A Numerical Solution Method for Nonlinear Shallow Water Waves

(Second Report)

by Takanori Hino*, *Member* Hideaki Miyata**, *Member*
Hisashi Kajitani**, *Member* Makoto Kanai**

Summary

The two-dimensional finite-difference method TUMMAC-III developed in the first report is applied to three problems of steep regular waves interacting with obstacles, i.e., run-up on a beach, deformation and breaking on a sandbar and diffraction by a rectangular floating body. Experiments are undertaken for these problems and the wave profiles and pressures on the body are compared with the computation. The agreement is satisfactory as far as the waves before breaking are concerned.

1. Introduction

A new numerical solution method for nonlinear shallow water wave problems was developed in the first report¹⁾. It was a two-dimensional version of the TUMMAC method and employed the donor-cell method in the finite-difference representation of the convective terms, the simultaneous iterative method in the solution of the Poisson equation for the pressure and the arbitrary body boundary condition. Some computed results of a solitary wave approaching a beach and walls were presented.

Recently some new numerical techniques have been developed and applied to practical problems of ocean engineering or naval hydrodynamics. For example, the finite-difference methods were applied to ship-motion problems²⁾³⁾. These methods adopt a too simple treatment of the exact nonlinear free surface condition and their applications are confined within restricted problems. The other example is a boundary element method also applied to a ship-motion problem⁴⁾. This method is surely useful to two-dimensional problems, but the flexibility of the technique is presumably not superior to the finite-difference method. Actually, Vinje⁴⁾ expresses the difficulty in dealing with the open boundary condition and in the application to a piercing body problem.

In this second report the TUMMAC-III meth-

od is applied to three problems of incoming steep regular waves interacting with various obstacles, i.e., a straight beach, a sandbar and a rectangular floating body. This can be done without difficulty simply by determining boundary cells depending on the location and the configuration of the obstacles. For the present method the difference between piercing and submerged body problems is rather small.

Steep regular waves are generated at the inflow boundary by sinusoidally oscillating the velocities as described in the first report. Therefore, the waves considered herein are confined within regular ones, although they are considerably deformed by the shallow water effect. However, the application to irregular wave problems can be easily achieved simply by giving irregular velocity fluctuation at the inflow boundary, as tried by Bourianoff³⁾. This is another advantage of the present method. The verification of the availability of the present method through the problems of very steep regular waves will promise us its availability to irregular wave problems.

Experiments of the three problems of shallow water waves interacting with obstacles were conducted at the Towing Tank of the University of Tokyo, whose dimensions were 86 m (length) $\times$ 3.5 m (width) $\times$ 2.4 m (depth). A wave-maker of flap-type was located at the end of the tank and a channel whose length and inner width were 2.4 m and 0.54 m, respectively, was located about 5 m apart from the wave-maker. The two-dimensional obstacles, i.e., a beach, a sandbar and a rectangular floating body were installed in the channel, as seen in Fig. 1. The walls of

* Graduate School, Department of Naval Architecture, The University of Tokyo

** Department of Naval Architecture, The University of Tokyo

Fig. 1 Setup of the experimental apparatus

the channel were made of transparent plastics for clear observation.

In the following chapters computed and experimental results are described, compared and discussed for the three problems of nonlinear behavior of shallow water waves. The conditions of computation and experiment are as listed in Table 1 and 2, respectively. At least one of the two conditions of experiment in Table 2 coincides with that of computation listed in Table 1.

The variables and symbols are as defined in the nomenclature of the first report, unless otherwise defined.

Table 1 Conditions of computation

	beach		sandbar	floating body	
	(a)	(b)	(a)	(a)	(b)
x (m)	10.00	2.5	2.7	2.00	2.00
d ₀ (m)	1.00	0.20	0.40	0.40	0.40
DX (mm)	1250	25.0	50.0	200	200
DZ (mm)	250	5.0	20.0	10.0	10.0
DT (sec)	0.01	0.01	0.01	0.005	0.005
λ (m)	4.0	1.0	1.0	1.0	0.6
h ₀ (m)	0.20	0.03	0.05	0.03	0.03
no. of marker in a cell	4	4	1	4	4
without relocation	after 401-th step	after 201-th step	non	non	non

Table 2 Conditions of experiment

	beach		sandbar		floating body	
	(A)	(B)	(A)	(B)	(A)	(B)
d ₀ (m)	0.2	0.2	0.4	0.4	0.4	0.4
λ (m)	1.0	1.0	1.0	1.0	1.0	0.6
h ₀ (m)	0.03	0.05	0.05	0.03	0.03	0.03

2. Run-up of steep waves on a beach

2.1 Condition of computation

Regular sinusoidal waves are generated at the inflow boundary by oscillating the fluid velocities according to Eqs. (13) and (14) of the first report. The slope of the beach is 1/5. The relaxation factor ω for the solution of the Poisson equation is 0.8, and the convergence of the pressure computation is assumed to be achieved when the difference of ϕ between the two iterative steps is less than $0.001 \text{ m}^2/\text{sec}^2$.

Two conditions under which computation is undertaken are listed in Table 1. The condition (b) coincides with the experimental condition (A) in Table 2. Four markers are used for each

Fig. 2 Computed velocity field of waves on a beach of 1/5 slope, $\lambda=4.0 \text{ m}$, $h_0=0.20 \text{ m}$, $d_0=1.0 \text{ m}$, vertical exaggeration is 2.49 : 1

Fig. 3 Details of deformation of waves on a beach (as Fig. 2), vertical exaggeration is 1.40 : 1

free surface cell, and they are ceased to be relocated after the 401-th step (case (a)) and the 201-th step (case (b)), respectively.

2.2 Computed results

Computed wave profile and velocity field are shown in Fig. 2. By the shallow water effect waves become steep at the crest and gentle at the trough. The wave that reaches to the shoreline has a gentle slope and great velocities to climb up the beach. The wave energy is concentrated near the free surface there. The details of wave deformation are shown in Fig. 3. The wave that runs up the beach loses its dynamic energy and is pulled down by the gravity and the velocities of the succeeding trough. And then the next wave approaching the beach, which is deformed into a profile similar to a solitary wave, has a steep forward face that is liable to break.

Velocity fields of another case are present in Fig. 4. On the shoreline the water climbs the beach at the 280-th time step and comes down at the 310-th step and finally the velocities of the wave crest collide with these downward velocities about 0.2 m ahead of the shoreline at the 340-th step. Consequently great vertical velocities are produced and they steepen and break the wave. The details of the wave profile are shown in Fig. 5. The wave profile immediately before breaking is obvious. The wave breaks, unlike a plunging breaker, with the water velocities in the direction normal to the slope of the beach.

2.3 Description of experiment

The wave-maker of flap-type is located at the end of the towing tank whose depth is 2.3 m. The immersion of the flap-plate in the water is 1.0 m. As the channel in which a two-dimensional beach model is installed is located about 5 m

Fig. 4 Computed velocity field of waves on a beach of 1/5 slope, $\lambda=1.0$ m, $h_0=0.03$ m, $d_0=0.20$ m, vertical exaggeration is 3.34 : 1

apart from the wave-maker, the waves do not suffer from shallow water effect until they are influenced by the bottom plate of the channel. Besides, the length of the channel may not be

Fig. 5 Details of deformation of waves on a beach (as Fig. 4), vertical exaggeration is 2.09 : 1

enough to complete the shallow water deformation. These are the respects that differ from the computational condition.

The time evolution of waves is recorded by a video-camera with a timer, and analysed from video-tape recording. A 35 mm camera is also used to clearly record the appearance of the non-linear wave motion.

2.4 Experimental results

Photographs of waves in the course of breaking are shown in Fig. 6. The time interval between the three pictures is about 0.1 sec. The waves propagate from right and they begin to break at the time of the middle picture. This wave deformation before breaking is in the same manner with the computation shown in Fig. 5.

Wave profiles near the beach analysed from

Fig. 6 Breaking and run-up of waves on a beach, $\lambda=1.0$ m, $h_0=0.03$ m, $d_0=0.2$ m

video-tape recording are present in Figs. 7 and 8. The condition of Fig. 7 is the same with that of Figs. 4, 5 and 6, and the three profiles at $T=20.45$, 20.55 and 20.65 sec. in Fig. 7 nearly accord with the three pictures in Fig. 6. Although

Fig. 7 Measured wave profiles on a beach, $\lambda=1.0$ m, $h_0=0.03$ m, $d_0=0.2$ m, vertical exaggeration is 3.34 : 1

Fig. 8 Measured wave profiles on a beach, $\lambda=1.0$ m, $h_0=0.05$ m, $d_0=0.2$ m, vertical exaggeration is 3.34 : 1

some detailed deformation of waves on the beach does not precisely accord with the computation, the behaviors of waves on the beach, namely, run-up and steep deformation of the forward face of the approaching waves, are well predicted by the computation. It is a little surprising that the peculiar wave deformation immediately before breaking shown in Fig. 5 is apparently very similar to the property of the wave in the middle picture of Fig. 6.

2.5 Discussion

In this problem of waves on a beach, the water depth is gradually decreased, and therefore the number of cells, in which the pressure is computed and through which the momentum is conveyed, is decreased. This means that the computation is proceeded with a relatively coarse cell division near the shoreline. Especially, the number of cell to interpret the run-up of waves on the beach is far from enough. It is almost impossible to determine the pressure in the cells of climbing water by the usual procedure. The approximate determination of the pressure of a cell containing both free surface and body

Fig. 9 Computed wave deformation on a sandbar, $\lambda=1.0$ m, $h_0=0.05$ m, $d_0=0.4$ m, vertical exaggeration is 1.59 : 1

boundary described in 3.7 of the first report is necessary, and the qualitative agreement described above demonstrates the validity of this approximation.

It is well known that the problem of waves approaching a beach contains very complicated phenomena such as surf-beat, and so the present method of solution is not always powerful enough. However, it is very useful for the understanding of nonlinear deformation of steep waves in the vicinity of the shoreline.

3. Deformation of waves on a sandbar

3.1 Condition of computation

The wave generation is in the same way with the problem of beach. The marker particle that tells the position of the free surface is placed only one for each free surface cell, and the relocation of the marker is not stopped for the sandbar problem.

The sandbar whose length and height are 1.0 m and 0.3 m, respectively, as seen in Fig. 1 is

Fig. 10 Computed pressure distribution on a sandbar, $\lambda=1.0$ m, $h_0=0.05$ m, $d_0=0.4$ m, vertical exaggeration is 1.59 : 1

shaped by two parabolic curves, and it is located 1.0 m apart from the inflow boundary where regular steep waves are generated.

3.2 Computed results

The computed wave deformation is shown in Fig. 9. The origin of the x -coordinate is the location of the numerical wave-maker and the regular steep waves propagate from left to right. When the wave crest approaches to the top of the sandbar where the water depth is shallowest, the forward face becomes steep until $T=2.90$ sec. Then, immediately after the crest pass the shallowest point, the profile of the crest becomes triangular and breaking is supposed to take place.

The normal pressure exerted on the surface of the sandbar is illustrated in Fig. 10, which is obtained from the same computational output as Fig. 9. The pressure distribution remarkably varies depending on the position and configuration of crest and trough. Obviously a wave crest renders high (inward) pressure and a wave trough

Fig. 11 Wave deformation on a sandbar, $\lambda=1.0$ m, $h_0=0.05$ m, $d_0=0.4$ m

Fig. 12 Measured wave profiles on a sandbar, $\lambda=1.0$ m, $h_0=0.05$ m, $d_0=0.4$ m, vertical exaggeration is 1.59 : 1

low (outward) pressure. The total force on the sandbar is sometimes to the upwave direction and sometimes downwave direction. The present method does not explain the high pressure caused by wave breaking, but it can only cope with the steep wave deformation immediately before breaking. Therefore, the force due to

breaking waves that may work in the upwave direction is not included in Fig. 10.

3.3 Description of experiment

The experiment was conducted under the conditions almost same with the case of a beach except for the depth of water that is increased to 0.4 m. The conditions are listed in Table 2. The wave of the condition (A) coincides with the computation and the other condition (B) is the case of more gentle wave.

3.4 Experimental results

Photographs by a 35 mm camera are present in Fig. 11, which show the process of a wave crest passing through the shallowest position (pointed by an arrow in the center of the pictures) and breaking into spilling breaker. The waves are propagating from right to left in this figure. A vertical rod placed at the point of breaking is to press the model of sandbar which has unexpected large buoyancy.

The time evolution of measured wave profile is shown in Figs. 12 and 13. The results in Fig. 12 under the same condition with the computa-

tion agree fairly well with the computed wave profiles in Fig. 9, although as a natural consequence the burst of a steep wave shown at 19.99 sec. in Fig. 12 cannot be seen in the computation. The case of much gentle waves of equal wavelength in Fig. 13 shows similar behavior on and behind the sandbar.

3.5 Discussion

The implementation of the open boundary condition often determines the availability of a numerical solution method. The present open boundary condition is substantially same with the TUMMAC-I method. The gradient of velocities along the x -axis is set zero at the open boundary. This simple condition seems to be successful for the present propagating wave problem, for no reflection of wave can be seen and the waves around the sandbar are scarcely influenced by the downwave open boundary.

The present solution method can be easily applied to the computation of forces due to regular and irregular waves on a submerged body of arbitrary configuration. The computation of drift forces due to waves and a study on optimum configuration of a body free from external forces can be undertaken.

4. Diffraction of waves by a rectangular floating body

4.1 Condition of computation

Regular waves are generated at the inflow boundary ($x = -1$ m) and propagate from left to right. Four markers are allocated on each free surface cell and they are always relocated at each time step after the wave configuration is updated. The open boundary condition on the right end of the computational domain is that no gradient of velocities is allowed in the x -direction, which is common throughout the TUMMAC-I and III methods.

A rectangular floating body is chosen, though the present solution method can cope with arbitrary configurations. It is rigidly fixed and its motion due to external forces is not permitted.

The two waves in the conditions (a) and (b) in Table 1 coincide with the experimental conditions (A) and (B) in Table 2, respectively. The difference between the two conditions is wavelength.

4.2 Computed results

Computed time evolutions of wave profiles and velocity fields are shown in Figs. 14 and 15 for the two conditions. For about one period of wave oscillation the evolution is successively followed by five figures in each case.

On the present condition of short wavelength in shallow water, a very small portion of the wave energy is transmitted across the floating

Fig. 13 Measured wave profiles on a sandbar, $\lambda = 1.0$ m, $h_0 = 0.03$ m, $d_0 = 0.4$ m, vertical exaggeration is 1.59 : 1

Fig. 14 Computed velocity field of waves around a rectangular floating body, $\lambda=1.0$ m, $h_0=0.03$ m, $d_0=0.4$ m

Fig. 15 Computed velocity field of waves around a rectangular floating body, $\lambda=0.6$ m, $h_0=0.03$ m, $d_0=0.4$ m

Fig. 16 Computed pressure amplitude distribution on a rectangular floating body, (A) $\lambda=1.0$ m, $h_0=0.03$ m, (B) $\lambda=0.6$ m, $h_0=0.03$ m

rectangular body, but most of the waves are reflected. Especially, wave motion is scarcely noticed behind the floating body in the case of shorter wavelength in Fig. 15.

The incoming regular wave first interacts with and is reflected by the floating body, and after a while by the fictitious wave-maker at the

inflow boundary, and hence amplification and cancellation of waves occur periodically. Waves are all cancelled out at $T=3.1$ sec. of Fig. 14, and then at $T=3.5$ sec. amplification begins to take place and it is achieved at $T=3.7$ sec. This resonant phenomenon is most intense when wavelength is close to the distance between

Fig. 17 Time evolution of computed drag and lift exerted on a rectangular floating body, (A) $\lambda=1.0$ m, $h_0=0.03$ m, (B) $\lambda=0.6$ m, $h_0=0.03$ m

the wave-maker and the floating body.

The computed amplitude of pressure on the body surface due to wave motion is shown in Fig. 16. The maximum amplitude approximately reaches to the value twice as much as the static pressure of the initial incoming wave.

The time evolution of drag and lift obtained by integrating the pressure distribution on the body surface is shown in Fig. 17. The interaction between incoming and reflected waves is reached

to the nearly stable state at around $T=2$ sec. for the case (A) and $T=3$ sec. for the case (B). The drag and lift are greater for the case of longer wavelength.

4.3 Description of experiment

The wave-maker was located about 5 m apart from the rectangular floating body. And therefore, it took considerable time for the interacting waves to reach to the steady state. The wave reflected by the floating body was propagated to the wave-maker and interacted with the wave-maker and then the steady state was reached. Wave recording was conducted more than 10 seconds after the wave-maker started to move. The waves in front of the floating body were recorded by a video-camera separately from those behind the floating body. Therefore, the phase of the waves before and behind the body cannot be matched.

In addition to recording of wave profiles, pressures on the floating body were measured. The setup of the apparatus was as shown in Fig. 1. The pressures working on the holes of 2 mm in outer-diameter were transferred through vinyl tubes of small length to pressure transducers and the output of the transducers was instan-

Fig. 18 Measured wave profiles before and behind a rectangular floating body, $\lambda=1.0$ m, $h_0=0.03$ m, $d_0=0.4$ m

Fig. 19 Measured wave profiles before and behind a rectangular floating body, $\lambda=0.6$ m, $h_0=0.03$ m, $d_0=0.4$ m

Fig. 20 Measured pressure amplitude distribution on a rectangular floating body,
(A) $\lambda = 1.0$ m, $h_0 = 0.03$ m, (B) $\lambda = 0.6$ m, $h_0 = 0.03$ m

taneously digitized and stored by the transient memory. A great care was taken to expel air containment in the water in the vinyl tube. Usually air was drawn by a ejector pump beforehand.

4.4 Experimental results

Wave profiles analysed from video-tape recording are present in Figs. 18 and 19. The experimental wave profiles are more deformed than the computation, which is presumably attributed partly to the great distance that reflected waves travel and partly to the poor resolution of the video-camera.

In spite of the difference of conditions between experiment and computation, the wave profiles are approximately accord with each other. Fig. 18 shows resonant phenomena with an amplified wave profile at $T = 13.90$ sec. and a cancelled wave profile at $T = 14.10$ sec. The transmittance is very small and only waves of very low amplitude can be observed behind the floating body, as predicted by the computation.

Distributions of measured pressure amplitude are shown in Fig. 20. In general their accordance with the computed results in Fig. 16 is good, but it is noted that some error of measurement is not negligible and besides the computation underestimates the pressure on the forward face of the body. The waves of longer wavelength exert greater pressure on the body.

4.5 Discussion

As the inflow boundary which is a numerical wave-maker cannot implement the radiation condition, the waves reflected by the floating body and propagated to the upwave direction interact with the numerical wave-maker. In the computation and experiment of this chapter the reflected waves are assumed to interact with the actual or fictitious wave-maker. However, in most of the practical engineering problems waves come from and radiate to the infinity. It is very difficult to give an open boundary condition on the numerical wave-maker, and this is one of the serious shortcomings of the present

numerical solution method. This shortcoming can be remedied by enlarging the distance between the wave-maker and the floating body, although the computation must be ceased when the reflected waves reach to the wave-maker and the required large computational domain is not advantageous from the economical standpoint.

However, it is evident that the present numerical method can predict nonlinear wave motions and their forces on a floating body fairly well. The present method can be applied to problems of bodies of arbitrary configuration and of irregular waves without difficulty. Besides, in case viscous effect cannot be ignored, it can be taken into account by some modifications of the method, because the present method is a direct computation of the Navier-Stokes equations.

5. Concluding remarks

The flexibility of the TUMMAC-III method is demonstrated by the three problems. Generally the agreement with experiment is satisfactory, although this method contains some shortcomings to be removed. The most substantial shortcoming is that this method cannot include the effect of wave breaking. The great pressure due to breaking of waves and the effect of dissipated wave energy cannot be estimated. However, the wave deformation shortly before breaking can be predicted and the present method will be useful to a large number of problems. Although the application of the TUMMAC-III method is confined to shallow water problems in these reports, its application to deep water problems can be easily achieved by extending the domain of computation and giving a proper condition on the bottom boundary. When this method is extended to a three-dimensional version, its availability will be considerably raised.

The computation was carried out by HITAC M-200H/280H at the Computer Center, the University of Tokyo. This research is partly sup-

ported by the Grant-in-Aid for Co-operative Research of the Ministry of Education, Science and Culture.

References

- 1) Hino, T. et al.: A Numerical Solution Method for Nonlinear Shallow Water Waves (First Report), J. Soc. Naval Arch. Japan, Vol. 153 (1983).
 - 2) Nichols, B. D. et al.: Nonlinear Hydrodynamic Forces on Floating Bodies, Proc. 2nd. Intern. Conf. Numerical Ship Hydrodynamics (1977).
 - 3) Bourianoff, G. I. et al.: Numerical Simulation of Ship Motion by Eulerian Hydrodynamic Techniques, Proc. 2nd. Intern. Conf. Numerical Ship Hydrodynamics (1977).
 - 4) Vinje, T. et al.: A Numerical Approach to Nonlinear Ship-Motion, Proc. 14th Symp. Naval Hydrodynamics (1982).
-