit of very long waves ($\lambda/h \gg 1$), the most appropriate wavemaker would be the one with a piston-type waveboard ($f_s/h = 1.00$) executing purely

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$$\begin{aligned}
Y &= Y_w + Y_i \\
&= A \sinh ky \cos(\omega t - kx) \\
&\quad + \sum_{n=1}^{\infty} A_n \sin k_n y \exp(-k_n x) \sin \omega t, \quad (10)
\end{aligned}$$

respectively.

It is evident that the fluid motion is composed of two parts: a wave part propagating in the positive x direction, and a part due to the local disturbances, i.e., a sum of terms of which each decays exponentially as $\exp(-k_n x)$. Obviously, the influence of the local disturbances virtually disappears when $x > 3h$ [Ref. (1)], but their role is important to the hydrodynamic loads on the waveboard and to the wave profile distortions very near the waveboard.

In connection with Eq. (10), the wave amplitude a far down the channel (say $x > 3h$) is^{1), 6)}

$$a = A \sinh kh. \quad (11)$$

With the aid of the Bernoulli's theorem, the hydrodynamic pressure p is (ρ is water density):

$$\begin{aligned}
p &= -\rho \frac{\partial \phi}{\partial t} \\
&= \rho g \tanh kh A \cosh ky \cos(\omega t - kx) \\
&\quad + \rho g \sum_{n=1}^{\infty} A_n \tan k_n h \cos k_n y \exp(-k_n x) \sin \omega t
\end{aligned} \quad (12)$$

The formulation so far is applicable to a general form of waveboard configuration, within the framework of linearization. The problem then narrows down to the determination of ω , k_n 's, A , and A_n 's provided that the constant water depth h , the desired wavelength λ , and the particulars of the waveboard motions are prescribed.

3. Specialization to an Articulated Double-flap Wavemaker

Let us now specialize the above formulation to an articulated double-flap wavemaker of Fig. 1. The flap sizes are $f_u = h - h_2$, and $f_L = h_2 - h_1$, for the upper and the lower flaps, respectively. The length of the overall waveboard is given by $f \equiv f_u + f_L = h - h_1$.

From Fig. 1, the contact surface equation $\xi(y)$ is given as

$$\xi(y) = \begin{cases} 0 & \text{for } 0 \leq y < h_1, \\ \alpha_1(y - h_1) & \text{for } h_1 \leq y < h_2, \\ \alpha_1(y - h_1) + \alpha_2(y - h_2) & \text{for } h_2 \leq y \leq h, \end{cases} \quad (13)$$

thus, either $\alpha_1 = 0$ or $\alpha_2 = 0$ corresponds to a single-flap wavemaker hinged at $y = h_2$ or $y = h_1$, respectively. In the present development, α_1 is assumed to be nonzero. From Eq. (13), it follows that the waveboard stroke at the mean water level S is

$$S = \alpha_1(h - h_1) + \alpha_2(h - h_2) \quad (14)$$

Substituting Eq. (13) into Eqs. (7) and (8), we have

$$\frac{A}{h} = \frac{2kh[\alpha_1 U(H_1) + \alpha_2 U(H_2)]}{\sinh kh \cosh kh + kh} \quad (15)$$

$$\frac{A_n}{h} = \frac{2k_n h[\alpha_1 U_n(H_1) + \alpha_2 U_n(H_2)]}{\sin k_n h \cos k_n h + k_n h} \quad (16)$$

where $H_1 \equiv h_1/h$, $H_2 \equiv h_2/h$, and the functions $U(r)$ and $U_n(r)$ are defined as

$$U(r) \equiv \frac{1-r}{kh} \sinh kh - \frac{1}{(kh)^2} \{ \cosh kh - \cosh[(kh)r] \}, \quad (17)$$

$$U_n(r) \equiv \frac{1}{(k_n h)^2} \{ \cos k_n h - \cos[(k_n h)r] \} - \frac{1-r}{k_n h} \sin k_n h. \quad (18)$$

From Eqs. (11) and (14), after some algebraic manipulation the transfer function a/S (the ratio of the wave amplitude a to the waveboard stroke at the mean water level S) is shown to be

$$\begin{aligned}
a/S &= \frac{2kh \sinh kh}{\sinh kh \cosh kh + kh} \\
&\quad \times \left\{ \frac{U^2(H_1) + \left[\frac{\alpha_2}{\alpha_1} U(H_2) \right]^2 + 2 \frac{\alpha_2}{\alpha_1} U(H_1) U(H_2)}{(1-H_1)^2 + \left[\frac{\alpha_2}{\alpha_1} (1-H_2) \right]^2 + 2 \frac{\alpha_2}{\alpha_1} (1-H_1)(1-H_2)} \right\}^{1/2}
\end{aligned} \quad (19)$$

As is in the case of a single-flap wavemaker, the transfer function a/S takes a limiting value of 2 for very short waves ($kh \gg 1$), regardless of the waveboard particulars.

On the other hand, the maximum waveform distortion η/a at $x=0$, which is obtainable from the local disturbances part of Eq. (10), may be expressed as

$$\begin{aligned}
\eta/a &= \frac{\sinh kh \cosh kh + kh}{2kh \sinh kh} \\
&\quad \times \sum_{n=1}^{\infty} \left[\frac{2k_n h \sin k_n h}{\sin k_n h \cos k_n h + k_n h} \right. \\
&\quad \times \left. \left\{ \frac{U_n^2(H_1) + \left[\frac{\alpha_2}{\alpha_1} U_n(H_2) \right]^2 + 2 \frac{\alpha_2}{\alpha_1} U_n(H_1) U_n(H_2)}{U_n^2(H_1) + \left[\frac{\alpha_2}{\alpha_1} U_n(H_2) \right]^2 + 2 \frac{\alpha_2}{\alpha_1} U_n(H_1) U_n(H_2)} \right\}^{1/2} \right]
\end{aligned} \quad (20)$$

Now, in order to compute the hydrodynamic load on the waveboard, the hydrodynamic pressure at $x=0$ is rewritten:

$$p|_{x=0} = p_w \cos \omega t + p_i \sin \omega t,$$

where

$$\begin{aligned}
p_w &= \rho g a \left[\frac{A}{a} \tanh kh \cosh ky \right] \\
p_i &= \rho g a \left[\sum_{n=1}^{\infty} \frac{A_n}{a} \tan k_n h \cos k_n y \right]
\end{aligned} \quad (21)$$

Clearly, the wave (or normal) pressure p_w denotes the wave part of the hydrodynamic pressure fluctuations, and p_i , designated as the inertia pressure by Biéssel¹⁾, represents the local disturbances in the

vicinity of the waveboard. The wave pressure p_w , in phase with the waveboard velocity, indicates that a finite amount of work is transferred over one cycle. The energy expenditure is consumed by the propagation of outgoing waves on the free surface. The inertia pressure p_i , in phase with the waveboard acceleration, does no work on the average, however, the principal role played by the inertia pressure turns up in the form of added mass effect of the waveboard motions. In the terminology of naval hydrodynamics, p_i and p_w are associated with the added mass and damping coefficients, respectively.

It follows that the theoretical mean power consumption of the wavemaker over one cycle, P , is related to p_w only, and can thus be shown to be^{1), 7)}

$$P \cong B \rho g a^2 (g/k)^{1/2} \quad (22)$$

which is, in fact, the energy transmission to a sinusoidal wavetrain of wavelength λ , amplitude a , and the propagation speed $c = (g/k)^{1/2}$. In arriving at Eq. (22), the finite bottom effect on the wave propagation speed has been neglected for the wavelength range of practical significance in towing tank operations.

By integrating the hydrodynamic pressure at $x=0$ over the flap lengths, we calculate the pure hydrodynamic loads F_u and F_L , acting on the upper and the lower flaps, respectively (the factor of 2 takes into account that water fills in both sides of the waveboard⁵⁾):

$$\begin{aligned} \frac{F_u}{2(\rho g a) B h} &= \left[\frac{A}{a} \frac{\tanh k h}{k h} (\sinh k h - \sinh k h_2) \right] \cos \omega t \\ &+ \left[\sum_{n=1}^{\infty} \frac{A_n}{a} \frac{\tan k_n h}{k_n h} (\sin k_n h - \sin k_n h_2) \right] \sin \omega t \quad (23) \\ \frac{F_L}{2(\rho g a) B h} &= \left[\frac{A}{a} \frac{\tanh k h}{k h} (\sinh k h_2 - \sinh k h_1) \right] \cos \omega t \\ &+ \left[\sum_{n=1}^{\infty} \frac{A_n}{a} \frac{\tan k_n h}{k_n h} (\sin k_n h_2 - \sin k_n h_1) \right] \sin \omega t \quad (24) \end{aligned}$$

In a similar manner, the hydrodynamically-induced torques T_u (for the upper flap $Q_2 Q_3$) and T_L (for the lower flap $Q_1 Q_2$) about their respective hinge points Q_2 and Q_1 may be calculated as:

$$\begin{aligned} \frac{T_u}{2B \rho g a h^2} &= \frac{A}{a} \tanh k h \\ &\times \left[\frac{1-H_2}{k h} \sinh k h - \frac{1}{(k h)^2} (\cosh k h - \cosh k h_2) \right] \cos \omega t \\ &+ \sum_{n=1}^{\infty} \left\{ \frac{A_n}{a} \tan k_n h \left[\frac{1}{(k_n h)^2} (\cos k_n h - \cos k_n h_2) \right. \right. \\ &\left. \left. + \frac{1-H_2}{k_n h} \sin k_n h \right] \right\} \sin \omega t \quad (25) \\ \frac{T_L}{2B \rho g a h^2} &= \frac{A}{a} \tanh k h \\ &\times \left[\frac{H_2-H_1}{k h} \sinh k h_2 \right. \end{aligned}$$

$$\begin{aligned} &- \frac{1}{(k h)^2} (\cosh k h_2 - \cosh k h_1) \left] \cos \omega t \right. \\ &+ \sum_{n=1}^{\infty} \left\{ \frac{A_n}{A} \tan k_n h \left[\frac{1}{(k_n h)^2} (\cos k_n h_2 - \cos k_n h_1) \right. \right. \\ &\left. \left. + \frac{H_2-H_1}{k_n h} \sin k_n h_2 \right] \right\} \sin \omega t \quad (26) \end{aligned}$$

4. Numerical Computation and Discussion

In the wavelength range of practical usefulness to towing tank operation, $0.1 < \lambda/h < 4.0$, numerical computations are carried out to investigate the dynamic characteristics of articulated double-flap wavemakers. In the initial design stages of a double-flap wavemaker, the entire waveboard length and the waveboard partition may be considered as variable parameters. Once a double-flap wavemaker is installed, this implies that f/h , f_u/h , and f_L/h are fixed, and only ω , α_1 , and α_2/α_1 are the parameters that can be varied during operations. Basically, in contrast to a single-flap wavemaker ($\alpha_2/\alpha_1=0$), a double-flap wavemaker has an additional degree of freedom, i.e., $\alpha_2/\alpha_1 \neq 0$, which is essentially the primary source of the added versatility of a double-flap wavemaker.

Fig. 2 shows a typical plot of the transfer function a/S of an articulated double-flap wavemaker (flap sizes are $f_u/h=0.2$, $f_L/h=0.3$ in this example). Note that the curve of $\alpha_2/\alpha_1=0.0$ corresponds to the transfer function of a single-flap wavemaker^{2), 8)} of flap length $f_s/h=f/h=0.5$. The variation of a/S with respect to the waveboard partition, i.e., the location of the upper hinge point H_2 , is displayed in Fig. 3, when the overall waveboard length is $f/h=0.5$ and the flap angle ratio $\alpha_2/\alpha_1=5.0$. Figs. 2 and 3 give support to the intuitively obvious predictions of the a/S -variation, that is, higher values of a/S would be attained if a greater amount of

Fig. 2 Variation of the transfer function a/S as functions of λ/h and α_2/α_1 . Flap sizes are $f_u/h=0.2$, $f_L/h=0.3$.

Fig. 3 Variation of the transfer function a/S as functions of λ/h and H_2 . Overall waveboard length is $f/h=0.5$, and the flap angle ratio is $\alpha_2/\alpha_1=5.0$.

water is accelerated by the waveboard movement. This, in turn, suggests that for a given waveboard length f/h , the upper flap size f_u/h should be larger and the flap angle ratio α_2/α_1 should be smaller, in order to obtain higher values of a/S . In other words, as long as the transfer function a/S alone is concerned, a single-flap wavemaker ($\alpha_2/\alpha_1=0$) is most effective for a given waveboard length.

Fig. 4 illustrates the variation of the wave pressure p_w with respect to the wavelength. At the same time, if the abscissa of Fig. 4 is replaced by X_w/a of Eq. (9), then it will also serve as the envelope of the maximum particle excursions in the x direction far down the channel. The wave pressure p_w is clearly the hydrodynamic pressure far down the channel, which is independent of the waveboard particulars. As is well-known, in the case of short waves p_w (or X_w) is appreciable only in a narrow layer adjacent to the surface and decays rapidly with increasing water depth. As the wavelength increases, p_w (or X_w) is felt into in-

Fig. 4 Plot of the wave pressure p_w vs. water depth.

creasing water depths, making the finite bottom effect nonnegligible.

Let us now turn to the question of the local disturbances, a sum of terms of which each decays exponentially as $\exp(-k_n x)$. For a given kh , there exist an infinite number of $k_n h$'s, but the first few terms are of dominant importance^{1), 6)}. In the present numerical computations, the series was terminated up to the first fifty terms to insure sufficient accuracy⁵⁾.

In contrast to the wave pressure p_w , the inertia pressure p_i displays marked dependence on the waveboard particulars as well as on wavelength. In Figs. 5~7, plots of the inertia pressure distribution are shown for varying design parameters. The following two points can readily be noticed: (1) unlike p_w which decays with increasing water depth, p_i sustains its presence throughout much of the water layer on the whole, (2) in the case of moderate and long waves, i.e., $\lambda/h \geq 2.0$, the overall magnitude of p_i is relatively small, however, p_i is immensely augmented when short waves are to be generated. From Figs. 5~7, it is found that, in the case of short waves (e.g., $\lambda/h=0.25$), employing a double-flap waveboard with large articulation (e.g., $\alpha_2/\alpha_1=5.0$) has a noticeable effect in cutting

Fig. 5 Plot of the inertia pressure p_i vs. water depth. Flap sizes are $f_u/h=0.5$, $f_L=0.2$, respectively.

down the inertia pressure. It is also interesting to note in Fig. 5 that, in the case of long waves (e.g., $\lambda/h=3.00$), p_i is slightly enlarged by use of a double-flap waveboard (e.g., $\alpha_2/\alpha_1=5.0$). This is an example of improperly utilized double-flap wavemakers.

As was pointed out earlier, the inertia pressure p_i arises because of the local disturbances in the immediate vicinity of the waveboard, which are created mostly by the differences between the waveboard motions and the actual water particle movements in waves. Accordingly, within the framework of linearized formulation, one crude way of making a rough estimate of the magnitude of the inertia pressure is to examine the difference between

$\xi(y)/a$ of Eq. (13) and the envelope of the maximum particle excursions in the x direction, that is, Fig. 4 with its abscissa replaced by X_w/a . Consequently, with an eye to reducing the inertia pressure, the following observations may be made: In the case of short waves (e.g., $\lambda/h=0.25$) the curve of X_w/a decays rapidly with increasing water depth, which suggests that the overall waveboard length f/h should be small and the flap angle ratio α_2/α_1 should be large. However, it should be remembered that all the arguments here are within the scope of linearized small-amplitude motions; therefore, excessively large values of α_2/α_1 , which would violate the underlying assumption that the flap angles are small, are excluded from the analysis. In the opposite situation, when moderate and long waves (e.g., $\lambda/h \geq 2.0$) are generated, the X_w/a curve decays much more mildly, thus a single-flap waveboard ($\alpha_2/\alpha_1=0$) of increased sizes is more appropriate. In the limit of extremely long waves ($\lambda/h \gg 1$), a piston-type wavemaker ($f/h=1.00$) with uniform horizontal stroke will be preferable.

The practical advantages of an articulated double-flap wavemaker is manifested by evaluating the wavemaker performance, such as the waveform distortion very near the waveboard and the hydrodynamic loads exerted on the waveboard, especially in the range of short waves (say $\lambda/h \leq 1.0$).

The maximum waveform distortion η/a at $x=0$, derived in Eq. (20), is shown in Table 1, for the case when the overall waveboard length is $f/h=0.5$. As to the short waves ($\lambda/h \leq 1.0$), the waveform distortion η/a diminishes as the upper flap size becomes smaller and the flap angle ratio increases. This is an expected result in view of the effectiveness of a double-flap wavemaker in reducing the inertia pressure (or the local disturbances), as was illustrated in Figs. 5~7. However, in the case

Fig. 6 Plot of the inertia pressure p_i vs. water depth, for the case of $\lambda/h=0.25$. Flap sizes are $f_u/h=0.3$, $f_L/h=0.2$.

Fig. 7 Plot of the inertia pressure p_i vs. water depth, for the case of $\lambda/h=0.25$, $f/h=0.5$, $\alpha_2/\alpha_1=5.0$.

of moderate and long waves (e.g., $\lambda/h=2.0$), the waveform distortion η/a actually increases by use of double-flap wavemakers. These findings are in line with our previous discussion that a single-flap wavemaker ($\alpha_2/\alpha_1=0$) with large waveboard lengths ($f/h \rightarrow 1.0$) is suitable for the generation of moderate and long waves. Obviously, if a single-flap wavemaker equipped with a waveboard of much larger size were utilized, then η/a would be much smaller for moderate and long waves. In summary, Table 1 indicates that for a given waveboard geometry, in order to attain small values of η/a , the flap angle ratio α_2/α_1 should be chosen roughly along the diagonal line, stretching from lower left to upper right corner, of Table 1.

The total hydrodynamic force acting on the entire waveboard $F(=F_u+F_L)$, of Eqs. (23) and (24), is shown in Table 2, for the case when $f/h=0.5$. Here F , nondimensionalized by $2Bh\rho g a$, is expressed in the form of $M \cos(\omega t - \beta)$. It is seen in Table 2 that the capability of double-flap wavemakers in reducing the hydrodynamic load F is pronounced for short waves. For instance, in the example of $\lambda/h=0.25$, use of a double-flap wavemaker (e.g., $H_2=0.8$, $\alpha_2/\alpha_1=5.0$) cuts down the hydrodynamic load F more than half in comparison with the single-flap wavemaker ($\alpha_2/\alpha_1=0.0$) with the same waveboard size $f/h=0.5$. However, in the case of moderate and long waves (e.g., $\lambda/h \geq 2.0$) not much reduction is achieved in hydrodynamic load F by use of double-flap wavemakers.

As was seen before, when short waves are generated, p_i overwhelms p_w (the phase angle β of Table 2 is close to $-\pi/2$), therefore, a significant reduction in terms of hydrodynamic loads is anticipated by use of double-flap wavemakers. On the contrary, in the case of moderate and long waves, p_w is the

Table 1. Maximum waveform distortion r/a at $x=0$, as computed from Eq. (21).

		$\alpha_2/\alpha_1=0.0$	$\alpha_2/\alpha_1=0.5$	$\alpha_2/\alpha_1=1.0$	$\alpha_2/\alpha_1=5.0$
$H_1=0.5$ $H_2=0.6$	$\lambda/h=0.25$	0.5751	0.5533	0.5411	0.5135
	$\lambda/h=0.50$	0.3839	0.3612	0.3483	0.3188
	$\lambda/h=1.00$	0.1428	0.1205	0.1076	0.0773
	$\lambda/h=2.00$	0.1317	0.1531	0.1658	0.1960
$H_1=0.5$ $H_2=0.7$	$\lambda/h=0.25$	0.5751	0.5348	0.5090	0.4401
	$\lambda/h=0.50$	0.3839	0.3441	0.3181	0.2458
	$\lambda/h=1.00$	0.1428	0.1064	0.0818	0.0095
	$\lambda/h=2.00$	0.1317	0.1645	0.1875	0.2587
$H_1=0.5$ $H_2=0.8$	$\lambda/h=0.25$	0.5751	0.5267	0.4908	0.3679
	$\lambda/h=0.50$	0.3839	0.3396	0.3058	0.1808
	$\lambda/h=1.00$	0.1428	0.1062	0.0769	0.0409
	$\lambda/h=2.00$	0.1317	0.1623	0.1877	0.2975

Table 2. Total hydrodynamic load $F(F_u+F_L)$ acting on the entire waveboard ($H_1 \leq y/h \leq 1.0$), nondimensionalized by $2B\rho gah$. Quantities are expressed in the form of $M(\cos(\omega t - \beta))$.

	$\alpha_2/\alpha_1=0.0$		$\alpha_2/\alpha_1=0.5$		$\alpha_2/\alpha_1=1.0$		$\alpha_2/\alpha_1=5.0$	
	M	β	M	β	M	β	M	β
	$H_1=0.5, H_2=0.6$							
$\lambda/h=0.25$	0.4830	-1.4883	0.4407	-1.4804	0.4169	-1.4752	0.3663	-1.4810
$\lambda/h=0.50$	0.1873	-1.1329	0.1708	-1.0870	0.1616	-1.0568	0.1411	-0.9729
$\lambda/h=1.00$	0.1545	-0.1702	0.1535	-0.1258	0.1531	-0.0998	0.1524	-0.0385
$\lambda/h=2.00$	0.2540	-0.0170	0.2539	-0.0094	0.2539	-0.0049	0.2539	-0.0060
$\lambda/h=3.00$	0.3205	-0.1138	0.3205	-0.1122	0.3205	-0.1113	0.3204	-0.1091
	$H_1=0.5, H_2=0.7$							
$\lambda/h=0.25$	0.4830	-1.4883	0.4150	-1.4757	0.3781	-1.4654	0.2689	-1.4223
$\lambda/h=0.50$	0.1873	-1.1329	0.1634	-1.0632	0.1483	-1.0056	0.1103	-0.7666
$\lambda/h=1.00$	0.1545	-0.1702	0.1532	-0.1083	0.1526	-0.0659	0.1526	0.0595
$\lambda/h=2.00$	0.2540	-0.0170	0.2539	-0.0063	0.2539	0.0013	0.2540	0.0247
$\lambda/h=3.00$	0.3205	-0.1138	0.3205	-0.1111	0.3204	-0.1092	0.3205	-0.1031
	$H_1=0.5, H_2=0.8$							
$\lambda/h=0.25$	0.4830	-1.4883	0.4224	-1.4765	0.3778	-1.4652	0.2248	-1.3929
$\lambda/h=0.50$	0.1873	-1.1329	0.1664	-1.0731	0.1508	-1.0160	0.1108	-0.6629
$\lambda/h=1.00$	0.1545	-0.1702	0.1534	-0.1203	0.1528	-0.0798	0.1523	0.0840
$\lambda/h=2.00$	0.2540	-0.0170	0.2539	-0.0085	0.2539	-0.0014	0.2540	0.0247
$\lambda/h=3.00$	0.3205	-0.1138	0.3205	-0.1114	0.3204	-0.1095	0.3201	-0.1006

dominant contributor to the hydrodynamic loads (the phase angle β of Table 2 is close to zero). In this situation, use of double-flap wavemakers, which can affect only p_i does not lead to an appreciable reduction in the hydrodynamic loads.

Finally, Table 3 contains numerical examples of the hydrodynamically-induced torques T_u (for the upper flap $\overline{Q_2Q_3}$) and T_L (for the lower flap $\overline{Q_1Q_2}$), of Eqs. (25) and (26), when the flap sizes are $f_u/h=0.2$ and $f_L/h=0.3$. In Table 3, the torques are made

dimensionless by $2Bh^2\rho ga$, and is expressed in the form of $M\cos(\omega t - \beta)$. First, let us consider short waves (e.g., $\lambda/h=0.25$). We can see that the wave pressure p_w is still largely unattenuated in the region of the upper flap $\overline{Q_2Q_3}$ ($0.8 \leq y/h \leq 1.0$). Therefore, is composed of contributions from both the wave pressure and the inertia pressure (the phase angle β assumes values roughly half-way between zero and $-\pi/2$). On the other hand, since the wave pressure p_w is rapidly decaying with in-

Table 3. Hydrodynamically-induced torques T_u (for the upper flap $\overline{Q_2Q_3}$ and T_L (for the lower flap $\overline{Q_1Q_2}$), nondimensionalized by $2B\rho gah^2$. Quantities are expressed in the form of $M \cos(\omega t - \beta)$. Flap sizes are $f_u/h=0.2$, $f_L/h=0.3$.

		$\alpha_2/\alpha_1=0.0$		$\alpha_2/\alpha_1=0.5$		$\alpha_2/\alpha_1=1.0$		$\alpha_2/\alpha_1=5.0$	
		M	β	M	β	M	β	M	β
$\lambda/h=0.25$	T_U	0.0096	-0.8449	0.0091	-0.7967	0.0088	-0.7576	0.0077	-0.5989
	T_L	0.0564	1.5708	0.0488	1.5708	0.0432	1.5708	0.0239	1.5708
$\lambda/h=0.50$	T_U	0.0101	0.0291	0.0101	0.0225	0.0101	0.0173	0.0101	-0.0015
	T_L	0.0230	1.5081	0.0197	1.4981	0.0172	1.4881	0.0080	1.3923
$\lambda/h=1.00$	T_U	0.0138	0.0772	0.0137	0.0547	0.0137	0.0366	0.0137	-0.0363
	T_L	0.0089	-0.5801	0.0083	-0.4383	0.0078	-0.3085	0.0078	0.2752
$\lambda/h=2.00$	T_U	0.0166	-0.1308	0.0166	-0.1522	0.0167	-0.1699	0.0169	-0.2451
	T_L	0.0182	0.0343	0.0182	0.0709	0.0182	0.1018	0.0186	0.2332
$\lambda/h=3.00$	T_U	0.0183	-0.2805	0.0184	-0.3004	0.0185	-0.3169	0.0190	-0.3888
	T_L	0.0252	-0.0450	0.0252	-0.0244	0.0252	-0.0077	0.0252	0.0693

creasing water depth for short waves, p_w is very small in the region of the lower flap $\overline{Q_1Q_2}$ ($0.5 \leq y/h \leq 0.8$). Hence T_L is determined principally by the inertia pressure p_i (the phase angle β is close to $\pi/2$). For this reason, double-flap wavemakers are seen to be more effective in reducing T_L , rather than T_u , in the case of short waves. Secondly, we turn to discuss the cases of moderate and long waves. The dominant contributor to the hydrodynamically-induced torques is the wave pressure p_w , which is not affected by the changes in waveboard particulars. Here again we see that a single-flap wavemaker with large waveboard lengths is desirable from the standpoint of having small torques.

Table 3 suggests that the setting of the operational variable α_2/α_1 should preferably be selected from roughly along the diagonal line of Table 3, from lower left to upper right corner. This is a similar result that we arrived at in Table 1, with a view towards having smallest waveform distortions.

Conclusion

A theoretical account is given of an articulated double-flap wavemaker, as a sequel to the previous paper²³ concerning the usual single-flap wavemaker. The additional degree of freedom in double-flap wavemakers is capable of reducing the local disturbances in the vicinity of the waveboard, which turns up in the form of much decreased inertia pressure especially in the case of short waves. Therefore, when short waves are to be generated, it is desirable that the overall waveboard length f/h be small and the flap angle ratio α_2/α_1 be relatively large. On the other hand, in the case of moderate and long waves, larger waveboards ($f/h \rightarrow 1.0$) with little or zero flap angle ratio ($\alpha_2/\alpha_1 \rightarrow 0$) are preferable.

The analysis clearly suggests that considerable improvement over single-flap wavemakers in performance curves, such as waveform distortion near the waveboard and the hydrodynamic loads, is feasible by suitable use of double-flap waveboards.

Needless to say, the results of the present study should be the subject of careful experimental verification.

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