A Study on the Flow around a Catamaran

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Summary

It is often experienced that the resistance of a catamaran hull form is more than the twice of that of the demihull. To investigate the reason for this resistance increase, the velocity components around a symmetrical catamaran form were measured and compared with those around the demihull. Further, the wake survey and wave pattern measurements were carried out.

As a result the wake resistance determined by means of wake survey was found to be more than the twice of that of the demihull. In the lower speed range this resistance increment is simply attributed to the increase of viscous resistance by the velocity augmentation around the catamaran. In the higher speed range, on the other hand, a resistance component due to the breaking waves is added further. Asymmetrical property of the velocity field around each demihull of the catamaran is relatively small and is considered to have less influence on resistance when comparing with the mean velocity augmentation from the demihull condition.

1. Introduction

Catamarans have large deck area and large restoring force in comparison with conventional single hull ships. Making the best use of these advantages of catamarans, various kinds of application have been considered, such as container ships, ferries, work boats and oceanographic survey vessels. When designing such catamarans, we must pay special attention to their hydrodynamic property because propulsive and sea-keeping performance of catamarans is sometimes inferior to that of single hull ships. For instance, resistance of a catamaran is more than the twice of that of the demihull and ship motion is larger than single hull ships.

The resistance characteristics of a catamaran have been studied so far by many investigators, for example by Yokoo and Tasaki¹⁾, Eggers²⁾, and Tasaki³⁾. Their studies are based on the linearized wave-making resistance theory and in most of them effects of wave interference have been studied. When we carry out resistance tests of catamaran models, it is often experienced that the resistance of a catamaran is more than the twice of that of the demihull even in low speeds where the wave resistance is negligibly small. This resistance increment in low speeds may be interpreted as an augmentation of the form factor and recalls us that the frictional

resistance is increased due to the velocity augmentation around a catamaran.

In the present study the flow around a symmetrical catamaran was investigated in order to know the reason for the resistance increase. Concerning the flow around a catamaran Pien⁴⁾ suggested that a cross flow across the keel line of each hull of a catamaran would occur and causes eddying resistance. No data, however, are available to support this phenomenon. In this study the velocity field in the relatively deep zone around a catamaran model was measured and compared with that around the demihull in isolation. Further the wake survey by use of pitot tube rake and the wave pattern measurements by means of longitudinal cut were carried out.

In addition to the above experimental study an analytical study on the flow around a pair of vertical circular cylinders in steady motion was carried out so as to help the interpretation of the results of the experimental study.

2. Experimental Study on a Catamaran Model

2.1 Velocity and Wave Height Measurement

The principal particulars of the catamaran model used in the present study are listed in Table 1. The demihull is symmetrical about her longitudinal center plane.

The velocity components were measured by a 5-hole pitot tube in the inner and outer regions around a twin hull from 10% of L_{pp} behind A. P.

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Table 1 Principal dimensions of models

L_{PP} (m)	4.000	L_{PP}/B	11.542
W (mm)	1,055.64	B/d	1.359
B (mm)	346.54	C_{bPP}	0.5543
D (mm)	555.09	$(W-B)/L_{PP}$	0.177
d (mm)	255.09	$(W-2B)/\delta$	1.050
V_0 (m ³) Twin Hull	0.39279	$S/V_0^{2/3}$ Twin Hull	9.802

I_1, I_2, E_1, E_2 : Position of Velocity Measurement

K_1, K_2 : Do in Twin Hull Cond.

I_1', I_2', E_1', E_2' : Position of Wave Height Measurement

Fig. 1 Positions of velocity and wave height measurements

to 10% of L_{pp} before F. P. The measurements were made also on the demihull at the corresponding positions. The vertical and transverse positions of measurement are shown in Fig. 1. The model ran at the speed of 2.00 m/s. The results are shown in Fig. 2a, b and c where V_x is the velocity component in the direction of advance, V_y in the inward direction and V_z upward.

In the first place, it is noticed in Fig. 2a that around the catamaran both in the inner and outer regions V_x is larger than that around the demihull. In the vicinity of hull surfaces at S. S. 5 the mean value of V_x at I_2 and E_2 exceeds about 5% of V over the value of V_x of the demihull. Further, it is noticed that the deviation of V_x at I_2 or E_2 from this mean value is about 2% of V . Therefore, it may be said that the asymmetric property is small in the velocity component V_x around each hull of the catamaran.

Next, the velocity component V_y is shown in Fig. 2b. The cross flow or lateral velocity component across the keel line is thus observed as suggested by Pien. In the entrance part it flows outwards, and in the run part it flows inwards. This tendency can be seen clearly at the closer position K_1 . The order of magnitude of V_y at K_1 is about 5-7% of model speed V . It seems to be difficult to conclude that this cross flow induces

Fig. 2a Comparison of measured V_x Fig. 2b Comparison of measured V_y

the eddying drag over such a flat bottom as the present catamaran form, and it may be more probable that the resistance increment experienced in catamaran form is mainly due to the velocity augmentation over the hull surface as observed in Fig. 2a.

The measured velocity component V_z is shown in Fig. 2c. In the outer region, the change of V_z in the longitudinal direction is small and similar to that around the demihull, but in the inner region of the twin hulls the change is relatively large.

Wave heights were also measured at the same transverse positions and at the same model speed as the velocity measurement. The measured wave heights are shown in Fig. 3. In this figure it is

noticed that in the inner region of the twin hulls steep waves appear which lead to breaking waves. In the outer region, on the other hand, the waves change in the similar way as those around the

demihull. Such steepness of the waves between the twin hulls is considered to be due to the interference of waves which are generated by both demihulls.

2.2 Wake Survey and Wave Pattern Measurement

In order to evaluate the influence of the velocity augmentation on the resistance components of a catamaran, wake survey by use of a pitot tube rake and wave pattern measurements by means of longitudinal cut were carried out.

The wake surveys were carried out at the two speeds, one at a relatively lower speed $V=0.98$ m/s ($F_n=0.155$), and the other at a higher speed $V=2.06$ m/s ($F_n=0.326$) where breaking waves were observed. The test results are listed in Table 2 and compared with the total resistance coefficients measured by a resistance dynamometer. Fig. 4a and b show the comparison of head loss distributions between the twin hull and the demihull at the both speeds. At the lower speed, the difference of head loss distributions is small,

Fig. 2c Comparison of measured V_z

Fig. 3 Comparison of wave heights

Table 2 Results of wake survey and wave pattern measurements

V (m/s)	FnWL	Twin Hull Condition						Demi Hull in Isolation					
		Cwake	Rwake	Cwave	Rwave	Cwake + Cwave	C _T	Cwake	Rwake	Cwave	Rwave	Cwake + Cwave	C _T
0.980	0.1549	0.0400	1.050				0.0404	0.0295	0.488				0.0290
1.961	0.3100			0.0007	0.073		0.0389			0.0009	0.058		0.0284
2.060	0.3256	0.0403	4.673	0.0012	0.135	0.0415	0.0409	0.0279	2.038	0.0011	0.084	0.0290	0.0292
2.158	0.3411			0.0035	0.447		0.0443			0.0033	0.262		0.0313

Remarks $C = R_{T.H.} / \frac{1}{2} \rho V^2 (\nabla_{T.H.})^{2/3}$ in Twin Hull Cond.

$C = R_{D.H.} / \frac{1}{2} \rho V^2 (\nabla_{D.H.})^{2/3}$ in Demi Hull Cond.

Rwake : Determined by $\iint_{\omega} \{ P_0 - P + \rho V(V-w) + \frac{1}{2} \rho V^2 (P_0 - P)^2 \} dy dz$ (See Ref. 6)

C_T : Resistance Coeff. measured by Resistance Dynamometer

Position of Wake Survey : 0.5 Lpp behind A.P.

Position of Wave Pattern Measurement: 3.215^m abreast from the Center Line.

Fig. 4a Comparison of head loss distributions
 $V=0.98$ m/s

but the wake resistance of the catamaran determined by the wake survey is about 2.15 times as much as (7.5% larger than twice) that of the demihull. At the higher speed the wake resistance is about 2.3 times as much as that of the demihull. The difference of head loss distributions between the catamaran and the demihull is distinctive both in the vicinity of the free surface and in the deeper zone. This is due to the breaking of waves observed in the inner region and due to the difference of the velocity on the hull surface of the catamaran and the demihull. The obtained wake resistance coefficients are shown in Fig. 5.

The wave pattern analyses based on Newman-Sharma method were carried out at three higher speeds. Wave pattern resistance coefficients are shown in Table 2 and Fig. 5. It is noticed that the wave pattern resistance is smaller than the twice of that of the demihull. This is because of wave breaking between the twin hulls.

The sum of wake resistance and wave pattern

Fig. 4b Comparison of head loss distributions
 $V=2.06$ m/s

resistance nearly equals that measured by a resistance dynamometer in towing test as shown in Table 2 and Fig. 5. It is found that the wave pattern resistance is relatively small when compared with the wake resistance. Therefore, if wake resistance is supposed to be proportional to the velocity squared, the mean increase of velocity on the hull surfaces of the catamaran compared with the demihull is estimated to be about 4% of V at $V=0.98$ m/s and about 7% of V at $V=2.06$ m/s respectively. This velocity augmentation corresponds approximately to the mean rise of velocity component V_x measured at the position I_2 and E_2 in the vicinity of S.S.5.

3. Study on the Flow around a Pair of Vertical Circular Cylinders

To get better understanding of the results from

Fig. 5 Resistance coefficient curves

Fig. 6 Coordinate system

the experimental studies in the previous section, an analytical study on the flow around a pair of vertical circular cylinders in steady motion was carried out. In this analysis the free surface is replaced by a rigid wall and only the body interference effect on the velocity field is considered, neglecting wave interference.

Suppose a pair of cylinders C_1 and C_2 of radius a in a uniform stream V as shown in Fig. 6. Let us introduce a parameter ϵ defined by, $\epsilon = a/l$ (< 1), where l is the half of the distance between the centers of cylinders. The parameter ϵ corresponds to the following relation with the total width W of the catamaran and the width B of the demihull:

$$\epsilon = \frac{1}{\left\{1 + \frac{(W-2B)}{B}\right\}}$$

The velocity components on the surface of the cylinder C_1 are expressed by the following expansions with regard to ϵ . On the surface of the cylinder C_1 :

$$\frac{V_x}{V} = \left(1 + \frac{\epsilon^2}{4}\right)(\cos 2\alpha - 1) + \frac{\epsilon^3}{4}(\sin \alpha - \sin 3\alpha) + \frac{\epsilon^4}{16}(4 \cos 2\alpha - 3 \cos 4\alpha - 1) + O(\epsilon^5), \quad (1)$$

$$\frac{V_y}{V} = \left(1 + \frac{\epsilon^2}{4}\right)\sin 2\alpha + \frac{\epsilon^3}{4}(\cos \alpha + \cos 3\alpha) - \frac{\epsilon^4}{16}(2 \sin 2\alpha + 3 \sin 4\alpha) + O(\epsilon^5). \quad (2)$$

The procedure for the derivation of the above expansion is explained in Appendix.

When a single circular cylinder is fixed in the same stream, the velocity components on the surface of the cylinder are expressed as:

$$\frac{V_x}{V} = \cos 2\alpha - 1, \quad (3)$$

$$\frac{V_y}{V} = \sin 2\alpha. \quad (4)$$

It is noticed in the first place that on the surfaces of a pair of vertical circular cylinders the velocity increases to about $(1 + \epsilon^2/4)$ times as much as that on the single cylinder. The velocity components at Q_1 , Q_2 , Q_3 and Q_4 as shown in Fig. 6 are expressed as follows respectively:

$$Q_1; \quad \frac{V_x}{V} = 0, \quad \frac{V_y}{V} = \frac{\epsilon^3}{2},$$

$$Q_2; \quad \frac{V_x}{V} = -2\left(1 + \frac{\epsilon^2}{4} - \frac{\epsilon^3}{4} + \frac{\epsilon^4}{4}\right), \quad \frac{V_y}{V} = 0,$$

$$Q_3; \quad \frac{V_x}{V} = 0, \quad \frac{V_y}{V} = -\frac{\epsilon^3}{2},$$

$$Q_4; \quad \frac{V_x}{V} = -2\left(1 + \frac{\epsilon^2}{4} + \frac{\epsilon^3}{4} + \frac{\epsilon^4}{4}\right), \quad \frac{V_y}{V} = 0.$$

It is found that the asymmetrical flow about the cylinder C_1 originates in the term of order $\epsilon^3/4$ in eqs. (1) and (2) and induces attraction force between the cylinders. Order of the asymmetrical term is higher than the term $\epsilon^2/4$ by ϵ .

It is shown that velocity increment on the surfaces of the twin cylinders is comparable with that obtained from the difference of wake resistance between the catamaran and the twice of that of the demihull. That is to say, in the case of the present catamaran model $\epsilon = 1/2.05$ and $(\epsilon^2/4)V = 0.06V$, on the other hand, the mean velocity increment is estimated to be about $0.04V$ at $V = 0.98$ m/s and about $0.07V$ at $V = 2.06$ m/s on the basis of the difference of wake resistance as mentioned before. There is certain difference of flow between the actual catamaran hull form and

cylinders due to the difference of form, but the mean velocity increment on the hull surface of a catamaran can be estimated approximately by $(\epsilon^2/4)V$.

By the use of the mean velocity increment of the flow on the surfaces of twin cylinders, the following relation is derived by assuming that the wake resistance is proportional to the square of the velocity:

$$R_{\text{wake(T.C.)}} \sim \left(1 + \frac{\epsilon^2}{2}\right) \cdot 2R_{\text{wake(S.C.)}} \quad (5)$$

where $R_{\text{wake(T.C.)}}$ and $R_{\text{wake(S.C.)}}$ denote the wake resistance of twin cylinders and that of the single cylinder respectively.

On the other hand, the wave resistance of the twin cylinders $R_{\text{wave(T.C.)}}$ can be obtained from eqs. (1) and (2) according to Baba⁵⁾, assuming that the Froud Number F_n defined by $F_n = V/\sqrt{2ga}$ is sufficiently small. $R_{\text{wave(T.C.)}}$ is expressed as follows:

$$\frac{R_{\text{wave(T.C.)}}}{\frac{1}{2}\rho V^2(2a)^2} \sim \frac{2 \times 8192}{315} \left(1 + \frac{3}{2}\epsilon^2\right) F_n^6,$$

where ρ is the density of fluid and g is the acceleration of gravity. Since the wave resistance of single cylinder $R_{\text{wave(S.C.)}}$ is expressed as:

$$\frac{R_{\text{wave(S.C.)}}}{\frac{1}{2}\rho V^2(2a)^2} \sim \frac{8192}{315} F_n^6,$$

the following relation is thus obtained:

$$R_{\text{wave(T.C.)}} \sim \left(1 + \frac{3}{2}\epsilon^2\right) \cdot 2R_{\text{wave(S.C.)}} \quad (6)$$

Eqs. (5) and (6) indicate that both resistance components of twin cylinders increase to more than the twice of respective components of the single cylinder due to the velocity increase on the twin cylinders.

4. Conclusions

Through the present study on the flow around a catamaran and her resistance components, the following results are obtained.

- (1) The wake resistance determined by means of wake survey is found to be more than the twice of that of the demihull, and this is due to the velocity augmentation on the flow around the catamaran.
- (2) In the lower speed range the increment of the wake resistance is simply attributed to the increase of the viscous resistance. In the higher speed range, however, a resistance component due to the breaking waves is added further.
- (3) Asymmetrical property of the velocity field around each demihull of the catamaran is relatively small, and is considered to have

less influence on resistance than the mean velocity augmentation from the demihull condition.

- (4) The analytical study on the flow around a pair of vertical circular cylinders supports above experimental results.

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Appendix

Suppose a pair of vertical circular cylinders C_1 and C_2 are placed in a uniform stream as shown in Fig. 6. The velocity potential $\phi_r(x, y)$ is written as follows:

$$\begin{aligned} \phi_r(x, y) = & -Vx + \frac{a}{2} \int_0^{2\pi} d\theta \cdot \sigma(\theta) \\ & \cdot \ln [(x - a \cos \theta)^2 + \{y - (l + a \sin \theta)\}^2] \\ & + \frac{a}{2} \int_0^{2\pi} d\theta \cdot \sigma(\theta) \cdot \ln [(x - a \cos \theta)^2 \\ & + \{y + (l + a \sin \theta)\}^2], \end{aligned} \quad (A.1)$$

where $\sigma(\theta)$ is a source distribution on the cylinder surface. $\phi_r(x, y)$ is rewritten in the following form:

$$\begin{aligned} \phi_r(x, y) = & -Vx + \frac{a}{2} \int_0^{2\pi} d\theta \cdot \sigma(\theta) \\ & \cdot \ln \{ [1 - 2A \cos(\theta - \alpha) + A^2] \\ & + \ln \{ 1 - 2B \cos(\theta + \beta) + B^2 \} \}, \end{aligned} \quad (\text{A.2})$$

where

$$\begin{aligned} A = \frac{a}{\sqrt{x^2 + (y-l)^2}}, \quad B = \frac{a}{\sqrt{x^2 + (y+l)^2}}, \\ \tan \alpha = \frac{y-l}{x}, \quad \tan \beta = \frac{y+l}{x}. \end{aligned}$$

Taking into consideration that the flow is symmetrical about the x -axis, let us put the source distribution $\sigma(\theta)$ as:

$$\begin{aligned} \sigma(\theta) = & \frac{V}{2\pi} \left\{ \sum_{n=1}^N a_n \cdot \cos(2n-1)\theta \right. \\ & \left. + \sum_{n=1}^N b_n \cdot \sin 2n\theta \right\}, \end{aligned} \quad (\text{A.3})$$

and use the following integral formulae:

$$\begin{aligned} \int_0^{2\pi} d\theta \left\{ \frac{\cos}{\sin} \right\} m\theta \cdot \ln \{ 1 - 2A \cos \theta + A^2 \} \\ = \begin{cases} -\frac{2\pi}{m} A^m \\ 0 \end{cases} \quad \text{for } m=1, 2, \dots, \end{aligned}$$

then $\phi_r(x, y)$ is expressed as follows:

$$\begin{aligned} \frac{1}{Va} \phi_r(x, y) = & -\frac{x}{a} - \frac{1}{2} \sum_{n=1}^N \frac{a_n}{2n-1} \\ & \cdot \{ A^{2n} \cos(2n-1)\alpha + B^{2n-1} \cos(2n-1)\beta \} \\ & - \frac{1}{2} \sum_{n=1}^N \frac{b_n}{2n} \{ A^{2n} \sin 2n\alpha - B^{2n} \sin 2n\beta \}. \end{aligned} \quad (\text{A.4})$$

Taking into account the boundary condition on the surface of C_1 , the following equation is obtained as to $\sigma(\theta)$ from eq. (A.1):

$$\begin{aligned} \sigma(\gamma) = & \frac{V}{\pi} \cos \gamma - \frac{\epsilon}{2\pi} \int_0^{2\pi} d\theta \cdot \sigma(\theta) \\ & \left(\frac{\sin \gamma + \frac{1}{2} \epsilon \{ 1 - \cos(\gamma + \theta) \}}{1 + \epsilon \{ \sin \gamma + \sin \theta \} + \frac{1}{2} \epsilon^2 \{ 1 - \cos(\gamma + \theta) \}} \right). \end{aligned} \quad (\text{A.5})$$

Since ϵ is always less than 1, we can expand the kernel function of eq. (A.5) asymptotically with regard to ϵ as follows:

$$\begin{aligned} \frac{\epsilon}{2} \sin \gamma + \frac{\epsilon^2}{4} \{ \cos 2\gamma - \cos(\gamma - \theta) \} \\ + \frac{\epsilon^3}{8} \{ -\sin 3\gamma + 2 \sin(2\gamma - \theta) - \sin(\gamma - 2\theta) \} \\ - \frac{\epsilon^4}{16} \{ \cos 4\gamma - 3 \cos(3\gamma - \theta) \\ + 3 \cos(2\gamma - 2\theta) - \cos(\gamma - 3\theta) \} + 0(\epsilon^5). \end{aligned}$$

Then using eq. (A.3) and (A.5), $\sigma(\theta)$ is written as:

$$\sigma(\theta) = \frac{V}{2\pi} \left\{ 2 \left(1 + \frac{\epsilon^2}{4} + \frac{\epsilon^4}{16} \right) \cos \theta - \frac{\epsilon^3}{2} \sin 2\theta \right.$$

$$\left. - \frac{3}{8} \epsilon^4 \cos 3\theta \right\} + 0(\epsilon^5) \quad (\text{A.6})$$

Next, the velocity components on the surface of C_1 is obtained from eq. (A.4) as follows:

$$\begin{aligned} \frac{1}{V} \phi_{rx}(x, y) = & -1 + \frac{1}{2} \sum_{n=1}^N a_n \left\{ \cos 2n\alpha + \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n} \right. \\ & \cdot \cos 2n\beta \left. \right\} + \frac{1}{2} \sum_{n=1}^N b_n \left\{ \sin(2n+1)\alpha \right. \\ & \left. - \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n+1} \sin(2n+1)\beta \right\}, \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \frac{1}{V} \phi_{ry}(x, y) = & \frac{1}{2} \sum_{n=1}^N a_n \left\{ \sin 2n\alpha + \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n} \right. \\ & \cdot \sin 2n\beta \left. \right\} - \frac{1}{2} \sum_{n=1}^N b_n \left\{ \cos(2n+1)\alpha \right. \\ & \left. - \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n+1} \cos(2n+1)\beta \right\}. \end{aligned} \quad (\text{A.8})$$

where ϕ_{rx} and ϕ_{ry} denote the velocity components in the direction of x -axis and y -axis respectively. Further with regard to ϵ , the following expansions are obtained:

$$\begin{aligned} & \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n} \cos 2n\beta \\ & = \begin{cases} \left(\frac{\epsilon}{2} \right)^2 \left(-1 + \epsilon \sin \alpha + \frac{3}{4} \epsilon^2 \cos 2\alpha \right) + 0(\epsilon^5), \\ \quad \text{for } n=1 \\ \left(\frac{\epsilon}{2} \right)^4 + 0(\epsilon^5), \quad \text{for } n=2, \end{cases} \\ & \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n+1} \sin(2n+1)\beta \\ & = \left(\frac{\epsilon}{2} \right)^3 \left(-1 + \frac{3}{2} \epsilon \sin \alpha \right) + 0(\epsilon^5), \\ & \quad \text{for } n=1, \\ & \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n} \sin 2n\beta \\ & = \begin{cases} \left(\frac{\epsilon}{2} \right)^2 \left(\epsilon \cos \alpha - \frac{3}{4} \epsilon^2 \sin 2\alpha \right) + 0(\epsilon^5), \\ \quad \text{for } n=1 \\ 0(\epsilon^5), \quad \text{for } n=2, \end{cases} \\ & \left(\frac{\cos \beta}{\cos \alpha} \right)^{2n+1} \cos(2n+1)\beta \\ & = -3 \left(\frac{\epsilon}{2} \right)^4 \cos \alpha + 0(\epsilon^5), \quad \text{for } n=1. \end{aligned}$$

Finally the velocity components on the surface of C_1 is expressed as:

$$\begin{aligned} \frac{1}{V} \phi_{rx} = & \left(1 + \frac{\epsilon^2}{4} \right) (\cos 2\alpha - 1) + \frac{\epsilon^3}{4} (\sin \alpha - \sin 3\alpha) \\ & + \frac{\epsilon^4}{16} (4 \cos 2\alpha - 3 \cos 4\alpha - 1). \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} \frac{1}{V} \phi_{ry} = & \left(1 + \frac{\epsilon^2}{4} \right) \sin 2\alpha + \frac{\epsilon^3}{4} (\cos \alpha + \cos 3\alpha) \\ & - \frac{\epsilon^4}{16} (2 \sin 2\alpha + 3 \sin 4\alpha). \end{aligned} \quad (\text{A.10})$$

In the same way the velocity components on the surface C_2 is expressed as:

$$\begin{aligned} \frac{1}{V}\phi_{rx} = & \left(1 + \frac{\epsilon^2}{4}\right)(\cos 2\beta - 1) - \frac{\epsilon^3}{4}(\sin \beta - \sin 3\beta) \\ & + \frac{\epsilon^4}{16}(4 \cos 2\beta - 3 \cos 4\beta - 1), \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} \frac{1}{V}\phi_{ry} = & \left(1 + \frac{\epsilon^2}{4}\right)\sin 2\beta - \frac{\epsilon^3}{4}(\cos \beta + \cos 3\beta) \\ & - \frac{\epsilon^4}{16}(2 \sin 2\beta + 3 \sin 4\beta). \end{aligned} \quad (\text{A.12})$$

It is easily confirmed that the obtained velocity satisfies the boundary condition on the surfaces of C_1 and C_2 respectively.