A simplified method to analyse the strength of double hulled structures in collision
(2nd Report)

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Summary
The strength of ship structures in collision is an important matter in terms of saving life, preventing pollution and from an economic point of view, especially in the case of hazardous cargo carriers, oil tankers and so on.

The authors investigated the strength of double hulled structures in collision, which were typical of the side structures of hazardous cargo carriers. In the former paper, the types of collisions were classified into five groups. Two types among those five groups were examined; one was the type where a stem collided against the ship's side, and the other was where a bulbous bow collided against it.

In this paper, another type in which a bulbous bow collided against the bilge part of a ship was examined. Such a type of collision is likely to occur between fully loaded ships, which means that the two ships have the greatest kinetic energy before collision. These three types are considered as the most critical ones among those five groups. Static destruction tests were carried out using simplified and detailed large scale models of a bilge structure representative of those used in double hulled ships, where the bow model was assumed to be rigid.

A simplified method to analyse the damage of bilge structures in collision was developed on the basis of the results from the experiments. The basic concept of it is similar to the former paper; a bilge structure is considered to consist of two main structural members, that is an outer shell which acts as a membrane and a trans web which supports such a membrane.

The analysis method was examined by comparing the calculated values with the experimental ones. From these, a good correlation emerged.

1. Introduction
The strength of hazardous cargo carriers in collision plays a very important role in preventing cargo leakage and in guaranteeing the safety of the crew.

The authors presented a simplified method to analyse the damage of a ship's side with double-hull construction, when a stem or a bulb of the other ship strikes against it1). The method showed a very good accuracy.

The types of collision studied in Ref.1) were (b) and (c) in Fig.1. Type (a) should also be investigated because the collision of type (a) is likely to occur between fully loaded ships, which means that the two ships have the greatest kinetic energy before collision. Therefore, this paper deals with the collision of type (a); collision strength is investigated experimentally using ship's side models, then a simplified method to predict the collision strength of a ship's side structure is proposed.

2. Bilge structure model tests
2.1 Test models
The following conditions were assumed as in Ref.1): -
(1) The bow of the colliding ship was rigid.
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Fig. 1 Classification of collision by type

Fig. 2 The bilge structure model

Fig. 3 Details of the model B2
One ship collided against the other ship perpendicular to the side shell.

The bilge structure was damaged in the vicinity of the point of collision. It did not collapse in its entirety.

The damage was not large in scale. Only that type of damage in which the bow reached the inner hull (but which was not damaged), was considered.

The shaded part in Fig. 2 was modelled into two test models (B1 and B2). In the B1 model, stiffeners and openings in trans web plates, side stringers and side girders were ignored, where their effects were compensated by adopting equivalent thickness in the web plates. In the B2 model, the stiffeners and the openings were modelled in their own shapes. Therefore, the B1 model was more simplified than the B2 model. The details of the B2 model are shown in Fig. 3.

The material used in the construction of the models was mainly JIS SS 41 mild steel. Table 1 shows their tensile test results. CO2 welding was used in the construction. Both ends of the models were firmly fixed using rigid frames to simulate the fixity of transverse bulkheads.

The bulb part of a bow was used as a bow indenter since the bulb part meets the ship’s side first as shown in Type (a) and (b) of Fig.1. The shape of the bulb indenter was simplified as shown in Fig.4, where the radius of the circle at the foremost part and the waterline incident angle were determined according to NKK’s experience. The indenter was scaled by the same proportion as the bilge models and cast to the shape.

### Table 1  Tensile test results of the material used in the bilge models

<table>
<thead>
<tr>
<th>Model</th>
<th>Nominal thickness (mm)</th>
<th>True thickness (mm)</th>
<th>Yield stress (kg/mm²)</th>
<th>Tensile strength (kg/mm²)</th>
<th>E₀ (%)</th>
<th>Eₙ (%)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1.6</td>
<td>1.56</td>
<td>26.7</td>
<td>36.2</td>
<td>24.0</td>
<td>46.6</td>
<td>Bilge shell</td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>2.19</td>
<td>25.1</td>
<td>34.9</td>
<td>23.5</td>
<td>44.8</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>1.2</td>
<td>1.17</td>
<td>18.5 *</td>
<td>31.2</td>
<td>26.2</td>
<td>48.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>1.65</td>
<td>32.8</td>
<td>39.4</td>
<td>19.8</td>
<td>34.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>2.22</td>
<td>29.4</td>
<td>36.1</td>
<td>24.0</td>
<td>43.6</td>
<td>Bilge shell</td>
</tr>
</tbody>
</table>

1. Test pieces are JIS NO.5.
   The gauge length is 50mm.

2. All data listed are the mean value of every 4 test pieces.

3. E₀ means the strain at maximum nominal stress.

4. Eₙ means the breaking strain.

5. The data marked with * are 0.2% proof stresses.

6. 1 kg/mm² = 9.80 MPa

Fig. 4 The bulbous bow indenter (rigid body)

2.2 Test conditions and test procedures

The bilge models were fixed at the positions of the transverse bulkheads, and simply supported along the lines of side girders and side stringers. The bulb indenter was set to meet the middle part between the trans bulkheads, which was in the middle of the neighbouring web frames, at an angle of 45° as shown in Fig.5. In this condition, the tangential force at the collided position was not calculated for sake of simplicity of the tests. The load was imposed statically using an oil jack. The measurements were done on the
2.3 The test results
2.3.1 Model B1

Fig. 6 shows the load and penetration relationship obtained from the test. The numbers in the figure indicate the measuring sequence. Five other curves on the points which correspond to the same position as the loaded point are also shown. The state of the damage after the test is shown in Photo 1. The details of the destruction are outlined as follows.

The bilge shell in only the loaded web space deformed to a V shape until $P_3$. The webs adjacent to the loading point started buckling between $P_3$ and $P_4$. The deformed length of the bilge shell grew from one web space to three. The next one web on each side of the loading point started buckling around $P_7$, consequently the deformed length grew to five web spaces. After that, the same pattern was repeated two more times. Finally, the deformation of the bilge shell reached the whole length of the model; i.e. one trans bulkhead length. Fig. 6 shows clearly the deformation development described above except for the

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Photo 1 The state of damage to the bilge model after the experiment (model B1)
The first buckling phenomenon.

Fig. 7 shows the longitudinal strain behavior measured on the bilge shell. The longitudinal strain in each web space increased enormously when the neighboring web buckled. From this figure, it seems that the left web of the loading point and the right one buckled at P3 and P4, respectively.

The bow model touched the side girder and the side stringer around P8 and P12, respectively. The load increment at P8 and P12 (Fig. 6) seems to have occurred for that reason. After that, the deformed area of the bilge shell expanded gradually as the bow indented it. The side girder and side stringer were crushed by the direct action of the bow model. However, the webs which supported the side girder or side stringer hardly deformed. Consequently, at P15, some cracks occurred at the intersections of the outer shell and the side girder, and the side stringer (see Fig. 8). The load wasn't decreased, though. At P18, another crack occurred at the butt joint of the bilge shell. After that, the load kept its value till the end of experiment.

On the other hand, the inner hull plate hardly deformed until P12, then the bilge hopper plate began to deform locally. Fig. 9 shows the deformation of it at FR.6 section.

2.3.2 Model B2

Fig.10 shows the load and penetration relationship obtained from the test. The longitudinal strain behavior on the bilge shell is shown in Fig. 11. Photo 2 shows the state of the damage after the test. The characteristics of the destruction were as follows:

The bilge shell deformed to a V shape supported by two center webs. Those two webs buckled during P3 to P4. The next webs on each side of the loading point began to buckle at P9. The
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Damage development was similar to that of the model B1 until this point. At P10, a crack occurred from FR.6 of the bilge shell (see Fig.12). It’s length was 150~200 mm. The crack length grew to 250 mm at P11. The bilge shell ruptured accompanied with a loud noise during P13 to P14, and consequently the load was instantly decreased. The crack expanded along the whole breadth of bilge shell.

The load was then increased again at P15. The crack advanced into the side girder and the bottom shell. After that, the crack grew as the bow indented.

The inner hull plate began to deform at P21; the bow seemed to have reached it. At P22, the deformation of the inner hull plate was increased.
considerably. It showed a bow shape assuming the shape of the indenter. The deformed length was only four web spaces. The difference of the deformed length in B1 and B2 is obviously for the reason that only the bilge shell of B2 ruptured, but B1 didn't. The buckling strength of the web in B2 appeared higher than that in B1 in comparison with the bilge shell tensile strength.

On the other hand, the deformation of the inner hull plate was very small until the bow indented it directly. Fig. 13 shows its deformation at FR. 5 section.

3. The method of analysis

A simplified method to calculate a load-penetration curve was developed, based on the results of experiments. A load-penetration curve could be used for estimating the maximum load and the amount of energy absorbed in a collision, which were the primary factors governing a collision strength. The present method was applicable as the bow was indenting the bilge shell until the point when it reached the inner hull.

3.1 Idealization of the structure

The processes of damage of models B1 and B2 are very similar. The main features of the damage processes are that the membrane tension of the outer hull in the longitudinal direction and the compressive strength of web plates, which supported the outer hull, determined the strength of the models. In view of the above results, the following conditions are assumed in the analysis:

1. The rigid-plastic deformation is assumed, that is, the elastic deformation is ignored.
2. Bilge shell, bottom shell, side shell, side girders, and side stringers are treated as membranes which carry membrane tension in the longitudinal direction.
3. Web plates support the above mentioned membranes, and they do not resist longitudinal displacements.
4. Post-buckling forces in the bottom shell, the side shell, the side girders and the side stringers just below the loaded part are considered when buckling occurs (see Fig. 18).
5. The deformations between web plates or web frames are linearly interpolated.
6. Deformations are symmetric about the loaded line.
7. The deformations of the whole structure can be obtained by adding those of substructures—Parts A, B and C—separately as shown.

Fig. 12 Cracks in the model B2

Fig. 13 The deformation of the inner hull (model B2)

Fig. 14 The idealization of the structure
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in Fig. 14. The side girder and the side stringer on the boundaries of the substructures belong to part A and part C, respectively.

The shape of the bilge part is simplified as shown in Fig. 15, where the names of structural elements are summarized.

The condition (7), which was mentioned above, has been based on certain considerations as follows:

Web plates are destroyed by the action of membrane tension in the outer hull. At this time, the bilge shell deforms laterally, and is stretched in the longitudinal direction, while shrinking in the direction normal to that. The destruction of web plate A develops to the point B and C; after that, the deformations at the points of B and C stop temporarily until web plates B and C are destroyed. The deformations of web plate A and the bilge shell proceed continuously without significant constraint by the web plates B and C. Consequently, the web plates A, B and C deforms almost independently, which justifies the condition (7). If the damage develops gradually from the loaded point and if web plate A is destroyed before the destruction of B and C, the condition (7) is available.

Next, we assume the shape of deformation as shown in Fig. 16(a), which shows the deformation of the part A after the (n-1)th web from the loaded point have been destroyed. The i-th region is the one between the (i-1)th web and the i-th web in Fig. 16. The deformed part of the i-th region is shown as the shaded area in Fig. 16(a). A similar shape of deformation is assumed in parts B and C (Fig. 16(b)). It should be noted that the numbers of destroyed web plates are not necessarily the same among parts A, B and C.
3.2 The analysis of the damage development

Under the conditions assumed in the previous section, the formulation of the damage development analysis is stated in this section. First, the nomenclatures are shown below:

\[ P = P_A + P_B + P_C \quad (\text{external load}) \]

\[ P_A, P_B, P_C = \text{loads on parts } \Delta, \beta \text{ and } \Omega, \text{ respectively} \]

\[ \delta_b = \text{the indentation length or the displacement of the loaded point} \]

\[ \delta_{A_0}, \delta_{B_0}, \delta_{C_0} = \text{the displacements of the } i \text{-th web plate at the points of } A, B \text{ and } C, \text{ respectively} \]

\[ \delta_{cr} = \text{the distance between point } A \text{ and the line } BC \text{ in the loaded section (see Fig. 15)} \]

\[ \delta_{max} = \text{the limit of the indentation length in the present analysis (see Fig. 15)} \]

\[ L = \text{the web frame space} \]

\[ R = \text{the radius of the bilge circle} \]

\[ \tau = \sigma_0 t \quad (\sigma_0 = \text{yield stress}, t = \text{plate thickness}) \]

The subscripts of \( \tau \) distinguish the structural element where it acts as follows:

\[ \tau_{BS} = \tau \text{ in the bilge shell} \]

\[ \tau_{MS} = \tau \text{ in the bottom shell} \]

\[ \tau_{SS} = \tau \text{ in the side girder} \]

\[ \tau_{SA} = \tau \text{ in the side stringer} \]

\[ \tau_{GB} = \tau \text{ in the side girder} \]

\[ \tau_{GC} = \tau \text{ in the side girder} \]

\[ \tau_{Q_1}, \tau_{Q_2} = \text{the vertical membrane forces in the } i \text{-th region acting on the web plates } \Delta, \beta \text{ and } \Omega \]

\[ \tau_{S_1}, \tau_{S_2} = \text{the horizontal membrane forces in the } i \text{-th region acting on the web plates } \Delta, \beta \text{ and } \Omega \]

\[ \Delta S_{A_0}, \Delta S_{B_0}, \Delta S_{C_0} = \text{membrane forces in the underformed structural elements of the parts } \Delta, \beta \text{ and } \Omega, \text{ respectively} \]

\[ \alpha_i = \text{a parameter which represents the amount of deformation of the web plate } \Delta \]

### 3.2.1 Membrane forces

The membrane force in a unit breadth of a plate, \( \tau \), is assumed as in Eq. (1). The resultant membrane forces (Q and \( S \)) acting on the parts \( \Delta, \beta \) and \( \Omega \) are given in Eq. (2).

\[ \tau = \sigma_0 t \quad (1) \]

(1) For the part \( \Delta \), defining \( \alpha_1 \) and \( \alpha_{cr} \) as

\[ \alpha_1 = \cos^{-1} \left( \frac{1 - \delta_{A_1}}{R} \right) \]

\[ \alpha_{cr} = \cos^{-1} \left( \frac{1 - \delta_{cr}}{R} \right) \]

\[ \delta_{A_1-1} < \delta_{cr} \text{ and } \delta_{A_1} < \delta_{cr} \]

\[ Q_{A_1} = 2\tau_{BS}R \alpha_1 \left\{ \frac{2}{L^2} + \left( \frac{\delta_{A_1-1} - \delta_{A_1}}{L} \right)^2 + 2\tau_{BS}R \right\} \]

\[ S_{A_1} = 2\tau_{BS}R \alpha_1 \left\{ \frac{2}{L^2} + \left( \frac{\delta_{A_1-1} - \delta_{A_1}}{L} \right)^2 + 2\tau_{BS}R \right\} \]

where \( i = 1 \sim n \).

### 3.2.2 The post-buckling strength of web plates

The web plate of a girder supports shell plates which are modelled into membrane plates. The web plate begins to buckle when the vertical component of a membrane force reaches the buckling load.

The deformations of the web plates \( \Delta, \beta \) and \( \Omega \) are assumed to be expressed by only the deformations at the points A, B and C, respectively,
A simplified method to analyse the strength of double hulled structures in collision as follows:

$$\begin{align*}
\text{(3a)} & \quad W_{A1} = W_{A1}(\delta_{A1}), \quad i=1 \sim n-1 \\
\text{(3b)} & \quad W_{B1} = W_{B1}(\delta_{B1}), \quad i=1 \sim l-1 \\
\text{(3c)} & \quad W_{C1} = W_{C1}(\delta_{C1}), \quad i=1 \sim m-1
\end{align*}$$

These relationships are illustrated in Fig. 17. According to the condition (7) in the section 3.1, the web plates $A$, and $C$ are buckled by the forces $Q_{A1}, Q_{B1}$ and $Q_{C1}$, respectively.

3.2.3 The post-buckling strength of the bottom shell and side girder, the side shell and side stringer

The rigid bow-indenter meets the bottom shell and side girder, the side shell and side stringer when the indentation length reaches $\delta_{er}$. These structural elements are not deformed at this point and they react to the indenter with their post-buckling strength. They are idealized into rectangular plates between webs, and a concentrated load acts on the middle of an edge of the rectangular plates, as shown in Fig. 18. Their reaction forces are used to form the equilibrium equations of parts $A$, $B$, and $C$.

3.2.4 The equations of equilibrium

The equations of equilibrium at the loaded point and on the web frames are given by Eqs. (5)-(7) (see Fig. 19).

Part $A$: \[ P_A = 2Q_{A1} \] (5a)

Part $B$: \[ Q_{B1} = Q_{B1} + W_{B1}, \quad i=1 \sim n-1 \] (5b)

Part $C$: \[ Q_{C1} = Q_{C1} + W_{C1}, \quad i=1 \sim m-1 \] (5c)

where $Q_{Ai}$, $Q_{Bi}$ and $Q_{Ci}$ are introduced to maintain the continuity of horizontal membrane forces. They correspond to the membrane forces in the longitudinal direction in the undeformed parts.

The $n$-th web plate in part $A$, the $l$-th web plate in part $B$, and the $m$-th web plate in part $C$ are not buckled at this point, so the following equations hold:

$$\begin{align*}
\text{(8a)} & \quad \delta_{A0} = 0 \\
\text{(8b)} & \quad \delta_{B0} = 0 \\
\text{(8c)} & \quad \delta_{C0} = 0
\end{align*}$$

Eqs. (2) through (8) give the relationships between the loads ($P_A$, $P_B$, $P_C$) and the indentation lengths ($\delta_{A0}$, $\delta_{B0}$, $\delta_{C0}$). As for the part $A$, for example, the number of unknowns is $5n$, which are $P_A$, $\delta_{A0}$ through $\delta_{A5}$, $Q_{A1}$ through $Q_{A5}$, $S_{A1}$ through $S_{A5}$, $W_{A1}$ through $W_{A5}$, and $S_{A0}$ through $S_{A5}$; the number of equations is $5n-1$, which are Eq. (2a), Eq. (3a), Eq. (5a), Eq. (6a), Eq. (7a), and Eq. (8a). Therefore, the relation between $P_A$ and $\delta_{A0}$ can be obtained numerically. And the equations for vertical equilibriums are separated from those for horizontal equilibriums, and they give $3n$ equations with $3n+1$ unknowns excluding $S_{A0}$ through $S_{A5}$ and $\delta_{A0}$ through $\delta_{A5}$. Similarly, the relationships between $P_B$ and $\delta_{B0}$, and between $P_C$ and $\delta_{C0}$ are obtained. The relation between the total load $P$ and the indentation length $\delta_0$ is given by summing the loads and the indentation lengths for the parts $A$, $B$, and $C$.

$$\begin{align*}
P = P_A(\delta_{A0}) + P_B(\delta_{B0}) + P_C(\delta_{C0})
\end{align*}$$ (9)
When the indentation length is smaller than \( \delta_{cr} \), the bow does not reach the parts B and C. Then \( P_B \) and \( P_C \) are equal to zero. Eqs. (9), (5a), and (6a) lead to the following equation:

\[
(10) \quad Q = 2Q_{An} + 2 \sum_{k=1}^{n-1} W_{Ak}
\]

Eq. (10) holds just before the \( n \)-th web is buckled. Then,

\[
(11) \quad Q_{An} < W_{An}(0)
\]

Consequently, Eqs. (10) and (11) lead to Eq. (12):

\[
(12) \quad P < 2 \sum_{k=1}^{n} W_{Ak}
\]

Eq. (12) shows that the damage develops to the neighbouring regions on both sides of the loaded point by the length of two web spaces at a time when the load is increased by the amount of \( 2 W_{Ak} \), if \( W_{Ak} \) is assumed to be constant for all \( k \). This corresponds to the experimental results mentioned in the second paragraph of section 2.3.1.

The values of \( W_{Ak}(\delta_{Ah}), W_{Bk}(\delta_{Bk}), W_{Ck}(\delta_{Ck}), G_B(\delta_{Bk}) \) and \( G_C(\delta_{Ck}) \) are determined approximately as mentioned in the following section.

### 3.3 Calculation of \( W \) and \( G \)

In order to execute the damage development calculation, \( W_A \), \( G_B \), etc. must be known. We calculated those values by using simplified methods because it is not easy to get them precisely. The methods used here should be, if necessary, substituted by other, better methods.

The web plate \( \Delta \) is fan-shaped and its boundary condition is difficult. So, we idealized the web plate \( \Delta \) as in Fig.20. The ultimate strength of the rectangular plate shown in Fig.20(a) was calculated by the ref. 2). On the other hand, the ultimate strength of the clumped beam which is shown in Fig.20(b) was calculated by applying the limit analysis method, where an effective breadth of the bilge shell was taken into account according to the ref. 5). The values of \( W_A \) were estimated 6,400 kg in the model B1, 9,700 kg in B2.

The values of \( W_A \) are also estimated from the destruction characteristic noted in Eq. (12) and the load-penetration relationship in the experiment. From Figs.6 and 10, the two web plates adjacent to the loaded line buckled at a load of 13 ton in the case of B1, and at a load of 21.7 ton in the case of B2. Therefore, each one web must have buckled at 6.5 ton and 10.8 ton, respectively. In the section 4, we used these values.

Then, we calculated the values of \( W_B \) and \( W_C \) as follows. In the case of the web plate \( \Delta \), for example, the problem is basically to get the ultimate strength of the stiffened panel as shown in Fig.21(a) in both B1 and B2. We substituted the three simplified conditions as shown in Fig.21(b), (c), (d) for (a), and we assumed the ultimate strength of (a) is given as the lowest value of the load in the bow loading direction among these three types. The ultimate strength of (b), (d) was calculated by the refs. 2), 3). On the other hand, the value of (c) was obtained by summing up the collapse loads of each truss member whose effective sectional area was calculated by applying the ref. 5). In the case of the web plate \( \square \), the same method was used.

Finally, the values of \( G_B \), \( G_C \) were calculated as follows. \( G_B \), \( G_C \) are the loads when both the bottom shell and the side girder, both the horizontal stringer and the side shell collapse, respectively (see Fig.18). They are considered as panels which are simply supported at all edges and are subject to:

\[
(13) \quad G_B = \sum_{k=1}^{n} W_{Ak}
\]

\[
(14) \quad G_C = \sum_{k=1}^{n} W_{Ak}
\]

Fig.21 Calculation of \( W_B, W_C \)
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3.4 Membrane rupture and damage development after the rupture

Cracks appear in the shell during the course of the damage development. In general, cracks are initiated at the place of maximum strain and stretch gradually until static equilibrium conditions are not satisfied, that is, a sudden rupture happens and the load decreases substantially. This process is verified in many kinds of experiments. The analysis of crack propagation, however, is so complex that a simplification is employed: the rupture happens as soon as an assumed condition is satisfied, and the process of crack propagation is ignored.

The time difference between the initiation of a crack and the rupture of a membrane depends on the situation where the crack appears. When it appears at the location where there is uniform strain, the difference may be small. On the other hand, when it appears at the location which is under local strain, the difference may be larger. Therefore, it seems reasonable to separate the conditions of membrane rupture into such two cases.

In the model B1, cracks appeared in the parts A, B, and C, but the crack along the butt joint seemed to be caused by a welding defect. In the model B2, a crack appeared in the part A. All these cracks except one crack in part A in the model B1 started from locations which were subject to local strain. These two types of cracks are due to the manner of destruction and the shape at the support point of the membrane. As Fig. 22 shows, the web plates of part A in the model B1 were crushed uniformly. The web plates of part B in the model B2 were not crushed uniformly. Some parts of them remained undestroyed and supported the bilge shell at their sharp points because there were stiffeners and a hole in the web plates. The web plates of parts B, C have this type of sharp point in nature. We refer to such point as the "hard point".

In the former paper, we separated the membrane rupture condition into (i) in the case where a membrane ruptures at the stem or bulb tip, and (ii) in the case where a membrane ruptures by uniform tension according to the ref. 6). In this paper, we added (iii) another condition in the case where a membrane ruptures at a hard point. The total membrane rupture condition is shown in Eq. (13).

The conditions (i), (ii) mentioned above are quite the same as in the former paper, which are given as in Eqs. (13-1) (13-3).

On the other hand, we considered the condition in the case of (iii) as follows. In the past, there have been several papers in which this type of membrane rupture was treated through the total strain value of local bending strain and uniform tension strain. For example, in the ref. 7), square plates which were fully clumped at all edges were penetrated at their center by a rigid body whose tip had a semi-sphere shape, and the maximum strain value which was the sum of the bending and tension strain was observed to be 0.338 when cracks appeared. In the ref. 8), ship side models which had two or three decks were indented by a wedge shaped rigid body whose tip was round, and the maximum strain value was 0.9 φ, (φ is the strain when the maximum load is reached in a tensile test), when the side shell ruptured. In the ref. 9), the same kind of experiments as in the ref. 8) were carried out, and the value was 0.3. In many other papers, the same way of thinking as mentioned above was adopted. Generally speaking, it seems that the membrane rupture conditions were determined in each paper by the way in which they correlated with the experiments. Therefore, a unified treatment or theory does not exist yet.

When we compare the situation of the part A now under discussion, with the experiments which were carried out in those references as to how the membrane ruptured, the main difference is as follows: The crack initiation and the membrane rupture happened at almost the same time in the latter, while they happened in a fairly large time-difference in the former. This difference was caused by the destruction characteristic of the bilge structure.

On the basis of the consideration mentioned above, we determined the membrane rupture condition at a hard point as follows. The initiation of a crack at a hard point is estimated by the value of the maximum strain which is the sum of local bending and uniform tension strains. However, the local bending strain is ignored because it is difficult to calculate. For its compensation, taking the time difference of the crack initiation and the membrane rupture into account,
the value of the uniform tension strain at the
membrane rupture was given \( (7/5) \varepsilon_u \) as in Eq.
\( (13-4) \), which is larger than the value obtained
for when a membrane ruptures due to only the
uniform tension.

(1) the membrane ruptures at the contact point
with the stem top (This condition is available
for \( r/t < 5/2 \varepsilon_u \)).

\[
2/\varepsilon_1 + r/t = 5/\varepsilon_u \quad \text{for} \quad r/t < 5/2 \varepsilon_u \quad (13-1)
\]

where

\( \varepsilon_1 \) = the maximum principal strain at the contact
point

\( r \) = the radius of curvature of the stem top in
the direction of \( \varepsilon_1 \)

\( t \) = the thickness of side shell plating

\( \varepsilon_u \) = the strain when the maximum load is reach-
ed in a tensile test

(2) i) the membrane rupture in the side
shell is caused by uniform extention
(a) When the deformation is limited to within
an area of one web frame space by one girder
space, the condition is expressed as follows (Fig. 23
(a)):

\[
\varepsilon = \varepsilon_u \quad (13-2)
\]

where

\( \varepsilon \) = the maximum of \( \varepsilon_1 \) and \( \varepsilon_2 \), where \( \varepsilon_1 \) and
\( \varepsilon_2 \) are the maximum principal strains in the areas
shown in Fig. 23 (a)

(b) When the deformation stretches over more
than two web frame or girder spaces, the condi-
tion is expressed as follows (Fig. 23 (b)):

\[
\varepsilon = (1/2) \varepsilon_u \quad (13-3)
\]

where

\( \varepsilon \) = the maximum of \( \varepsilon_1 \) and \( \varepsilon_2 \), where \( \varepsilon_1 \) and
\( \varepsilon_2 \) are the maximum principal strains aver-
ged over the areas shown in Fig. 23 (b)

ii) the membrane ruptures at a hard point

\[
\varepsilon = (7/5) \varepsilon_u \quad (13-4)
\]

In general, it is difficult to decide in what way
the bilge webs of actual ships are destroyed: They
might be destroyed uniformly, or might
be destroyed except for some parts which become
hard points. Since the bilge structures of actual
ships are almost the same as the model B 2, how-
ever, it seems better to assume that the hard
points occur in almost all cases, and that the
points where hard points occur are similar geometri-
cally to those in the model B 2 (see Photo 2 and
Table 2).

The membrane is assumed to rupture as soon
as one of the conditions in Eqs. (13-1)~(13-4)
is satisfied, and its tension vanishes. Therefore,
only webs, girders and stringers act against the
indentation of a bow after the rupture happens.
Here, the deformation difference between the be-
inning the rupture and after the rupture is as-
sumed to be negligible.

So far, we treated the parts \( \text{A}, \text{B} \) and \( \text{C} \)
independently, but such treatment is not valid
for the phenomenon of membrane rupture. In
the model B 2, for example, the crack which ap-
peared in the part \( \text{A} \) propagated into the parts \( \text{B} \)
and \( \text{C} \), while there were no original cracks in
them. Such phenomenon is not taken into account
unless we examine a crack propagation mechanism,
but it is not easy. So, for the present, as far
as the parts \( \text{B} \) and \( \text{C} \) are concerned, the mem-
brane is assumed to rupture as soon as the mem-
brane of the part \( \text{A} \) ruptures. On the other hand,
the buckling resistances \( G_B \) and \( G_C \) are assumed
to keep the value of the initial buckling loads
to compensate for the effect of the membranes.

4. The comparison of the experimental
results with the calculated results

Numerical calculations corresponding to the two
tests were done by means of the analysis method
presented in chapter 3. Figs. 24 and 25 show the
relationship between the calculated results and the
experimental ones, where the structural strength
data which were used here are listed in Table 2.
The calculations were done assuming two different
possibilities (see Fig. 26): i.e.,

Calculation (1)...Assuming that the post-buckling
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reaction forces of the webs, side girders, side stringers etc. were constant, and

Calculation ③...Assuming that the post-buckling reaction forces of them were reduced linearly to zero value at the time when they collapsed up to their depth.

From Figs. 24 and 25, some points became clear as follows:

(1) Both calculation ① and ② show a good correlation with the test results, especially calculation ③. They express the deformation characteristics of the load and penetration relationship well, due to webs buckling until the rupture of the bilge shell.

(2) The assumption of the membrane rupture conditions is reasonable.

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(1) Both calculation ① and ② show a good correlation with the test results, especially calculation ③. They express the deformation characteristics of the load and penetration relationship well, due to webs buckling until the rupture of the bilge shell.

(2) The assumption of the membrane rupture conditions is reasonable.

Table 2 Data used in the calculations

<table>
<thead>
<tr>
<th>Item</th>
<th>Model B1</th>
<th>Model B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>333.3 mm</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>6.00 mm</td>
<td></td>
</tr>
<tr>
<td>C BS</td>
<td>55.0 kg/mm</td>
<td>65.3 kg/mm</td>
</tr>
<tr>
<td>C MS</td>
<td>55.0 kg/mm</td>
<td>65.3 kg/mm</td>
</tr>
<tr>
<td>C SG</td>
<td>41.7 kg/mm</td>
<td>54.1 kg/mm</td>
</tr>
<tr>
<td>C SS</td>
<td>55.0 kg/mm</td>
<td>65.3 kg/mm</td>
</tr>
<tr>
<td>C HS</td>
<td>55.0 kg/mm</td>
<td>21.6 kg/mm</td>
</tr>
<tr>
<td>W a</td>
<td>6500 kg</td>
<td>10800 kg</td>
</tr>
<tr>
<td>W b</td>
<td>5780 kg</td>
<td>7380 kg</td>
</tr>
<tr>
<td>W C</td>
<td>6860 kg</td>
<td>6870 kg</td>
</tr>
<tr>
<td>G s</td>
<td>6650 kg</td>
<td>10560 kg</td>
</tr>
<tr>
<td>G C</td>
<td>7850 kg</td>
<td>8950 kg</td>
</tr>
<tr>
<td>δ cr</td>
<td>200 mm</td>
<td>205 mm</td>
</tr>
</tbody>
</table>

(3) The calculation method presented in chapter 3 underestimates the load after the rupture of the bilge shell.

The absorbed energy and penetration curves are shown in Figs. 27 and 28. According to these figures, the difference between the calculations and the tests become a little greater after the rupture. The accuracy, however, seems satisfactory.

From the results mentioned above, it is considered that the analysis method presented in this paper is useful both in calculating the load-penetration relationship and in examining other characteristics of the damage done to a bilge structure in collision.

As far as these calculations are concerned, calculation ③ is recommended. This recommendation is opposite to the one in the case of a double hulled side structure examined in the ref. 1).

5. Conclusions

Static destruction tests were carried out using simplified and detailed large scale models of a bilge structure representative of those used in double hulled ships. Next, an analytical investigation was done using the experimental results. The main conclusions are as follows:

(1) A bilge structure can be modelled into two main parts in the case of a minor collision; one is the bilge shell which acts as plastic mem-
branes and reacts to the forces in the longitudinal direction only; the other is web plates which may be regarded as buckling members which support the bilge shell.

(2) Parts of bilge web, on the margin of the opening or at the end of a stiffener, form cusps and they promote the rupture of the outer hull.

(3) The inner hull scarcely deforms during the damage process until that moment when it is actually struck by the bow.

(4) A simplified method of analysis was formulated from which the relationship between the amount of destruction and the destruction load in bilge structures in collision can be calculated.

Acknowledgement

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References


7) Japan Nuclear Shipbuilding Research Committee: Study on the influence of external forces and countermeasure for collision or grounding of nuclear ship, (1962).
