Scale Effects on Wake Distribution and Viscous Pressure Resistance of Ships

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Abstract

Assuming that no appreciable separation of either two-dimensional or three-dimensional type occurs at the stern of a body or a ship, the scale effects of the boundary layer and wake have been studied with the purpose of finding the \( R_n \)-dependency of the wake distribution at propeller location. The \( R_n \)-dependency on pressure resistance has been also investigated.

It is pointed out the scale effects of the velocity distribution of the boundary layer and wake as well as the pressure resistance may be slightly different from each other and also between a two-dimensional body, a body of revolution, and a ship. The upper and lower limits of \( R_n \)-dependency are described for the wake distribution and the pressure resistance.

1. Introduction

This paper describes the scale effects of the wake distribution in the vicinity of a propeller and the pressure resistance of a ship with the assumption that there occurs no serious separation either of the two-dimensional or three-dimensional type at the stern. Here the word "wake" is used to mean the velocity defect at or near the propeller plane, as is conventionally used in naval architecture. But in some places where the more clear terminology is appropriate, the "near wake" is used. The wake at far downstream is called the far wake.

As to the wake distribution, Sasajima and the present author proposed a simple method in 1966.\(^1\) The method was derived with the assumption that the velocity distributions of the wakes of a model ship and a full scale ship are similar to each other. The result was that the velocity defect ratio is the same between the model and the full scale ship and the thickness (wideness) of the wake is proportional to the frictional resistance coefficient. The method has been successfully applied to many cases so far and has shown its usefulness for estimating the full scale wake distribution from the model wake. It is, however, also true that there remain several points to be checked in a more detailed discussion. Two of them are pointed out here. First, the derivation of the method was made assuming that the flow field is somehow two-dimensional. Second, it is assumed that the boundary layer characteristics are preserved at the location of the propeller where the boundary layer changes into the wake. The following two questions, therefore, motivated the present study. (1) How does the three-dimensionality of the flow field affect the result obtained before? (2) How do the characteristics of the far wake enter into the scaling law?

The same questions are also pertinent to the problem of the scale effects of pressure resistance. Sasajima et al. including the present author presented a paper on this problem before.\(^2\) Here a further discussion on this subject is added based on the same motivation said above.

In the following, the discussions are made by taking up a two-dimensional body and a body of revolution as the limiting cases of a ship form. To study the nature of the solution for the ship, an approach is made from the two limiting flow fields, the boundary layer and the far wake. A more detailed discussion on the characteristics of the two-dimensional boundary layers than that of the previous paper\(^3\) is also attempted. Turbulent flow without wave making phenomena is assumed throughout the paper.

2. Wake of two-dimensional bodies

First, the near wake of a two-dimensional body is discussed. To be compatible with the assumption that no appreciable separation occurs at the tail of the body, the tail configuration would
be a cusp form as shown in Fig. 1. In the figure schematical distributions of velocities in the boundary layer and wake as well as pressures on body surfaces are shown, together with the notations to be used.

2.1 If the velocity distribution in the boundary layer, which is non-dimensionalized by the velocity at the edge of the boundary layer, \( U \), and by the thickness, \( \delta \), is independent of the Reynolds number, \( Rn \), as was done in the literature 1, it is written as

\[
\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad (1)
\]

where \( f \) is a function.

In the far wake, the velocity distribution is assumed to be similar by similar non-dimensionalization, i.e.,

\[
\frac{U-u}{(U-u)_{\text{max}}} = g\left(\frac{y}{\delta}\right) \quad (2)
\]

where \( g \) is another function. In the far wake the velocity defect is usually a quantity to be considered similar.

Now the concern is centered on how the relative thickness of the boundary layer (and wake), \( \delta/L \), and the velocity defect ratio, \( (U-u)/U \), behave as \( Rn \) changes. Here \( L \) is the length of the body. To know this, first the characteristics of the boundary layer and far wake are investigated. Then the characteristics of the near wake could be estimated by interpolating those of the boundary layer and the far wake.

For the boundary layer

\[
\frac{\delta}{L} \propto \frac{\delta^*}{L} \propto \frac{\theta}{\delta} \propto C_F \quad (3)
\]

where \( \delta^* \) is the displacement thickness, \( \theta \) the momentum thickness, and \( C_F \) the frictional resistance coefficient of the body. \( C_F \) can be replaced by \( C_{Fr} \), the value of \( C_F \) for the corresponding flat plate, or by \( C_F \), the viscous resistance coefficient, because the difference between them is of second order and does not affect the essential part of the following discussion. The relation (3) was obtained in literature 1.

In the far wake

\[
\delta \propto (\beta xd C_F)^{1/2} \quad (4)
\]

\[
\frac{(U-u)_{\text{max}}}{U} \propto \frac{(d C_F)^{1/2}}{\beta x} \quad (5)
\]

where \( \beta \) is a constant, \( d \) the breadth of the body, and \( x \) the distance in downstream direction from the tail end.

Hence

\[
\frac{\delta}{L} \propto \sqrt{C_F} \quad (6)
\]

\[
\frac{(U-u)_{\text{max}}}{U} \propto \sqrt{C_F} \quad (7)
\]

When the body is a flat plate, the flow field immediately after the end of the plate is still under strong effect of surface shear stress, \( \tau_0 \), in the neighborhood of the end. So, the characteristics of the near wake are rather close to those of the boundary layer than the far wake. Hence \( \delta/L \propto C_F \) will be a good approximation. But, at the tail of bodies with fullness, the roll of \( \tau_0 \) to drag the boundary layer gradually decreases, so that the nature of the flow at the near wake is probably more and more influenced by the far wake. From this reasoning, the following law is conceivable.

The strongest and weakest limits of scale effects for the wake distribution near the tail are given by the following. (i) The relative thickness, \( \delta/L \), of the wake is proportional to \( C_F \propto C_F^{1/2} \). (ii) The velocity defect ratio, \( (U-u)/U \), at same non-dimensional distance, \( y/\delta \), is proportional to \( C_F \propto C_F^{1/2} \). The strongest limit corresponds to the result obtained earlier in the literature 1. If the body becomes fuller at the tail the weaker limit in \( Rn \)-dependency will be more probable. This means that the thickness of the wake is larger and the velocity distribution is fuller in this method than in the previous method, at least qualitatively.

2.2 In the above discussion the similarity of shape of the velocity distribution was assumed. It is, however, a well known fact that the shape of the boundary layer changes with \( Rn \). In this section this effects are discussed, first for the case of a flat plate flow, then for the case with pressure gradient.

The velocity distribution in the boundary layer of a flat plate is approximately expressed by the following log-law:

\[
\frac{u}{u_*} = a + \frac{1}{K} \ln \frac{u_* y}{\nu} \quad (8)
\]
where \( u_0 \) is shear velocity, \( \sqrt{\tau_0/\rho} \), and \( a \) and \( \kappa \) are both constants. To be exact this distribution should be modified in the sublayer and at the outer part of the boundary layer. For the present purpose, however, the modification does not play an important role in the following discussion, so it is omitted.

At the edge of the boundary layer

\[ \frac{U}{u_0} = a + \frac{1}{\kappa} \ln \frac{u_0 \delta}{\nu} \]  (9)

The velocity defect law is given by

\[ \frac{U - u}{u_0} = -\frac{1}{\kappa} \ln \frac{y}{\delta} \]  (10)

while, for the flat plate

\[ \frac{\partial \theta}{\partial x} = \frac{u_0^2}{U} \]  (11)

where \( x \) in this equation is the downstream distance from the leading edge of the plate.

Using eq. (8), \( \theta \) is written as

\[ \theta = \delta \frac{u_0}{U} \left( 1 - 2 \frac{u_0}{\kappa U} \right) \]  (12)

From eq. (9)

\[ \delta = \frac{y}{u_0} e^{\left( \frac{u_0}{\kappa U} - 1 \right)} \]  (13)

Using eqs. (11), (12), and (13)

\[ \frac{U x}{\nu} = \kappa \left( \frac{u_0}{U} \right)^2 \left[ 1 - 4 \frac{u_0}{\kappa U} + 6 \frac{u_0^2}{\kappa^2 U^2} \right] \]  (14)

This gives the relation between \( u_0 \) and \( x \) and is not a new one, of course. For the present purpose, it is appropriate to further obtain the relation between \( u_0/U \) and \( C_{Fz} \) where \( C_{Fz} = \left( \frac{\int_0^x \tau_0 dx}{\frac{1}{2} \rho U^2 x} \right) \).

\( C_{Fz} \) and \( \theta \) are connected by the following equation.

\[ C_{Fz} = \frac{2 \theta}{x} \]  (15)

By the use of eqs. (12), (13), (14), and (15)

\[ C_{Fz} = \left( \frac{u_0}{U} \right)^2 \left[ 1 + 2 \frac{u_0}{\kappa U} - \cdots \right] \]  (16)

Inversely

\[ \frac{u_0}{U} = \sqrt{\frac{C_{Fz}}{2}} \left[ 1 - \frac{1}{\kappa} \sqrt{\frac{C_{Fz}}{2}} + \cdots \right] \]  (17)

Thus, as the first approximation

\[ \frac{u_0}{U} = \sqrt{\frac{C_{Fz}}{2}} = \sqrt{\frac{C_{F0}}{2}} \]  (18)

Here, the expression was obtained by approximating \( C_{Fz} \) by \( C_{F0} \), the value of \( C_{Fz} \) at \( x = L \), in the vicinity of the end of the plate.

From this and eq. (10), the following conclusion is obtained: The velocity defect ratio \((U-u)/U\) of the boundary layer at same \( y/\delta \) is proportional to \( C_{Fz}^{1/2} \) in contrast with \( C_{F0}^0 \) as obtained in 2.1.

Next, \( \delta \) is expressed by \( u_0/U \) and \( x \) using eqs. (13) and (14) as follows.

\[ \delta = \frac{C_{F0} x}{U} \left( 1 + 4 \frac{u_0}{\kappa U} - \cdots \right) \]  (19)

Approximately

\[ \frac{\delta}{x} = \frac{C_{F0}}{U} = \kappa \sqrt{\frac{C_{F0}}{2}} \]  (20)

which shows \( \delta/x \) is proportional to \( C_{F0}^{1/2} \) instead of \( C_{F0} \).

Obviously this result on the boundary layer coincides with that in the far wake. The result produces more flat, full distribution of velocity than the previous result even in the boundary layer. However, for the usual range of \( R_n \) differences between models and ships, the difference in velocity distributions obtained from two methods is not large, but rather delicate. With the ratio of about two between the \( C_{F0} \) of a model and a ship, the estimated velocity distributions of the ship from the model following the two methods cross each other with different slopes, but in an average, they do not differ much (see Fig. 3 which appears later). It may be stated, therefore, that the result obtained in the present paper does not alter appreciably the result obtained by the previous method in an average. However, in the present derivation, theoretical straightness based on the boundary layer theory is more stressed. Fig. 2 is the comparison of the measured velocity distributions between a model and a ship including the comparison between the estimated and measured values. Probably due to the fact that this particular velocity distributions are close to those of a flat plate, agreement is satisfactory in practical purpose.

![Fig. 2 Comparison of velocity distributions between the measured full scale experiment and the predicted distribution from model experiments (Hampton Maru, SR 107)](image-url)
It is also to be noted that the present derivation matches the discussion of the far wake. Because the wake behavior is determined by the initial condition, which is given by the boundary layer upstream of the wake, the fact that the same \( Rn \)-dependency is obtained between the boundary layer and the far wake is an indication of consistency of the theories for the two fields.

To be more accurate, the case with pressure gradient should be discussed. If this is done rigorously it is evident that the above mentioned approach from the two limiting cases will not be necessary. However, it is obviously very difficult in the present knowledge. So, in the following, only the case of moderate pressure gradient is discussed.

An approximate expression of \( \theta \) in the boundary layer with pressure gradient is\(^1\)

\[
\frac{\theta}{\left( \frac{U}{v} \right)^{1/4}} = \frac{a}{U^b} \int_0^x U^n dx \tag{21}
\]

where \( n, a, b \) are constants and the boundary layer is assumed to be turbulent from the nose of the body. As is easily shown, this is written as\(^3\)

\[
\frac{\theta}{L} = C_{F0} \cdot f \left( \frac{x}{L} \right) \propto C_{F0} \tag{22}
\]

To derive eq. (21), the shape factor, \( H \), is assumed to be constant, disregarding that \( H \) varies along \( x \). While, from the standpoint of \( Rn \)-dependency, the variation \( H \) with \( Rn \) is considered to be small.

Hence, \( \delta^* \), which is equal to \( H \theta \), is given by

\[
\frac{\delta^*}{L} \propto C_{F0} \tag{23}
\]

As to \( \tau_0 (=\rho u^2) \), Ludwieg—Tillmann's formula is usually written as

\[
\frac{\tau_0}{\rho U^2} = 0.246 \left( \frac{U}{v} \right)^{10-0.67H} \tag{24}
\]

Since the change of \( H \) with \( Rn \) is small, it is known that the \( Rn \)-dependency of \( \tau_0 \) comes from the term \( U \theta U \). The combination of eqs. (21), (22), and (24), with the notice of that the number \(-0.268 \) in eq. (24) essentially corresponds to \( 1/\mu \) in eq. (21), yields the \( Rn \)-dependency of \( \tau_0 \) to be the same as the flat plate. Thus \( (u_\infty/U)^2 \propto C_{F0} \) in respect of scale effects.

With this preparation, the scale effects \( \delta \) and \( (U-u)/U \) in the flow with pressure gradient are found as follows. According to the present knowledge, the velocity defect law is expressed as\(^3\)

\[
\frac{U-u}{u_\infty} = f \left( \frac{y}{\delta}, I \right) \tag{25}
\]

where \( I \) is the parameter given by

\[
I = \int_0^x \left( \frac{U-u}{u_\infty} \right)^2 \frac{y}{\delta} \int_0^x \left( \frac{U-u}{u_\infty} \right) \frac{y}{\delta} \tag{26}
\]

\( I \) depends only on the pressure gradient along the streamline and does not change according to \( Rn \). This implies the scale effect of the velocity defect ratio in the flow with pressure gradient is the same as that of a flat plate.

As to \( \delta \), using

\[
\frac{\delta}{L} = \frac{U}{U} \int_0^x \left( \frac{U-u}{u_\infty} \right) \frac{y}{\delta} \tag{27}
\]

with eqs. (23) and (26), it is found that the scale effect of \( \delta \) is also the same as that of a flat plate.

In summary, for the two—dimensional body

(i) \( \delta/L \) is proportional to \( C_{F1/2} \)

(ii) \( (U-u)/U \) at same \( y/\delta \) is proportional to \( C_{F1/2} \)

3. Wake of bodies of revolution

The boundary layer of a body of revolution can be divided into two according to its characteristics, i.e., that of the tail and that of the most portion except for the tail. For the latter, the nature of the boundary layer is the same as the two-dimensional case, because \( \delta \) is smaller than the radius of the body and there is no effect of the radius on the boundary layer development. For the former, however, the boundary layer characteristics will be more or less different from those of the upstream boundary layer, because the boundary layer at the tail is no more thin as compared with the radius which tends to zero.

In the far wake the theory shows that\(^3\)

\[
\frac{\delta}{L} \propto \left( \frac{\beta A x C_{F}}{L} \right)^{1/3} \propto C_{F}^{1/3} \tag{28}
\]

\[
\frac{(U-u)}{U} \propto \left( \frac{A C_{F}^{1/3}}{\beta^2 \gamma^2} \right)^{1/3} \propto C_{F}^{1/3} \tag{29}
\]

where \( \gamma \) is some area. This suggests that, for bodies of revolution, the scale effects of the wake are weaker than two—dimensional bodies both in thickness and in velocity defect.

Thus, considering the characteristics of the boundary layer and far wake, one can summarize the strongest and weakest limits of scale effects for the wake of bodies of revolution as follows:

(i) \( \delta/L \) is proportional to \( C_{F1/2}-C_{P1/2} \)

(ii) \( (U-u)/U \) at same \( y/\delta \) is proportional to \( C_{P1/2} \)

4. Discussion on ship wake

The flow field near the stern of a ship is more complicated than a two—dimensional body or a body of revolution. However, it can be imagined that the flow characteristics of the ship will be of intermediate nature between the two bodies. For this estimation to be probable, appreciable formation of three—dimensional separation vortices has to be omitted. When the vortices develop in the boundary layer, many
changes occur in the characteristics of the boundary layer-vortex system. For example, the induced velocities of vortices will deform the growth of the boundary layer. Separated vortices have tendency to shoot out into the outer potential field when their circulations are strong. This causes the change of the structure of flow field at the stern. Thus, tentatively in this paper, these phenomena are all neglected.

Then the remaining points to be considered will be the existence of the cross flows in the boundary layer and the change of convergence rate of flow in the girthwise direction of ships. It may be further assumed that the effects of the cross flows will not be large, provided that the magnitude of the cross flows is small. With this assumption, it is conjectured that the ship boundary layer has the characteristics of intermediate nature between the two-dimensional body and the body of revolution.

Thus, the wake distribution of a ship is given inside the following limits.

(i) $\delta/L$ is proportional to $C_F^{1/2} - C_F^{1/3}$

(ii) $(U-u)/U$ at same $y/\delta$ is proportional to $C_F^{1/2} - C_F^{1/3}$

In both relations, the first quantity $C_F^{1/2}$ corresponds to the two-dimensional case and the second one $C_F^{1/3}$ does to the body of revolution. It is still difficult to determine how locally, and also how grossly, the two-dimensional- or body-of-revolution-like nature is inherent in the near wake. The existence of the strongest and weakest limits of scale effects on ship wakes is only pointed out here. To look at the differences between the velocity distributions for a ship predicted from the velocity distribution of a model by different methods, three curves are written in Fig. 3: the previous method (line 1 in the figure), the strongest scale effect method (line 2), and the weakest scale effect method (line 3). Possible zone of velocity distribution for a ship is indicated in the figure. Here the potential wake is assumed constant for the sake of brevity. The ship wake of line 1 is written by contracting the model wake in the direction of thickness in the ratio of frictional resistance coefficients of the ship and the model, which is assumed to be 0.5, following the previous method. The lines 2 and 3 are obtained by contracting the model wake both in thickness and in velocity defect following the discussion developed above. These methods of contraction express that the contraction is, so to speak, a diagonally contracting type instead of a horizontally contracting type of line 1.

To check the present result for the case of ship form, the comparison is attempted between the measured velocity distributions and the predicted ones. Fig. 4 shows the comparison for the result of SR 107 and Fig. 5 is the similar comparison for the data of Namimatsu et al.

Fig. 3 Illustrative figure of comparison between ship wakes predicted by three different methods from a model wake

[Diagram of wake distribution showing lines 1, 2, and 3 with key annotations and legend.

Fig. 4 Comparison of velocity distributions between the measured full scale experiment and the predicted distributions from model experiments (Nizuru Maur, SR 107)
In both figures, the measured distributions are extrapolated to the full scale values following the concept of the present paper. In the extrapolation only the case of CF1/2, the strongest Rn-dependency, is attempted, because for these particular data, this gives slightly better agreement with the experiments. As is shown in the figure it is still difficult to judge the result if this explains the scale effects of wakes of ships correctly. Probably several other factors may influence the similarity and these are the items to be studied further.

5. Pressure resistance of two-dimensional bodies

The pressure distribution on the surface of a body in viscous flow is calculated usually based on the concept that it is the same as that in the potential flow around the new body which consists of the original surface plus the displacement thickness. For the purpose of more precise discussion, it is necessary to consider the change of pressure inside the boundary layer along the normal to the surface. According to a simplified study on this effect,2),8) however, the characteristics of scale effects of pressure resistance seem to be unchanged even when the change of pressure inside the boundary layer is considered. So, in this report, only the effect of the displacement thickness is considered.

The pressure difference between the surface potential pressures on the new body and the original body is denoted by $\Delta p$. Slopes of the surfaces of both bodies are assumed to be small. Then $\Delta p$ can be easily calculated by a linearized concept and the following approximate relation is derived.

$$\frac{\Delta p}{\frac{1}{2} \rho U^2} \propto \text{slope of } \frac{\delta}{L} \propto \frac{\delta}{L} \propto C_F$$  \hspace{1cm} (30)

It is noted that the replacement of the second term by the third term is only permitted in respect of the $R_n$-dependency. Thus it is concluded that the pressure resistance coefficient, which is given by the integration of $\Delta p/\frac{1}{2} \rho U^2$ at the stern, is proportional to $C_F$ in the first approximation.

6. Pressure resistance of bodies of revolution

For the bodies of revolution, the relation between the pressure change and the body surface slopes is the same as for the two-dimensional bodies. However, the relation between the surface slope and the Reynolds number is expected to be different, because the manner of development of the boundary layer of the bodies of revolution is different from the two-dimensional case at the stern.

For a portion of the body whose radius is large as compared with the thickness of the boundary layer, the characteristics of the boundary layer are practically the same as the two-dimensional case. Thus the pressure change caused by the addition of the displacement thickness is the same as the two-dimensional bodies.

While, in the far wake, the total resistance, $R$, is expressed by

$$R = \rho n \delta^* U^2$$  \hspace{1cm} (31)

whence

$$\frac{\delta^*}{L} = \sqrt{\frac{C_F S}{2 \pi L^3}} \propto \sqrt{C_F}$$  \hspace{1cm} (32)

where the approximation that the total resistance is nearly equal to the frictional resistance is used.

Since the boundary layer except for the tail behaves like the two-dimensional one, as explained earlier, eq. (32) suggests that $\delta^*/L$ near the tail gradually changes its form from the two-dimensional nature of being proportional to $C_F$ to the bodies-of-revolution-type of being proportional to $C_F^{1/2}$. From this it is anticipated that the pressure resistance of the bodies of revolution has probably the following upper and lower limits: the pressure resistance coefficient is proportional to $C_F \sim C_F^{1/2}$. In other words, for the bodies of revolution the pressure form factor, $K_F$, has the possibility to change with $R_n$, where

$$K_F = \frac{C_F}{C_F^0}$$  \hspace{1cm} (33)

$$C_F = \frac{\text{pressure resistance/} \frac{1}{2} \rho U^2 S}$$
S is the surface area
More discussions on this point are made in the next section.

7. Discussion on pressure and total resistance of ships

In a similar manner as for the wake distribution, the pressure resistance of ships will be affected by the fullness of the stern as well as the shape of framelines. When the shape of framelines is strongly different from that of the bodies of revolution, it is necessary to consider the effect of three-dimensional separation vortices on pressure resistance. However, if no appreciable separation exists at the stern, the scale effects of pressure resistance will be explained as follows referring to the results obtained in the previous sections.

When the form of ship stern is close to a two-dimensional body the pressure resistance is proportional to $C_F$. While, if the ship itself is very similar to the body of revolution, the pressure resistance has the tendency to have a term to be proportional to $C_F^{1/2}$. It is, however, difficult to describe more specifically the relation between ship forms and pressure resistance characteristics at this stage. In addition, ships are in any case three-dimensional bodies even when their local stern forms are of two-dimensional nature. So, this may suggest that the pressure resistance of ships may be more likely body-of-revolution-type.

Thus the pressure resistance coefficient will be of the nature proportional to $C_F \sim C_F^{1/2}$. Writing the pressure resistance coefficient in the form of the pressure form factor, $K_p$,

$$K_p = a + \frac{b}{\sqrt{C_F}}$$  \hspace{1cm} (34)

$a$ and $b$ are functions of ship form.

Now a short comment is added on the total resistance of ships. If strong three-dimensional separations such as bilge vortices occur, it is anticipated that the pressure resistance will include another term in respect of the scale effects. This component is probably of the type of velocity-squared law, although the final conclusion should be reached after more complete study on bilge vortices formation and their resistance.

Hence

$$K_p = a + \frac{b}{\sqrt{C_F}} + \frac{c}{C_F}$$  \hspace{1cm} (35)

is a possible form of $K_p$, where $c$ is again a function of ship form. It is obvious that the more precise discussion is necessary to insure this relation, especially for the case of $Rn \to \infty$, but the relation will be practically admitted as the first approximation of pressure resistance of ships at the present stage.

In Fig. 6, the similarity law of total resistance of ships is shown based on the above discussions. $K_F$ in the figure is the frictional form factor. It is, in principle, affected by the occurrence of separation, but the effect is probably small, so that $K_F$ is considered to be constant. The upper and lower limits of the total resistance coefficient come from the possible variation of the limits of pressure resistance corresponding to the strongest and weakest dependency on $Rn$.

8. Conclusion

1. Assuming that no appreciable separation of

"Smooth surface is assumed. Edge effects and wire effects (slenderness effects) are not considered."
either two-dimensional or three-dimensional type occurs at the stern of a body or a ship, the scale effects of the boundary layer and wake have been discussed with the purpose of finding the $Rn$-dependency of the wake distribution at propeller location. The $Rn$-dependency on pressure resistance has been also investigated.

2. For two-dimensional bodies, the near wake behaves as follows:
\[
\frac{U-u}{U} \text{ at same } \frac{y}{\delta} \propto C_F^{1/2}
\]

3. For the bodies of revolution
\[
\frac{\delta}{L} \propto (C_F^{1/3} \sim C_F^{1/3})
\]
\[
\frac{U-u}{U} \text{ at same } \frac{y}{\delta} \propto (C_F^{1/3} \sim C_F^{1/3})
\]

4. For ships, the strongest and weakest limits of $Rn$-dependency of wake characteristics will be the same as those of 3, the bodies of revolution.

5. As to pressure resistance, the following results are anticipated.

For two-dimensional bodies, $C_P \propto C_F$

For the bodies of revolution and ships, $C_P \propto (C_F^{1/3})$

6. As to the form factor of ships, the following relations are anticipated.

\[K_F: \text{ independent of } Rn\]
\[K_F = a + \frac{b}{\sqrt{C_F}} + \frac{C}{C_F}\]

where $a$, $b$, and $c$ depend on ship forms. The upper and lower limits of total resistance coefficient are proposed.

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References


Addenda (November 9, 1979)

1) After my sending the advance copy of this paper to Mr. T. Nagamatsu, Nagasaki Experimental Tank, MHI, I obtained a letter from him telling that a similar approach is described in a report by H. P. Rader*. Several days later, on November 5, I received the copy of Rader’s paper through the kindness of Mr. Nagamatsu, who had obtained it from Mr. K. Tamura, deputy manager of the Institute. After having read the paper, I express here my great pleasure to find the researcher who considered a similar matter as mine and pay my high regard to him for his excellent paper.

It may be appropriate to add a little note on the contents. we hit upon the same idea to use the characteristics of 2D bodies and bodies of revolution. The utilization and development of this idea are, however, slightly different between us. He concentrates his discussion on wakes. Mean values are discussed with the purpose of practical usefulness. My paper is concerned with the velocity distribution and discussion is made from both the boundary layer and the wake. I also made the discussion from the standpoint of the upper and lower limits.


2) A part of another idea in the present paper about the effect of non-similar velocity distribution on $Rn$-dependency of scaling law was discussed in the following paper by the present author.