Towed Ship Motion Test in Directional Spectrum Waves in a Long Tank (Part 2)

by Seiji Takezawa*, Member Tsugukiyio Hirayama*, Member Shivashis Acharyya**, Member

Summary

As a continuation to our previous report on tank testing results of ship motions in directional spectrum waves this paper presents further test results under 'following sea' condition. Uniform irregular waves with arbitrary directional spectrum have been generated in the whole area of towing tank of Yokohama National University by the newly installed snake type wave maker and running tests have been carried out at Froude numbers 0.2 and 0.275. The experimental data have been analyzed at encounter frequency base using both Fast Fourier Transformation (FFT) and Maximum Entropy Method (MEM) to obtain the power spectrum of motions and the results are presented along with the theoretical estimates. Directional transfer functions of motions have been calculated using well known New Strip Method (NSM) and analyzed directional wave spectra obtained from experimental data by Maximum Likelihood Method (MLM) have been used to estimate theoretical motion spectra. The compared results show fairly good agreement.

This paper also presents short term prediction of motions calculated for both 'head sea' and 'following sea' conditions over various range of wave periods.

1. Introduction

This paper is in continuation to our previous report1 where some experimental results on ship motions in directional spectrum wave were presented. These experiments were mainly carried out in head sea condition. This paper discusses further test results under 'following sea' condition. Running test results in long crested wave under following sea condition are available in a few reports2,3; but experiments in directional spectrum wave have not yet been carried out because of non availability of sufficiently uniform experimental area. With the development of techniques to generate directional spectrum wave with sufficient accuracy2,3 laboratory simulation of actual sea state has become possible today. Taking advantage of this merit running tests under following sea condition have been carried out at the towing tank of Yokohama National University and this paper is a report of the test results.

2. Laboratory simulation of directional waves and ship motions

2.1 Generation of directional spectrum waves

The towing tank in Yokohama National University is 100 m long, 8 m wide and 3.5 m deep. The wave generator consists of 16 unit plunger type wave makers which are driven individually by electric servo controller. The input time series data for a particular wave spectrum and a particular directional distribution function are calculated by a main frame computer and are transferred to a micro computer which converts the digital data into analog signals for the wave maker units every 100 milliseconds (for details see ref. 2, 3). In the present study wave signal data were prepared using single summation method for P-M type ITTC spectrum with mean period (T02) of 11.3 sec (1.21 sec in model scale) and significant wave height of 6.7 m (7.6 cm in model scale). Four kinds of directional distribution including one of long crested wave have been chosen. Two commonly used models of directional distribution function are:

\[ D_1(\chi) = C_1 \cos^{2n} \frac{\chi}{2} \quad \text{where} \quad -\frac{\pi}{2} \leq \chi \leq \frac{\pi}{2} \quad (1) \]

\[ D_2(\chi) = C_2 \cos^{2s} \left( \frac{\chi}{2} \right) \quad \text{where} \quad -\pi \leq \chi \leq \pi \quad (2) \]

Using this \( D(\chi) \), we can express wave spectrum as:

\[ S(\omega, \chi) = S(\omega) D(\chi) \]

The following distributions are considered to prepare input wave signals:

a) Using equation (1) with \( n=1 \); commonly known as cos-square distribution.

b) Using equation (2) with s-parameter as 10 and as 25. Approximate relation between s and n is given by \( s = 4.3n \).

In following sea condition we have assumed mean wave direction as 0 degree and wave distribution from
2.2 Description of model and experimental set up.

The model used in this experiment was well known 'SRIO8' the model of a container ship and was the same as was used in ITTC comparative study. The principal particulars of the ship and the model has been given in Table 1; the conditions being kept same as was used in 'head sea' experiment. Experiments have been carried out with this model at Froude numbers 0.2 and 0.275 which correspond to speeds of 0.885 m/s (16.1 knots in Ship scale) and 1.217 m/s (22.1 knots). Experimental set up has been shown in Fig. 1. Waves have been measured by a servo-needle type wave probe placed transverse to the model along the centre of gravity (c. g.) line. In order to estimate directional distribution of encounter waves a wave probe array consisting of five capacitance type wave probes arranged in a pattern as shown has been placed transverse to the model. Heave has been measured by position sensor which consists of a camera that generates electrical signals of vertical movement of a L. E. D. target placed along the vertical line of c. g. of the model. Pitch and roll have been measured by potentiometers placed at c. g. of the model. The vertical accelerometer has been placed port bow at a height equivalent to the top most container level above deck. Though a freely floating ship has six degrees of freedom, considering practicability of experimental set up surge, sway and yaw motions have been restricted during the experiment. The voltage signals from the above instruments have been fed into a micro computer for analog-digital conversion. Apart from the running test mentioned above measurement of waves only, at 50 m from the wave maker with model not in position has also been carried out. These wave data have been used in theoretical estimation of motion spectra as discussed later.

2.3 Analysis of experimental data

Digitally converted data of the analog signals can be plotted in the form of time histories as shown in Fig. 2. One set of test result in long crested wave (left) and another set in short crested wave with 'cos-square' directional distribution (right) obtained under running condition of \( F_n = 0.275 \) in following sea are shown. It is observed from this fig. that the encounter time periods of wave, heave and pitch have become shorter in short crested wave than that in long crested wave. It has been found in the analyzed results that this reduction is around 10 %. As far as the orders of heave and pitch amplitudes are concerned not much of change is observed from this example. But this being the result of one test run, no final conclusion can be drawn from this. However we have observed 17.3 % increase in heave and 7.8 % decrease in pitch in short crested wave of present condition. The order of roll amplitude is seen to be considerably large in short crested wave with maximum single amplitude reaching 20 degrees. As shown in Fig. 1 the position of accelerometer being off-centric there is obvious effect of roll on vertical acceleration apart from the effect of pitch and heave. It may be seen from time history of acceleration in short crested wave (Fig. 2) that low frequency oscillation exists at time intervals of larger roll amplitude.

### Table 1 Principal particulars

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<th>ITEMS</th>
<th>SPECIFIED VALUE</th>
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<td>BLOCK COEF.</td>
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<td>( F_n = 0.275 )</td>
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<tr>
<td>7.07 (sec)</td>
<td>7.02 (sec)</td>
<td>7.04 (sec)</td>
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Fig. 1 Arrangement for model experiment in following sea condition.
Running tests with same speed and same wave condition have been carried out 8-14 times to obtain more than 200 cycles of motion from time histories. One set of time history obtained from single run has been analyzed by Fast Fourier Transformation (FFT) as well as by Maximum Entropy Method (MEM) to obtain 1-dimensional spectrum of wave and motions. The length of data used in single analysis varied from 6.2 minutes (39.6 sec in model scale) to 9.8 minutes (62.5 sec) depending on speed conditions. The final spectrum has been obtained from averaging of spectra correspond to same wave and speed condition. To obtain directional wave spectrum, wave data measured by array of five wave probes in rested condition without model have been analyzed by Maximum Likelihood Method (MLM) which is known to have highest directional resolution of analysis\(^\text{1,7}\). The length of data used for this analysis was around 40 minutes (4.3 min in model scale). Directional power spectrum at 5 degree intervals has been analyzed. The sampling time in all analyses mentioned above running or rested condition has been taken as 0.468 sec (0.05 sec in model scale).

The wave data measured in running condition consist of wave energy in encounter frequency base. Wave frequency and encounter frequency are related by the following equation:

\[ \omega_e = \omega_w - \frac{g}{2} \sqrt{V \cos \chi} \]  

(3)

In following sea condition encounter wave angle \(\chi\) varies from -90 deg to +90 deg and hence encounter frequency becomes zero at \(\omega_w = g/V \cos \chi\) and needs special attention. The distribution of encounter frequency along with that of wave energy over encounter angle in short crested irregular wave under following sea condition has been shown in Fig. 3. One of the contour curves of a directional spectrum wave mea-
Fig. 4 Analyzed directional wave spectrum (a) from measured time history at $F_n=0.0$ and corresponding contour curves (b). Theoretical transfer functions and corresponding contour curves of heave (c, d) and vertical acceleration (g, h); Estimated motion spectra and corresponding contour curves of heave (e, f) and vertical acceleration (i, j); all at $F_n=0.275$
Fig. 5 Analyzed directional wave slope spectrum (a) from measured time history at $F_n = 0.0$ and corresponding contour curves (b). Theoretical transfer functions and corresponding contour curves of pitch (c, d) and roll (g, h); Estimated motion spectra and corresponding contour curves of pitch (e, f) and roll (i, j); all at $F_n = 0.275$. 

Towed Ship Motion Test in Directional Spectrum Waves
sured at \( F_N = 0 \) has been shown in (4). The vertical lines represent wave frequencies and the horizontal lines correspond to wave directions. At \( F_N = 0.275 \) the wave frequency changes to encounter frequency as per equation (3) and the vertical lines of (a) follow a parabolic distribution over directions as shown in (b); the frequency level at ±90 deg., however, remain unchanged. The corresponding points of the contour curve from (a) have been placed on (b) at same frequency-direction intersections and new form of the contour obtained. It’s quite clear from this figure that wave energy overlaps over certain range of frequency and encounter angle in following sea. Hence it is difficult to distinguish from a set of wave data measured in following sea under running condition the actual correspondence of absolute wave frequency and wave energy. For this reason direct application of MLM to obtain directional encounter spectrum in following sea is not possible. In the present study we have thus used wave data at \( F_N = 0 \) to estimate motion spectra as discussed in next article.

### 3. Experimental and theoretical results

#### 3.1 Theoretical estimation of motion spectra.

Transfer functions of motion at 5 degree intervals of encounter wave angle and at Froude numbers 0.2 and 0.275 have been calculated by the well known New Strip Theory (NSM). The degrees of freedom in sway, yaw and surge have been restricted in the calculation in accordance with experimental condition. Two dimensional motion spectra have been estimated using transfer function data and directional wave spectrum analyzed from experimental data at \( F_N = 0 \) by the following equation:

\[
S_x(\omega_0, \chi, V) = [H_x(\omega_0, \chi, V)] S(\omega_0, \chi)
\]

where \( S(\omega_0, \chi) \) = wave spectrum at \( V = 0 \)

In Fig. 4 and Fig. 5 examples of calculated transfer function of heave (4c and 4d), vertical acceleration (4g and 4h), pitch (5c and 5d) and roll (5g and 5h) for \( F_N = 0.275 \) have been shown. For calculation of roll we have introduced damping coefficient obtained from free roll experiment. Transfer function of vertical acceleration (4h) is not symmetrical to \( \chi = 0 \) line, because of off centric position of accelerometer. The transfer functions of pitch and roll have been non-dimensionalized by wave slope. Hence in order to estimate motion spectra of pitch and roll by equation (4) we need to use wave slope spectrum instead of wave spectrum. Fig. 4 shows an example of analyzed directional wave spectrum (a) and corresponding contour curves (b), estimated spectrum of heave (e) and corresponding contour curves (f), estimated spectrum of roll (i) and corresponding contour curves (j) also drawn on wave absolute frequency base. Wave characteristics in the above figures are same as to that in Fig. 2. It may be seen from these figures that spectrum of heave follows almost the same pattern as that of wave, however spectrum of pitch has a narrower directional spreading as expected from distribution of transfer function. Spectrum of roll has peak at the region of 50-55 deg. of encounter angle and reaches zero at 0 deg. Also in case of roll it may be seen that the frequency range within which transfer functions of roll vary, covers considerable energy region in wave spectrum. This results very high order of roll in following sea condition. In head sea condition, the order of roll is considerably small and hence pitch has the maximum effect on vertical acceleration. We have seen that in head sea, acc. spectrum follows same pattern as that of pitch. But in following sea the order of roll being very high, effect of roll dominates at certain encounter angles and we can observe similarity between spectrum of roll and vertical acceleration.

The motion spectra thus estimated on wave absolute frequency base have been transformed to encounter spectra by the following equation:

\[
S_x(\omega_0, \chi) = \frac{S_x(\omega_0, \chi, V)}{1 - \frac{2\omega_0^2}{g} V \cos \chi}
\]

The resultant encounter spectra have then been integrated over wave encounter angle to obtain one dimensional encounter spectra which can be directly compared with the experimental results. Apart from motion spectra, the analyzed directional wave spectra at \( F_N = 0 \) have also been transformed into one dimensional encounter spectra in the same manner to establish accuracy of encounter wave spectra measured in running condition. An example of directional encounter wave spectrum obtained by equation (5) has been shown in Fig. 6. The original wave spectrum at \( F_N = 0 \) is same as that shown in Fig. 4. If compared with Fig. 3, the spike like peaks in the present figure would be the results from insufficient width of \( \Delta \omega \) and \( \Delta \chi \) in the mesh. The encounter frequency and encounter spectral ordinates depend on wave encounter angle (as can be seen in equation (3) and (5)) and thus distribution of spectral peaks over encounter angle does not follow a straight line. It can be seen from Fig. 6 that in case of following wave, the distribution of spectral peaks of encounter spectrum take concave form over encounter angle and it is clear from this figure that this spectrum when integrated over encounter angle will maintain relatively wider distribution over encounter frequency than that in case of long crested wave. This has been observed in figures of one dimensional spectra mentioned later and so the mean period is altered from that of long crested wave.
3.2 Comparison of experimental and theoretical results

3.2.1 On Motion spectra

Encounter spectra of wave and motions in long crested wave at $F_n=0.275$ with same wave characteristics as that shown in Fig. 2 have been presented in Fig. 7 where experimental results analyzed by FFT (marked by *), experimental results analyzed by MEM (marked by $\bigcirc$) and calculated result by NSM (marked by X) have been compared. Analyzed frequency range has been 0.08-1.00 (rad/s) and energy integration has been made in this region. Among them the wave spectrum marked by ‘CAL’ corresponds to that transformed from $F_n=0$ and used in theoretical estimates. The spectral density ordinates in these figures have been normalized (divided by $m_o$ of wave spectrum) to take into account any variation of wave energy measured in running condition from that in rested condition. It may be seen from these figures that the calculated results are very close to analyzed results by MEM. The peak levels of encounter spectra in following sea are expected to be very high due to the presence of singularity at $\omega_n=g/2V \cos \chi$ (ref: equation 5). However this fact is not clearly established in the analyzed results by FFT. It should be noted that during the analysis, FFT has been applied to the whole time history of single run so that the frequency resolution in Fig. 7 is maximum.

Fig. 8 shows similar results in directional spectrum wave for two speed conditions with $F_n=0.2$ and $F_n=0.275$ (the frequency range is wider than that in Fig. 7).

In general these spectra are found to be widely distributed over encounter frequency compared to that in long crested. This is justified by the fact that encounter frequency varies with wave encounter angle and the spectral density ordinates in these figures represent summation over all encounter angles (as discussed in article 2.3). If we compare the results, in most cases the calculated results are closer to analyzed results of FFT. However the results of MEM analysis maintain relatively higher ordinates at peak in most cases. Calculated heave spectra are well in order with that obtained from experiment for both speed conditions (this was observed in head sea results also). The order of calculated pitch is slightly higher than that of experiment,
Fig. 8 Analyzed and estimated motion spectra in short crested irregular waves. Wave conditions are same as of Fig. 4 & Fig. 5. $T_{oz}$ of wave is 11.34 sec (1.2 sec in model scale)
but the deviation is less than that in head sea condition. In head sea roll spectrum, the frequency level at peak corresponds to natural frequency of roll, because peak frequency of wave is away from roll natural frequency. However in following sea condition we find that frequency level corresponding to peak of wave spectrum also plays an important role as this frequency is very close to the natural frequency of roll. In case of $F_n = 0.275$ we can observe two peaks in the roll spectrum corresponding to peak of wave spectrum and natural frequency of roll. This can be seen also in Fig. 5 (1) as two dimensional description. Vertical acceleration is a combined effect of heave, pitch and roll with pitch playing the prominent role in head sea condition. In the present case we can observe considerable energy level in acceleration spectrum near natural frequency of roll suggesting greater effect of roll on vertical acceleration. However, as we can see later, the energy level or significant value of vertical acceleration in following sea is very small compared to that in head sea. Apart from this the static effects of roll and pitch angle have also entered into the measured results of experiment; because we have used strain gauge type accelerometer. These effects are given by

$$a_p(t) = g(1 - \cos \theta(t)),$$

$$a_r(t) = g(1 - \cos \phi(t)).$$

(6)

Especially when roll angle is very large this effect results low frequency oscillation which can be observed in analyzed spectra. This static effect is not considered in calculation of transfer functions of vertical acceleration and the deviation in the calculated spectra is thus justifiable. In the following sea condition, non linear roll motion like parametric oscillation would be possible from theoretical point of view, but this phenomena have not been studied in detail in the present paper.

3.2.2 On Statistics of Significant Motion

In order to investigate distribution of significant motion over directional spreading of wave Fig. 9 has been prepared. In this figure the ratio of root mean square (RMS values) of motion and wave elevation has been plotted against angular spreading factor which is given by inverse of $s$-parameter of equation (2). The experimental results (marked by *) and calculated results (marked by $\bigcirc$) for speed conditions of $F_n = 0.2$ and $F_n = 0.275$ have been presented. $1/s = 0$ represents the condition of long crested wave and $1/s = 0.2, s = 5$ has been taken equivalent to 'cos-square' spreading. In general the calculated results are in better agreement with experiment than those observed in head sea condition, for this case of mean period ($T_{ave} = 11.3 \ sec$).

In case of $F_n = 0.2$ we find that heave initially reduces up to a spreading factor of around 10 ($1/s = 0.1$) and then gradually increases with widening of spreading and reaches almost the same level as that of long crested at around $s = 5$. This trend however has not been observed in calculated result where there is a gradual increase in heave with widening of spreading. In case of $F_n = 0.275$ the calculated results are almost in line with the experimental results and we observe continuous increase in heave over directional spreading. Calculated results of pitch in short crested wave are around 18% higher than experimental results with $F_n = 0.2$ and around 9% higher with $F_n = 0.275$. Pitch initially reduces from long crested wave to a directional spectrum wave with a spreading factor of around 10 and then slowly increases with spreading. The order of pitch at a spreading factor of 5 (considerably wider spreading) is found to be less than that in long crested wave. Roll doesn't exist within long crested wave but sharply increases with directional spreading of wave. However initial trend of sharp increase does not continue with widening of spreading. We have observed in Fig. 5 that transfer function of roll has a peak at around 50 deg. of wave encounter angle. Hence a higher degree of roll is expected even in relatively narrow spreading wave (with wave energy ranges from $-60 \ deg.$ to $+60 \ deg.$). However widening of wave energy distribution beyond $\pm 60 \ deg.$ does not effect roll amplitude significantly. This trend has been observed in this figure. Apart from their trend, differences between experiment and calculation seem to be large, and further investigation is needed for large rolling in following directional spectrum waves. On distribution of acceleration response over angular spreading factor, a combined effect of heave, pitch and roll can be observed clearly. The initial increase in vertical acceleration from angular spreading factor of 0 (long crested) to around 0.05 is justified by sharp increase of roll over this region. The declining trend between angular spreading factors of 0.05 and 0.1 would be the effect of heave and pitch over this region. Beyond this region, with widening of spreading we can observe a gradual increase in vertical acceleration. The calculated results of vertical acceleration are slightly higher than experimental results in case of $F_n = 0.2$; but follow the similar distribution over angular spreading. The calculated results in case of $F_n = 0.275$, however, are in good agreement with experimental results. On the whole it may be said that trends of both experiment and calculation are similar, and the agreement is better at $F_n = 0.275$ than that at $F_n = 0.2$ for the present case of observation. Angular spreading effect cannot be neglected both in roll and vertical acceleration apart from heave and pitch. The effect of wave height is not so significant if we consider the points within the triangle in Fig. 9.

3.2.3 On statistics of period of motion

In short crested waves, as can be seen in equation (5) statistics of period depends on the wave spreading factor and ship speed. In order to investigate variation of period of motion with wave spreading Fig. 10 has been prepared where the ratio of encounter period ($T_{ave}$)
Fig. 9 Ratio of root mean square of motion and wave elevation (in ship scale) against angular spreading factor \((=1/s)\). Significant wave height is around 6.7 m. 
\(\triangle\) correspond to experimental data with significant wave height of 3.5 m.
in short crested and that in long crested have been plotted against angular spreading factor \((\frac{1}{s})\). The cases in head sea which was not presented in previous report have also been shown in the same figure. In this figure the mean periods obtained from spectral analysis has been shown. Statistical analyses of time histories measured during the experiment have also been carried out and deviations in results from that obtained by spectral analyses have been found to be below 5%. We have thus considered the results of spectral analyses only in this paper. Cases of wave and heave with speed condition of \(F_n = 0.275\) have been shown here as an example. Wave considered in this case, has a mean period \((T_0)\) of 11.2 sec (1.2 sec in model scale). As expected while considering spectral deformation, in both the cases (wave and heave) we can observe a gradual decrease in mean period over angular spreading factor just opposite to the results obtained in head sea condition. Similar trend has been observed in other modes of motion and other speed conditions. About period, both MEM and FFT methods give almost the same results in spite of spectral difference. Furthermore the agreement with calculation is fairly good.

### 4. Statistical prediction of ship motions

We have carried out experiments on ship motions in long crested and in short crested waves covering different models of spreading and with three different wave periods in head sea condition and with single wave period in following sea condition. The test results have been presented in our previous report and in the first half of the present paper. However with these results in hand it is difficult to conclude on behaviour of ship in all possible period of encounter in sea waves taking into account the effect of directional spreading. To investigate this topic, short term predictions have been carried out using transfer functions of motions calculated by NSM. As we have observed while comparing calculated results with experimental results, this calculation will lead to over estimation of heave, pitch and vertical acceleration in head sea and under estimation of roll in both head and following sea. The results of heave, pitch and vertical acceleration in following sea however, will not differ much. Anyhow the calculated results will need corrections to the order of actual transfer function. Right now research is in hand to estimate directional transfer functions from experimental results in directional spectrum wave and we are going to present revised results of statistical prediction in future opportunity.

#### 4.1 Short Term Prediction

A P-M type ITTC spectrum has been chosen for short term prediction. The directional distribution function has been estimated considering equation (2) with different spreading parameter. The results in case of \(F_n = 0.275\) have been presented in Fig. 11 where ratio of significant motion and significant wave height has been plotted against mean wave period \((T_{02})\). Curves with continuous line represent results in head sea and that with broken line represent results in following sea. L.C. (marked by *) corresponds to Long Crested wave, N. S. (marked by 0) corresponds to Narrow Spreading wave and W. S. (marked by X) corresponds to Wide Spreading wave. In head sea, order of heave increases sharply over mean wave periods of 4.0 sec-10.0 sec and almost stabilizes over 10.0 sec-18.0 sec. This trend is similar to frequency-transfer function relation. In following sea there is gradual increase in heave with increase in mean wave period. Pitch is maximum in the region of 8.0-10.0 sec of mean wave period in both head sea and following sea, the order in head sea is around double that in following sea. Over a wave period of 4.0 sec-9.0 sec pitch increases with directional spreading, but the relation changes and it decreases over wave period of 9.0 sec-18.0 sec. The higher order of heave and pitch over the wave period of 8.0-10.0 sec in head sea is due to the fact that peak of encounter wave spectrum at this region is closer to the natural frequencies of heave and pitch. Vertical acceleration follow the same pattern as that of pitch, especially in case of head sea. Also
order of acceleration in following sea is almost negligible compared to that in head sea. In transient water wave experiment we have observed that NSM calculation over-estimates transfer functions of heave and pitch near natural frequencies of the corresponding motions. In head sea, encounter wave energy covers natural frequency of heave and pitch, but in following sea encounter wave energy ranges over fairly lower frequencies than the natural frequencies of above motions. This justifies why calculated results in following sea are in better agreement with experimental results. Also in head sea with longer wave period where encounter wave energy drifts away from natural frequencies of heave and pitch and the calculated results are closer to experimental results. Thus theoretical estimation of transfer function needs further investigation.

Roll gradually increases with wave period in head sea, however in following sea it reaches maximum at around 8.0 sec of mean wave period where encounter wave energy has a peak in the region of natural frequency of roll. The most significant feature of roll response is that it becomes very large in following sea condition, and this is very much in contrast to heave, pitch and vertical acceleration. Furthermore, the effect of directional spreading on roll cannot be neglected in following sea.

5. Conclusion

In this paper we have presented, for the first time, laboratory test results of running ship motions in directional spectrum wave under following sea condition and theoretical estimation of motions by New Strip Method has been compared with it. The used model is of a container ship and only heave, pitch and roll have been made free in the experiment. Measured vertical acceleration is at off-centric point. Theoretical calculations have also been introduced under similar condition. Furthermore we have presented short term prediction of motions for both head sea and following sea conditions using theoretically estimated transfer functions and the effect of short crestedness have been discussed. To sum up above mentioned discussions we can summarize the conclusions as follows.

a) About spectrum, analyzed one dimensional encounter wave spectrum obtained by taking mean of spectra analyzed from time history of single run in following sea showed good agreement with encounter wave spectrum transformed from analyzed 2-D spectrum at $F_n=0$, suggesting accuracy of analyses method (MLM) applied in the present study.

b) In following sea where number of encounter wave peaks for analysis is very few, experimental results analyzed by MEM showed better estimation of spectral peak than FFT method especially in long
c) Theoretically estimated encounter motion spectra using NSM show fairly good agreement with analyzed spectra (by MEM) from experimental data in following long crested wave.

d) Theoretically estimated encounter heave spectra and significant values in following short crested wave are in good agreement with analyzed results from experimental data. In case of encounter pitch spectra and significant values in following short crested wave, calculated results show slight over estimation if compared with experimental results.

e) Theoretically estimated encounter roll spectra and significant values in following sea are not in good agreement with analyzed results from experimental data and further investigation on this subject is needed.

f) About effect of directional spreading, in following sea with widening of directional spreading of wave, heave showed increasing path and pitch showed slightly decreasing path judging from experiment and calculation. The agreement between experiment and calculation is better at $F_n = 0.275$ than that at $F_n = 0.2$. On the whole, spreading effect is not so large on heave and pitch amplitude.

g) On the contrary, in following sea, roll sharply increases with widening of wave spreading and order of roll is very high compared to that in head sea. So the spreading effect cannot be neglected for estimation of roll response. Higher order of roll in following sea effects vertical acceleration considerably, the magnitude of acceleration however is very small.

h) With widening of directional spreading of wave, mean period of wave and motions gradually decreases in following sea and increases in head sea reaching about 10 to 20% at 'cos-square' distribution and so this variation is not negligible.

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