Examination of a 2-Equation Model of Turbulence for Calculating the Viscous Flow around Ships

Kenji Muraoka*, Member

Summary

This paper is concerned with the calculation method of viscous flow around ship based on the assumption of partially parabolic flow and the $K$-$\epsilon$ model of turbulence (Spalding 1975, Muraoka 1979). Inlet conditions for the $K$-$\epsilon$ model of turbulence are improved in comparison with the previous paper (Muraoka 1979). The validity of $K$-$\epsilon$ model of turbulence is examined through calculating the flow around the axisymmetric bodies and comparing the results with the experiment of Huang et al. (1978). The results give fine agreements with the experiment in not only velocity profile and pressure variation but also turbulent properties when it adopts reasonable conditions at inlet plane. The examination is extended to the flow around ship, and the results are that the agreement of velocity profile between calculation and experiment is better than that in the previous paper and that the turbulent properties seem to represent the phenomena. It can be believed that the 2-equation model of turbulence is more useful tool than any other 0- or 1-equation model in considering the turbulence around ships.

1. Introduction

The turbulent phenomena of the flow around ships have become important in the field of naval hydrodynamics. (Townsin 1969, Ikehata et al. 1971) For examples, there are the unstable phenomena in selfpropulsion test (Watanabe 1969), the great irregular vibrating propeller force, the unsteady propeller cavitation and so on. Hitherto, while many calculation methods for 3-dimensional viscous flow around ships have been proposed and examined their reliability from the coincidence of velocity and pressure with the experiment (Himeno and Tanaka 1976, Mori 1978, Soejima and Yamazaki 1978, and others), these have not been examined from the coincidence of turbulent properties. The reason lies partly that the detailed experiment studying turbulent properties around ship is very difficult except few examples (Ikehata et al. 1971, and others), and partly that calculation methods are little paid attention to the modelling of turbulence.

On the other hand, there are some examinations around axisymmetric bodies. Among them, Huang et al. (1978) used Cebeci and Smith model of turbulence (1974) to axisymmetric bodies assuming the thin boundary layer approximation with the displacement effect, and presented good coincidence with the detailed experiment in velocity profile and pressure variation, but not well in turbulent viscosity or turbulent properties. Next, Patel and Lee (1978) used Bradshaw et al.'s model of turbulence (1967) for bodies of revolution assuming the thick boundary layer approximation. They showed good results in velocity and shear stress profile with the elaborate work of mixing length modifications from original profile. And finally they advised to test the 2-equation model of turbulence judging from the phenomena that the length scale of turbulence $l$ changed rapidly at the stern. In addition to the Patel's recommendation, the necessity of 2-equation model of turbulence comes from the reason that turbulent model of 0- and 1-equation level are not applied to the recirculating flow or the complicated flow around the stern of body where $l$ cannot be described apriori. Among the 2-equation models of turbulence, the $K$ and $K^{n/2}/l$ (denoted $\epsilon$) model has been favoured (Launder and Spalding 1972) because the exact partial differential equation for the turbulent kinetic energy dissipation rate $\epsilon$ can be derived easily and $\epsilon$ appears directly as an unknown parameter in the partial differential...
equation for $K$.

Abdelmeguid and others with author (1978) and Muraoka (1979) used the $K$-$\varepsilon$ model of turbulence to calculate the viscous flow around ship stern assuming the partially parabolic flow concept (Spalding 1975). In these papers, the assumption for the Navier-Stokes equation was studied and proved its validity through comparing the velocity profile and pressure variation, but the $K$-$\varepsilon$ model of turbulence was not examined. About the inlet conditions for $K$ and $\varepsilon$ in these papers, it was assumed fully turbulent from the hull to outer boundary according to the concept for the internal flow. But such an assumption may not be reasonable because the turbulent region is usually confined within the boundary layer and the outer boundary is located far from the boundary layer edge even if the intermittency near the boundary layer edge is existed.

Here, in order to examine the validity of $K$-$\varepsilon$ model of turbulence for turbulent phenomena, this is concerned of improving the inlet condition for $K$ and $\varepsilon$, calculating the flow around the axisymmetric bodies and comparing the results with the detailed experiment by Huang et al. (1978). And, the examination is also extended to the flow around ship.

2. Theoretical Basis

2.1 Basic equations

The governing equations for turbulent viscous flow are described in tensor notation as follows:

Continuity Equation:

$$\frac{\partial}{\partial x_j}(\rho u_j)=0,$$

(1)

Momentum Equations:

$$\frac{\partial}{\partial x_j}(\rho u_i u_j)=-\frac{\partial P}{\partial x_i}-\frac{\partial}{\partial x_j}(\sigma_{ij}),$$

(2)

where $u_{i,j}$ ($i,j=1,2,3$) are velocity components in Cartesian coordinates $x_{i,j}$ ($i,j=1,2,3$); $P$ and $\rho$ are the pressure and density, respectively; $\sigma_{ij}$ ($i,j=1,2,3$) are the stresses in the fluid and defined by

$$\sigma_{ij}=-\mu_{eff}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right).$$

(3)

In equation (3), $\mu_{eff}$ is effective viscosity and defined by

$$\mu_{eff}=\mu_{i}+\mu_{t}=\mu_{l}+C_{D} \rho \cdot K^2/\varepsilon,$$

(4)

where $\mu_{l}$ and $\mu_{t}$ are the laminar and turbulent viscosity, respectively; $C_{D}$ is a proportionality constant; $K$ is the turbulent kinetic energy and $\varepsilon$ is its dissipation rate. The equation for $K$ and $\varepsilon$ are given as follows:

$$\frac{\partial}{\partial x_j}(\rho u_j K)=-\frac{\partial}{\partial x_j}(J_{K,j})-G_{\varepsilon}+\rho \varepsilon,$$

(5)

$$\frac{\partial}{\partial x_j}(\rho u_j \varepsilon)=-\frac{\partial}{\partial x_j}(J_{\varepsilon,j})+C_{1}G_{\varepsilon}+C_{2}^\varepsilon \varepsilon,$$

(6)

where

$$J_{K,j}=-\left(\frac{\mu_{eff}}{P_{r,K}}\right) \frac{\partial K}{\partial x_j},
$$

$$J_{\varepsilon,j}=-\left(\frac{\mu_{eff}}{P_{r,\varepsilon}}\right) \frac{\partial \varepsilon}{\partial x_j},$$

$$G_{\varepsilon}=\mu_{l}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \frac{\partial u_i}{\partial x_j}.$$ 

In these equations, $P_{r,K}$ and $P_{r,\varepsilon}$ are Prandtl/Schmidt numbers; $C_{1}$ and $C_{2}$ are proportionality constants. The values of $C_{D}$, $C_{1}$, $C_{2}$, $P_{r,K}$ and $P_{r,\varepsilon}$ are shown in Table 1. (Launer and Spalding 1972)

2.2 Equations for axisymmetric body

Here, it is assumed that the axisymmetric body moves straight with constant speed $V$. The flow around the axisymmetric body is considered two-dimensional, but the flow is assumed to be three-dimensional in order to account for the three-dimensional turbulence fluctuation. And it is adopted a partially parabolic flow concept to consider the thick boundary layer around the stern of axisymmetric body (Spalding 1975). It is taken a cylindrical polar coordinate system $0-xr\theta$ fixed to the body where $x$ denotes the axial direction and it is considered the quarter of the whole flow region cutting off by the horizontal plane, $\theta=\theta_{0}=0$ and the vertical plane, $\theta=\theta_{r}=-\pi/2$ as indicated in Fig. 1. The radius of axisymmetric body is denoted by $r=r_{s}(x)$ and the radius of outer plane.

Table 1 Proportionality constants

| $C_{D}$ | 0.09 |
| $C_{1}$ | 1.44 |
| $C_{2}$ | 1.92 |
| $P_{r,K}$ | 1.00 |
| $P_{r,\varepsilon}$ | 1.23 |
Examination of a 2-Equation Model of Turbulence for Calculating the Viscous Flow boundary is denoted by \( r = r_0 = \text{constant} \). For the sake of convenience, the coordinate system \((x, r, \theta)\) is transformed into the system \((\xi, \eta, \zeta)\) by the following equations (Muraoka 1979):

\[
\begin{align*}
\xi &= x, \\
\eta &= \frac{r - r_s}{r_o - r_s}, \quad r_s \leq r \leq r_o, \\
\zeta &= \frac{\theta - \theta_0}{\theta_v - \theta_0}, \quad \theta_0 \leq \theta \leq \theta_v.
\end{align*}
\]

Then, the following transformations are derived:

\[
\begin{align*}
\frac{\partial}{\partial x} &= \frac{\partial}{\partial \xi} \left( \frac{\partial \xi}{\partial x} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \eta}{\partial x} \right) \\
\frac{\partial}{\partial r} &= \frac{\partial}{\partial \xi} \left( \frac{\partial \xi}{\partial r} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \eta}{\partial r} \right) \\
\frac{\partial}{\partial \theta} &= \frac{\partial}{\partial \xi} \left( \frac{\partial \xi}{\partial \theta} \right) + \frac{\partial}{\partial \eta} \left( \frac{\partial \eta}{\partial \theta} \right) \\
\end{align*}
\]

where

\[
\begin{align*}
\lambda &= r_o - r_s, \quad \psi = \theta_v - \theta_0 = \pi/2, \\
F &= \left\{ \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial x} \right\}.
\end{align*}
\]

By using the partially parabolic flow concept and the transformations, the governing equations are summarized in the following form:

\[
\begin{align*}
\rho \left[ \frac{1}{r \lambda} \frac{\partial}{\partial \xi} (r \lambda u \phi) + \frac{1}{r \lambda} \frac{\partial}{\partial \eta} ((v - u) F \phi) \right] + \frac{1}{r \lambda} \frac{\partial}{\partial \xi} (\lambda \omega \phi) &= \frac{1}{r \lambda} \frac{\partial}{\partial \eta} (r \lambda \phi) + S_\phi, \\
\end{align*}
\]

where \( u, v, w \) are the velocity components in the \( x, r, \theta \)-directions; \( \phi, \Gamma_\phi, S_\phi \) denote flow variables, viscosity and additional sources or sinks of the corresponding property which is being transported.

For continuity equation:

\[
\phi = 1, \quad \Gamma_\phi = 0, \quad S_\phi = 0.
\]

For \( u \)-momentum equation:

\[
\begin{align*}
\phi &= u, \quad \Gamma_\phi = \mu_{\text{eff}}, \\
S_\phi &= -\frac{\partial P}{\partial \xi} + \frac{F \partial P}{\lambda \eta}.
\end{align*}
\]

For \( v \)-momentum equation:

\[
\begin{align*}
\phi &= v, \quad \Gamma_\phi = \mu_{\text{eff}}, \\
S_\phi &= -\frac{1}{\lambda} \frac{\partial P}{\partial \xi} + \frac{\phi u^2}{r} - \frac{2 \mu_{\text{eff}} v}{r^2} \\
&\quad + \frac{1}{\lambda} \frac{\partial}{\partial \eta} \left( \mu_{\text{eff}} \frac{\partial v}{\partial \eta} \right) + \frac{1}{r \lambda} \frac{\partial}{\partial \xi} \left( \mu_{\text{eff}} \frac{\partial w}{\partial \xi} \right) \\
&\quad + \frac{\mu_{\text{eff}}}{r \lambda} \frac{\partial v}{\partial \xi} + \frac{2 \mu_{\text{eff}}}{r \lambda \eta} \frac{\partial w}{\partial \xi} \\
&\quad - \frac{1}{r \lambda} \frac{\partial}{\partial \eta} \left( \mu_{\text{eff}} \frac{\partial \phi}{\partial \eta} \right).
\end{align*}
\]

For \( w \)-momentum equation:

\[
\begin{align*}
\phi &= w, \quad \Gamma_\phi = \mu_{\text{eff}}, \\
S_\phi &= -\frac{1}{r} \frac{\partial P}{\partial \xi} + \frac{\phi v^2}{r} + \frac{\mu_{\text{eff}}}{r \lambda} \frac{\partial w}{\partial \xi} \\
&\quad + \frac{1}{r \lambda} \frac{\partial}{\partial \eta} \left( \mu_{\text{eff}} \frac{\partial v}{\partial \eta} \right) + \frac{1}{r \lambda} \frac{\partial}{\partial \xi} \left( \mu_{\text{eff}} \frac{\partial w}{\partial \xi} \right) \\
&\quad + \frac{\mu_{\text{eff}}}{r \lambda} \frac{\partial v}{\partial \xi} + \frac{2 \mu_{\text{eff}}}{r \lambda \eta} \frac{\partial w}{\partial \xi} \\
&\quad - \frac{1}{r \lambda} \frac{\partial}{\partial \eta} \left( \mu_{\text{eff}} \frac{\partial \phi}{\partial \eta} \right) \
\end{align*}
\]

For \( K \)-equation:

\[
\begin{align*}
\phi &= K, \quad \Gamma_\phi = \mu_{\text{eff}}, \\
S_\phi &= G_e - \rho e, \\
S_\phi &= G_e [K - G_e \mu_e]/K,
\end{align*}
\]

where

\[
G_e = \mu \left[ \frac{1}{r^2} \left( \frac{\partial v}{\partial \eta} \right)^2 + \left( \frac{1}{r} \frac{\partial u}{\partial \eta} + \frac{\partial v}{\partial r} \right)^2 \right] \\
+ \left( \frac{1}{r} \frac{\partial u}{\partial \eta} - \frac{w}{r} \right)^2 + \left( \frac{1}{r} \frac{\partial u}{\partial \eta} \right)^2
\]

2.3 Boundary conditions

The flow field in interest is surrounded by hull surface, outer boundary, horizontal and vertical plane, inlet plane and exit plane (see Fig. 1). In order to solve the basic equations, the conditions at these boundaries are defined as follows:

(1) Hull surface

\( u=v=w=K=e=0, \) at \( r=r_s \),

and near the surface the 'wall function method' (Patankar and Spalding 1972) is applied to account for the damping effect of the wall on turbulence.

(2) Outer boundary

The flow velocities \( u, v, w \) and the pressure \( P \) at the outer boundary are obtained from the calculation of the potential flow around the body, and

\[
\frac{\partial K}{\partial r} = \frac{\partial e}{\partial r} = 0, \quad \text{at } r = r_0.
\]

(3) Horizontal and vertical plane

\( w=0, \)

\[
\frac{\partial u}{\partial \eta} - \frac{\partial v}{\partial \eta} - \frac{\partial K}{\partial \theta} = 0, \quad \text{at } \theta = \theta_v, \theta_v.
\]

(4) Inlet plane

It is assumed that the boundary layer is uniform around the body and the boundary layer thickness \( \delta \) is calculated by Schlichting’s formula for flat plate:
\[ \delta = 0.37 \left( \frac{V \cdot \delta}{\nu} \right)^{-1/5}, \]

where \( V \) is uniform velocity; \( \nu \) is kinematic viscosity; \( \delta \) is the length from the leading edge. The velocity components \( u, v \) and \( w \) are as follows:

\[
\begin{align*}
    u &= \begin{cases} 
        V \left( r - r_s \right) / \delta^{1/3} & \text{at } r - r_s \leq \delta \\
        V & \text{at } r - r_s > \delta 
    \end{cases}, \\
    v &= w = 0.
\end{align*}
\]

As the turbulent properties seem to be confined within the boundary layer, these are treated separately inside and outside of the boundary layer thickness \( \delta \) which is determined above. The turbulent properties \( K \) and \( \epsilon \) are assumed as follows:

For \( K \):

\[
K \left\{ \begin{array}{l}
    = f(r-r_s) \cdot V^2 & \text{at } r - r_s \leq \delta \\
    = 0 & \text{at } r - r_s > \delta,
\end{array} \right.
\]

where the function \( f(r-r_s) \) is determined from the experimental data of Klebanoff for flat plate. (For example, Rotta 1972)

For \( \epsilon \):

The mixing length \( l_m \) is determined using the value \( g(r-r_s) \) which Bradshaw et al. (1967) have given for thin boundary layer.

\[
l_m = \left\{ \begin{array}{l}
    g(r-r_s) & \text{at } r - r_s \leq 1.2 \delta \\
    g(1.2 \delta) & \text{at } r - r_s > 1.2 \delta.
\end{array} \right.
\]

The value of \( \epsilon \) is determined from the mixing length \( l_m \) as follows:

\[
\epsilon = \frac{C_p \delta^{1/4} \cdot K^{3/2}}{l_m}.
\]

(5) Exit plane

The pressure \( P \) is obtained from the calculation of the potential flow.

2.4 Calculation procedure

The calculation procedure has mentioned in detail in the previous paper (Muraoka 1979), here it is described in brief.

First, the finite difference algebraic equations are obtained by integrating the governing partial differential equation over the many small subdivisions of the flow field in interest, which are formed of the intersecting grid lines along directions parallel to \( \xi \), \( \eta \)- and \( \zeta \)-direction. The unknowns in the finite difference equations are three velocities \( u, v, \) and \( w \), the pressure \( P \), the turbulent properties \( K \) and \( \epsilon \). The pressure is stored as three dimensional array and the other variables are stored as two dimensional arrays at every \( r-\theta \) plane. At the start of the calculation, the pressure field is guessed from the potential flow solution. The three velocities are solved at any \( r-\theta \) plane by using the guessed value of the pressure and evaluating the coefficients of the finite difference equations on the basis of flow properties at the immediate upstream plane. Since the three momentum equations are coupled mutually through the unknown pressure field, it is necessary to employ iterative procedure at any \( r-\theta \) plane for their solution in which the pressure field is guessed and corrected so as to bring the velocities in conformity with the continuity equation. In each iteration, the previous solutions of the velocities and pressure are used as the values of them in the coefficient of finite difference equations. And the iterations are repeated until the continuity errors are practically zero. Next, the equation for \( K \) and \( \epsilon \) are solved so as to provide \( K-\epsilon \) distributions appropriate to next downstream plane. Then, the same manner is repeated at every downstream plane in the flow field in interest. Reaching the last downstream plane, one marching is completed. The marchings are performed many times, and the calculation is terminated when the pressure field does not alter comparing the previous solution.

3. Calculation of Axisymmetric Bodies

By using the method in the previous section, calculations are performed for two axisymmetric bodies which are named Afterbody 1 and 2 by Huang et al. (1978). The length \( L \) of the body is 3.066 m and the maximum radius \( r_{max} \) is 0.1397 m. The stern configuration of two bodies are shown in Fig. 2. The radius of outer boundary \( r_s \) is twice as large as maximum radius of the body. The grids at \( r-\theta \) plane are 12 \times 12 and fine near the hull surface, whose configuration is shown in Fig. 3. The grids along \( x \)-direction are 30 whose length is every 0.02 \( L \). The inlet plane is located at 0.5 \( L \) from the leading edge and the exit plane is located at 0.1 \( L \) behind the body. The marchings from upstream to downstream are performed 40 times. Calculations are performed under the uniform velocity \( V = 30.48 \text{m/s} \) which is the just same as the experiment. Below, the experimental data are all quoted from Huang et al. (1978).

Fig. 2 Stern configurations of axisymmetric bodies
3.1 Calculation results

First, the effect of the inlet condition is tested through calculating the viscous flow around Afterbody 1. In the left hand side of Fig. 4, the values of $K$, $l$ and $u$ at inlet plane are shown comparing the present method with the previous one (Muraoka 1979). In the previous method, $K$ is $0.004V^2$ and $\varepsilon$ is determined from the Escudier’s distribution of mixing length. In the right hand side of Fig. 4, the numerical results at $x/L=0.846$ are also shown. The present method gives more reasonable results in turbulent properties $K$ and $l$ than the previous method. The velocity profile of the present method is closer to the experiment.

Next, the reliability of the $K$-$\varepsilon$ model of turbulence is examined by calculating the stern flow around Afterbody 1 and 2. The calculated and measured velocities, $u$ and $v$ are shown in Fig. 5. The results are good coincidences with the experiment except those at $x/L=0.934$. The numerical results represent well the difference of two bodies. In Fig. 6, the pressure variations in the flow field in interest are shown comparing with the measured ones and the numerical results of the inviscid flow. The viscous effect on pressure is appeared only near the vicinity of hull surface around the stern region and the results of present method follow well, but the result of $x/L=1.057$ is not good because the exit plane may be close behind the body. Fig. 7 shows comparison of turbulent properties, the turbulent kinetic energy $K$ and the length scale of turbulence $l$, determined by $C_DK^{3/2}/\varepsilon$. The measured $K$ is determined by the definition $K=1/2(u'^2+v'^2+w'^2)$ where $u'$, $v'$ and $w'$ are the velocity fluctuations in $x$-, $r$- and $\theta$-directions. The measured $l$ is determined by $l=\nu/K^{1/2}$. The coincidences of $K$ and $l$ between calculation and experiment are fairly good, but the coincidence of $K$’s distribution behind the ship, at $x/L=1.057$ is slight different. The calculation represents almost the difference of two axisymmetric bodies.
Though it is assumed three-dimensionality in this calculation, the numerical results of $w$ are approximately zero and the numerical values of velocity, pressure and turbulent properties are not changed in $\theta$-direction. Namely, the numerical results show two-dimensionality.

### 3.2 Comparison of turbulent viscosity

Here, the comparison of turbulent kinematic viscosity $\nu_t$ around the axisymmetric bodies are shown in Fig. 8 and 9. In these figures, the values of $U_f$, $\delta_p^*$ and $\delta_r$ are all derived from the experiment of Huang et al. The numerical results of Huang et al.'s method are coincident with one line by non-dimensionalizing because of 0-equation model of turbulence, but the experimental results are not coincident with one line and diminish their values towards the stern. The numerical results of the present method follow well the variation of $\nu_t$ around the stern and present fine agreement with the experiment. Fig. 10 shows comparison of velocity profiles. There is little difference of numerical results, but the present method seems to be closer to the experiment.

Under the above examination, it is concluded that if the distribution of length scale of turbulence is not estimated apriori in the flow around the ship stern, the 2-equation model of turbulence is good choice among many turbulent models for providing turbulent viscosity. And, it can be said that the higher level the turbulent model is, the closer to the phenomena the result is.

### 4. Application to Ship Form

The application of the improved inlet condition gives fine results of the flow around axisymmetric bodies, in not only velocity and pressure but also turbulent properties. Here the present method is also applied to the ship
model of Model B (Muraoka 1979) which is a liner model and the same as the one used in SR138. The length L of the model is 6 m and the block coefficient $C_B$ is 0.572. The calculation method is the same as the previous method except the value at inlet plane. The grids at $r-\theta$ plane are $12 \times 12$ and those near the hull surface are twice coarser than those of axisymmetric bodies. The grids along main flow direction are 30. Fig. 11 shows the numerical results along two diagonal lines in the flow at St. 1/2, where the circles show the numerical values at grids nodes. The present result is closer to the experiment. It is assumed that the turbulent properties $K$, $l$ and turbulent kinematic viscosity $\nu_t$ of the present method are more reasonable than those of the previous method because the turbulent phenomena are restricted almost within the boundary layer. But, it is very pitty that there are no experimental data of turbulent properties. It is desired to study the turbulence in the flow around ship stern in more detail and prompt the calculation taking into account the turbulence.

5. Concluding Remarks

By the present examination of the axisymmetric bodies and ship, the following remarks can be derived:

(1) Through improving the inlet conditions for turbulent properties $K$ and $\epsilon$, the numerical results of turbulent properties are much improved and are well coincident with the experiment.

(2) The improvement of the coincidence in the turbulent properties provides the improvement of the agreement in velocity and pressure distribution.

(3) The $K-\epsilon$ model of turbulence provides better coincidence in turbulent viscosity with the experiment than 0-equation model of turbulence, and is assumed to be very effective in considering the turbulence in the flow around ship stern.

As the future work, the detailed experiment of studying the turbulent properties around the ship stern is desired.

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Reference


