Numerical Simulation of 3-D Nonlinear Water Wave by Boundary Element Method

—In the case of submerged bodies—

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Abstract

A boundary element method is developed for the 3-D non-linear water wave problems. Triangular panels are used which can represent general 3-D shapes easily and accurately. All the variables are linearly approximated on each panel. The non-linear free surface conditions are satisfied by finite difference approximation. The finite difference calculation is carried out in a mesh system which is finer than that used for the boundary element calculation. A new condition for the downstream open boundary is proposed. After several pilot computations for accuracy analysis, the waves and forces of a submerged sphere and a submerged ship-like body are calculated. Through the comparisons of the results with the analytical solution and those of experiment, it is found that the present method is accurate and efficient.

Nomenclatures

U_0 : steady flow velocity
 $U(t)$: velocity at time t
 L : length of the HSV model
 D : diameter of the sphere
 H : depth of submergence
 S_B : area of body surface
 t : time
 T : nondimensional time
 g : gravitational acceleration
 n : normal direction of boundary, or time step when it appears as superscripts
 l, m : tangent directions of the body surface
 ϕ : perturbation velocity potential
 ϕ_n, ϕ_l, ϕ_m : differentiation of ϕ with respect to n, l, m
 ξ : wave height
 S_{pq}, D_{pq} : influence coefficients
 K_0 : g/U_0^2
 λ_0 : wave length ($2\pi/K_0$)
 F_n : Froude number
 C_w : wave resistance coefficient ($F_z/\frac{1}{2}\rho U_0^2 S_B$)
 C_L : lift force coefficient ($F_z/\frac{1}{2}\rho U_0^2 S_B$)
 T_q : tent function
 r : distance between field point and integrating point
 i, j : index for finite difference mesh along the x and y direction respectively
 p, q : index for the grid points in boundary panel

system

Ω_q : region consisting of all the panels connected with point q

1. Introduction

To calculate nonlinear free-surface flows and forces acting on bodies is an important problem in the marine hydrodynamics.

Recently nonlinear flow simulations have been carried out by solving the Navier-Stokes equation directly by the finite difference method ([1], [5]). Although the method is almost general for any type of flows, it has turned out difficult to evaluate high Reynolds-number flows. And the computational domains used in the usual finite difference method are so limited that there still remains a suspicion whether the flow is thoroughly solved.

The assumption that the fluid is inviscid and irrotational assures the existence of velocity potential whose use provides more realistic method and sometimes more reliable results for the nonlinear free surface problems. The so-called Rankine source method has explored a new stage in ship wave resistance calculation, [6]. However, satisfactory results have not been obtained yet due to some numerical immaturities and the limitation of the double-hull linearization of the free-surface conditions. The use of the constant-source-panel method for the double-hull flow sometimes yields non-zero resistance even for double model flows.

On the other hand, attempts have been made to simulate the fully nonlinear free-surface flows by boundary element methods. The results for the bounded flows such as sloshing problems are celebrated (e. g., [2]).

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Some nonlinear infinite flow problems have been challenged by the methods (e. g. [3], [4]), but the numerical methods proposed in those papers are usually not applicable to general 3-D water wave problems.

In the present paper, transient and steady nonlinear free surface flows are simulated by boundary element method. The boundary surfaces are divided into triangular panels and the variables concerned in the problem are assumed to vary linearly across each panel. The finite difference calculation is carried out in a mesh system which is finer than the grid system used in the boundary element calculation. We call it double mesh system. By this method the effect of numerical viscosity can be eliminated and the efficiency of calculation is greatly improved.

The problem is solved by time-marching method to satisfy all the nonlinear free-surface conditions. As numerical examples, the free-surface flows around a submerged sphere and a submerged ship-like body are calculated.

2. Basic Equations

We neglect the viscosity of fluid and assume the flow is irrotational, so that the velocity potential exists. Here we consider the flow around a body which accelerates gradually from the rest state and finally moves at a constant speed U_0 . Then the problem is described as follows in the moving coordinate system shown in Fig. 1.

The governing equation :

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad (1)$$

Here ϕ is the perturbation velocity potential.

The body surface condition :

$$\phi_n = -U(t)n_x \quad (2)$$

The free-surface conditions :

$$\frac{\partial \zeta}{\partial t} = \phi_z - U(t)\zeta_x - \phi_x \zeta_x - \phi_y \zeta_y; \quad \text{at } z = \zeta \quad (3)$$

$$\frac{\partial \phi}{\partial t} = -g\zeta - U(t)\phi_x - \frac{1}{2}(\phi_x^2 + \phi_y^2 + \phi_z^2); \quad \text{at } z = \zeta \quad (4)$$

The initial condition :

$$\zeta = 0, \phi = 0, \text{ at } t = 0 \quad (5)$$

By the Green's theorem, the Laplace equation (1) is

transformed into the integral form as follows :

$$c\phi = \iint_S \phi_n \frac{1}{r} ds - \iint_S \phi \frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds \quad (6)$$

where r is the distance between field points and integrating points, S is the boundary surfaces surrounding the fluid domain, and c is the solid angle divided by 4π .

Now the problem is to determine the velocity potential by solving the integral equation (6) with its boundary values and initial values given by (2)~(5).

3. Numerical Method

3.1 Discretized expression of the governing equation

For the numerical computation, a finite computing domain is taken as shown in Fig. 1 whose boundary surfaces consist of body surface, free-surface, upstream and downstream surfaces, side surface and bottom. All the boundary surfaces are divided into triangular panels and the integration over the surfaces is decomposed into the summation of the contributions from each panel. ϕ and ϕ_n are assumed to vary linearly in each panel. Then (6) is discretized as follows.

$$\sum_{q=1}^N \phi_q D_{pq} - \sum_{q=1}^N \frac{\partial \phi_q}{\partial n} S_{pq} = 0, \quad p = 1, 2, \dots, N \quad (7)$$

where N is the total number of the nodes, S_{pq} , D_{pq} are influence coefficients expressed by the following integrations.

$$S_{pq} = \iint_{\Omega_q} T_q \frac{1}{4\pi r} d\Omega; \quad D_{pq} = \iint_{\Omega_q} T_q \frac{\partial}{\partial n} \left(\frac{1}{4\pi r} \right) d\Omega \quad (8)$$

where Ω_q is the integration region consisting of all the panels connected with point q . T_q is a tent function which takes the value of one at the point q and zero at the other nodes of the region.

The influence coefficients are calculated by analytical formulae when point p is near to point q . Approximate formulae are used to simplify the calculation when point p is far from point q .

All the influence coefficients should be renewed at each time step according to the change of the shape of the free-surface and the body surface. In order to save computing time, however, they are renewed only in a sub-region where the relative change of the boundaries is significant, and the renewal is carried out after every two or three time steps.

Fig. 1 Coordinate System

Fig. 2 Integration region for the calculation of the influence coefficients

3.2 Free-surface boundary conditions

The velocity potential on the free-surface and the free-surface elevation can be determined iteratively by (3) and (4).

An implicit time-marching method is employed as follows:

$$\frac{\zeta_{ij}^{n+1} - \zeta_{ij}^n}{\Delta t} = \frac{\left(\frac{\partial \zeta}{\partial t}\right)_{ij}^{n+1} + \left(\frac{\partial \zeta}{\partial t}\right)_{ij}^n}{2} \quad (9)$$

$$\frac{\phi_{ij}^{n+1} - \phi_{ij}^n}{\Delta t} = \frac{\left(\frac{\partial \phi}{\partial t}\right)_{ij}^{n+1} + \left(\frac{\partial \phi}{\partial t}\right)_{ij}^n}{2} \quad (10)$$

where $\left(\frac{\partial \zeta}{\partial t}\right)_{ij}^{n+1}$, $\left(\frac{\partial \zeta}{\partial t}\right)_{ij}^n$ and $\left(\frac{\partial \phi}{\partial t}\right)_{ij}^{n+1}$, $\left(\frac{\partial \phi}{\partial t}\right)_{ij}^n$ are the finite difference forms of r. h. s. of (3) and (4) respectively. In (9) and (10), the differentiations with respect to x are given by the third-order-upstream difference, while those with respect to y are by the central difference:

$$\left(\frac{\partial f}{\partial x}\right)_{i,j} = \frac{2f_{i+1,j} + 3f_{i,j} - 6f_{i-1,j} + f_{i-2,j}}{6\Delta x} \quad (11)$$

$$\left(\frac{\partial f}{\partial y}\right)_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y} \quad (12)$$

where f is ζ or ϕ .

A different grid system is used in the finite difference calculation to reduce the numerical viscosity. The grid size is much finer than the boundary elements.

The values of ϕ_n at the nodes of the finite difference mesh are obtained by the interpolation of those at the grids of the boundary elements.

The upstream condition for ζ is

$$\left(\frac{\partial \zeta}{\partial x}\right)_{ij}|_{i=1} = 0 \quad (13)$$

3.3 Open boundary conditions

On the upstream boundary surface, the side surface and the bottom, the Neumann condition is assumed.

$$\phi_n = 0 \quad (14)$$

As the downstream condition, the Sommerfeld condition is commonly used to give the value of ϕ . It is found, however, (7) is very sensitive to the term ϕ because the coefficient of ϕ_p is far greater than that of $\frac{\partial \phi_q}{\partial n}$.

Therefore here we use

$$\phi_n = \phi_n|_{z=0} e^{K_0 z} \quad (15)$$

as boundary condition, where $\phi_n|_{z=0}$ is calculated by using the value of ϕ on the free surface, and $K_0 = g/U_0^2$. The condition (15) can be derived from the linear steady wave theory, the result of which may be a good approximation at the far downstream.

3.4 Velocity and pressure on the body surface

Suppose l and m are two tangent directions at any node on the body surface. The velocity components ϕ_l and ϕ_m can be determined by the central finite differences of ϕ on the nodes of body surface. Then the velocity components ϕ_x , ϕ_y and ϕ_z can be determined from ϕ_l and ϕ_m together with ϕ_n given by (2). The pressure acting on the body surface is determined by the Bernoulli's equation.

4. Numerical Results

4.1 Flows without free-surface

To check the method, the potential flow around a sphere without free-surface is first calculated. Fig. 3 shows the computed results of the velocity and pressure on the horizontal symmetric plane together with the analytical solution (THETA is the angle from the stagnation point). The number of the nodal points and panels are 283 and 512 respectively. The maximum error in the pressure is about 1.6 % which can be further decreased if we increase the number of nodal points.

Fig. 4 shows the arrangement of elements for Series-

Fig. 3 Computed velocity and pressure on the horizontal symmetric plane

Fig. 4 Panels and the velocity vectors on the hull surface of Series-60, $C_B=0.6$

60, $C_s=0.6$ model and the computed velocity vectors on the hull surface. The number of nodal points is 272. The velocity is continuous along the hull. The resistance coefficient due to the surface pressure turned out to be 0.06×10^{-3} which is as less as about 5 % of the measured wavemaking resistance coefficient. The computing time was 15sec by HITAC 680H.

It can be concluded that the present method can be used to calculate the potential flow around the double-model accurately.

4.2 Free-surface flows

The wave generated by a submerged sphere is calculated under the condition $H/D=1$, $F_n=U_0/\sqrt{gD}=0.5774$, where D is the diameter and H the submergence depth measured from the rest free-surface to the centre of the sphere. To verify the present method and to investigate the non-linear characteristic of the problem, calculations under both the linear and the nonlinear free-surface conditions are carried out. The computational domain is

$$x: -7.5 \times D \sim 10 \times D; \quad y: -6.5 \times D \sim 6.5 \times D; \quad z: \zeta \sim -4 \times D$$

The number of the nodes is 73 on the body surface and 649 on the free surface.

Fig. 5-a shows the computation domain, Fig. 5-b the boundary elements on the free surface and Fig. 5-c the fine mesh for the finite difference calculation. The minimum size in the x direction of the boundary elements on the free surface is $\lambda_0/10$, while that of the finite difference mesh is $\lambda_0/60$, where λ_0 is the wave length

Fig. 5 (a): Computational domain, (b): boundary elements and (c): finite difference mesh on the free-surface

Fig. 6 The development of the wave by sphere ($F_n=0.5774$)

given by $2\pi U_0^2/g$.

Fig. 6 shows the time history of the development of non-linear wave (for clarity the wave height is enlarged by factor 4), while Fig. 7 shows that of the wavemaking resistance and the lifting force coefficients. No wave reflection from the downstream boundary is observed. The average linear results by the numerical computation are slightly smaller than the analytical solution by Bessho [7]. The wave length obtained by the nonlinear numerical calculation is larger than $\lambda_0=2\pi/k_0$ by about 3.7 %.

Although the wave near the sphere seems to have attained a stationary state around $T=15$, the wave resistance and lifting force are still oscillating with a large amplitude.

Fig. 7 Time histories of the wave resistance and the lift force coefficients of the sphere ($F_n=0.5774$)

Fig. 8 Sketch of computational domain and grid system for HSV

As a more general case, calculations are carried out for the main hull of HSV, a high speed submerged ship, under the condition $H/L=0.15$, $F_n=U_0/\sqrt{gL}=0.42\sim 0.70$. (L is the length of the hull, and H is the depth of submergence, measured from the rest free-surface to the center of the maximum section of the body). The geometry of HSV and its detailed experimental data can be found in [8].

Fig. 8 shows a part of the computational domain and the grid system for HSV. Fig. 9 and Fig. 10 show the time histories of the development of wave (for clarity the wave height is enlarged by factor 15) and the resistance and the lift force coefficients at $F_n=0.66$. Being different from the case of the sphere, the force coefficients converge monotonously and quickly. The potential $\phi/(\frac{1}{2}U_0L)$, the velocity u/U_0 and pressure distributions along the upper centerline and keel line

Fig. 9 The development of the wave by HSV ($F_n=0.66$)

Fig. 10 Time histories of the wave resistance and the lift force coefficients of HSV ($F_n=0.66$)

Fig. 11 Potential, velocity and pressure distributions along the upper centerline and the keel line of HSV ($F_n=0.66$)

are shown in Fig. 11, where the Froude number is 0.66. It can be seen that near the bow and the stern the pressure acting on the upper side changes rapidly and the regions where the pressure is large are quite limited.

Fig. 12 is an example of the comparison between the computed longitudinal wave profile and the measured one (at $F_n=0.66$, $y/L=0.621$; the body is located from $x/L=0.0$ to 1.0). The numerical result agrees well with that of the measurement except near the second crest and the second hollow (around $X/L=1.7\sim 4.1$). The difference is perhaps partially caused by the free-surface piercing part of the model which was used in the experiment to support the model.

In Fig. 13 the calculated resistance and lift force

Fig. 12 Comparison between the computed wave profile and the measured ($F_n=0.66$, $y/L=0.621$)

Fig. 13 Comparisons between the numerical results and the experimental results of HSV

coefficients are compared with the measured. The calculated wavemaking resistance is a little larger than that by wave-analysis. This trend is very common. On the other hand, the lift force coefficient is positive while that of experiments is negative. This is because no circulation is included in the present calculation. The calculated lifting force is only the result of the free surface effect.

The computing time for the case of HSV is about 30 to 90 minutes per speed by HITAC 680H (without IAP) and the total required memory size is about 10 MByte.

Although the computed cases are limited, it can be concluded that the present method is prospective for ship wave problems.

5. Concluding Remarks

A numerical method for 3-D non-linear water wave problems is proposed which is based on the boundary element method. The numerical techniques contrived here such as the double mesh method, the downstream boundary condition and the method of the local renewal of the influence coefficients have been successful.

The numerical examples show that the method is comparatively efficient and accurate. It can be a practical method for 3-D non-linear water wave problems.

The method is applicable to the wave problems of

surface-piercing bodies. It can be also extended to the ship motion problems.

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References

- 1) Inui, T., Kajitani, H. and Miyata, H.: Nonlinear Free-Surface Waves and Computational Hydrodynamics, read at 75 Jahre HSVA Internationales Symposium, (1988).
- 2) Shinkai, A., Nozu, Y., Yamaguchi, K. and Fukuda, J.: Numerical Analysis of Three-Dimensional Sloshing Problems, Trans. of the West-Japan Soc. of Naval Arch., No. 76, pp. 103-115, (1988) (in Japanese).
- 3) Liu, Y. Z. and Yang, C.: Nonlinear Radiation Problem of an Axisymmetric Cylinder, Advances in Hydrodynamics, Vol. 2 No. 1, pp. 81-101 (1987).
- 4) Lin, W.-M., Newman J. N. and Yue, D. K.: Nonlinear Forced Motions of Floating Bodies, 15th Symp. on Naval Hydrodynamics, (1985)
- 5) Hino, T.: Numerical Simulation of a Viscous Flow with a Free Surface around a Ship Model, Jour. of the Soc. of Naval Arch. of Japan, Vol. 161, pp. 1-9, (1988).
- 6) Dawson, C. W.: A Practical Computer Method for Solving Ship-Wave Problems, Proc. of 2nd Intern. Conf. on Numerical Ship Hydro., pp. 30-38, (1977).
- 7) Bessho, M.: On the Wave Resistance Theory of a Submerged Body, 60th Anniversary Series, Soc. of Naval Arch. of Japan, Vol. 2, pp. 135-172, (1957).
- 8) Mori, K., Hotta, T., Ebira, K. and Xu Q.: A Study on Semi-Submergible High Speed Ship with Wings, Jour. of the Soc. of Naval Arch. of Japan, Vol. 162, pp. 1-8, (1988) (in Japanese).