Directives for the Design of a Linear Quadratic Autopilot for Minimum Fuel Consumption

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Summary

A performance index for an optimal Linear Quadratic (LQ) autopilot for minimum fuel consumption was determined. This performance index and resulting system closed-loop eigenvalues were compared with several indices (and eigenvalues) suggested by different researchers. Directives for the ideal positioning of the eigenvalues regarding the reduction of the propulsive losses and fuel consumption are discussed.

Key Words : Autopilot, Directives for Design, Minimum Fuel Consumption, Optimal Linear Quadratic Regulator, Performance Index.

1. Introduction

A basic aspect of a good product or service is its cost. In the maritime transport service, fuel costs can be roughly estimated between 20 to 35% of the operational cost of a vessel. It is in the interest of every armateur to reduce the cost of fuel consumption, which will allow them greater competitiveness in the market.

Among the alternatives to reduce fuel consumption, there are a few which have very low installation and application costs, such as the autopilot. Since the steep increase of the petroleum price in 1973, fuel consumption became a major factor in a ship's operational cost and the autopilot concept was looked at from another perspective. This examination added an extra parameter to the autopilot project : the reduction of propulsive losses or the reduction of the fuel consumption. Briefly, the autopilot should keep the ship's heading through a procedure that minimizes the propulsive losses generated from a controller actuator, such as the rudder.

Independent from the control technique, a fundamental point to the autopilot is the formulation of the performance index. Since the autopilot has the goal of minimizing this index, it is essential to the autopilot's good performance that the performance index is a trustworthy indicator regarding fuel consumption.

In previous reports⁽¹⁾⁽³⁾ an extensive review of the literature on the subject was presented. Also a map of the performance index was then determined, using deterministic optimal control techniques (Linear Quadratic) and simulating the ship operation in a straight

course through deep non-restricted waters and with different external disturbances ; waves, wind and current approaching from different directions. This procedure was repeated both for a slender-form liner type vessel, Mariner class, and for a full-form type vessel, the tanker "Esso Osaka".

The proposed performance indices in the literature were then compared to the obtained map of performance index.

The potentialities and limitations of tuning the performance index regarding the reduction of the propulsive losses and fuel consumption were also discussed.

This report summarizes the earlier results and, additionally, describes directives for the design of an autopilot for minimum fuel consumption. These directives were obtained from the analysis of the closed-loop eigenvalues.

The paper is organized as follows***.

- the optimal linear quadratic regulator is described in chapter 2 ;
- the vessel movement mathematical modelling and external disturbances are presented in chapter 3 ;
- the proposed performance indices in the literature are grouped in chapter 4 ;
- the procedure adopted for the performance index mapping is shown in chapter 5 ;
- a few simulation results and the obtained maps are shown in chapter 6 ;
- comparison with other performance indices are shown in chapter 7 ;
- the closed-loop eigenvalues are analysed in chapter 8 ;
- directives for the design are discussed in chapter

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*** Any additional piece of information can be found in references 1 & 2, except where indicated.

9 ;

- conclusions are presented in chapter 10.

2. Optimal Linear Quadratic (LQ) Regulator

Consider a system described by the following linear mathematical model :

$$\dot{x}(t) = A * x(t) + B * u(t) \quad (1)$$

where $x(t)$ is an n-dimensional vector of the state variable ;

$u(t)$ is an m-dimensional vector of the control ;

$$\dot{x}(t) \triangleq dx(t)/dt;$$

A, B are known matrices.

A cost function that penalizes the system error and the energy required for the controller action is the quadratic performance function :

$$P. I. = \int_{t_0}^{\infty} [x^T(t) * Q * x(t) + u^T(t) * R * u(t)] dt \quad (2)$$

where Q is a symmetric matrix at least semi-defined positive ;

R is a symmetric matrix defined positive.

The optimal control law is given by :

$$u(t) = -R^{-1} * B^T * P * x(t) \quad (3)$$

where P is obtained from :

$$P * A - P * B * R^{-1} * B^T * P + Q^T + A^T * P = 0 \quad (4)$$

Equation (4) is called the Algebraic Riccati Equation and can be numerically calculated by different procedures, such as the backward in time integration, Kleinman Algorithm, or Signal Matrix Algorithm. The adopted procedure was the Signal Matrix Algorithm.

3. Vessel Movement Mathematical Modelling

There are two distinct mathematical models of the ship movement. A linear model is used as a design tool for the project of the autopilot and it is called the Work Model. A non-linear mathematical model is used as a simulation tool of the ship movement and it is called the Evaluation Model.

3.1 Work Model

Considering that the ship's movement in the horizontal plane is non-coupled to the movement in the other plane and that the ship sails through open and non-restricted waters, then the following state equations for forces and moments are obtained :

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\delta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a16 & a17 & a18 & 0 \\ a19 & a20 & a21 & 0 \\ 0 & 0 & a1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \delta \\ \psi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -a1 \\ 0 \end{bmatrix} * \delta_c$$

Fig. 1 Movement in the Plane

Table 1 Work Model Coefficients

Coef	Cargo Ship	Cargo Ship	Tanker	One-Oil		Tanker	Tanker
	Mariner	Serie 60	190,000 ton	Ballast	Full	Ballast	Full
a16	-0.7692	-0.8937	-0.5963	-0.2978	-0.4238	-0.4538	-0.4310
a17	-0.3319	-0.2857	-0.3721	-0.2803	-0.3619	-0.4327	-0.4482
a18	0.1707	0.1062	0.1029	0.1159	0.1476	0.0956	0.1379
a19	-3.3823	-4.3663	-3.6544	-4.3692	-2.9571	-4.0081	-1.9827
a20	-1.6428	-2.7179	-1.8704	-1.6388	-1.7095	-1.8201	-1.7586
a21	-1.6301	-0.9157	-0.7923	-0.7722	-1.0047	-0.8071	-1.1465
a1	-6.9965	-6.8294	-12.3434	-13.0221	-12.1915	-14.3532	-13.1467

$$+ \begin{bmatrix} af1 & af2 \\ af3 & af4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F2 \\ F6 \end{bmatrix}$$

$$\dot{u} = a2/a5 * u + a4/a5 * \delta - (a2 - a3)/a5 - F1/a5 \quad (5)$$

where as shown in Fig. 1., ship's movement characteristic variables are :

V is the ship's translatory velocity ;

u & v are the velocity components related to the x and y axis respectively ;

ψ is the heading angle ;

β is the drift angle ;

δ is the rudder angle ;

δ_c is the commanded rudder angle ;

ai 's are the ship movement linear model coefficients ;

$a1$ is equal to $-1/\tau$;

τ is the time constant (approx 3 sec) ;

afi 's are the force and moment coefficients ;

Fi 's represent force and moment.

All values involved were made non-dimensional, according to the SNAME procedure. The linear coefficients for six different vessels are given in Table 1. Note that the surge equation is non-coupled with the others - sway and yawing.

3.2 The Evaluation Model

A non-linear model can be obtained in the same way as the Work Model just by the inclusion of the Taylor Expansion superior order terms. Usually, the non-linear model includes terms up to and including the third

order, since superior order terms do not increase the model precision.

The study of the rudder machine dynamic is not one of the purposes of this paper. However, a good representation is necessary for the Evaluation Model. The following model was adopted for the rudder machine dynamic⁵⁾:

$$\dot{\delta}(t) = (-1)^n * V_{rud} * \{1 - e^{-(1^{(n-1)}) * (\delta_c - \delta) * t/c}\} (6)$$

where V_{rud} is the rudder speed, considered equal to 3 degrees/sec ;

δ is the rudder angle ;

δ_c is the commanded rudder angle ;

n is equal to 1 for $\delta_c - \delta < 0$;

is equal to 2 for $\delta_c - \delta \geq 0$;

c is a coefficient that adjusts the function inclination at the origin.

The maximum rudder angle is constrained to a range of $\pm 35^\circ$.

3.3 External Disturbances

Any vessel sailing on the sea surface is subjected to the combined action of wind, wave and current. However, in this paper only the wind action is considered, what is sufficient for the ultimate aim of this paper, which is establishing directives for the design of an autopilot. The performance index mapping considering the combined environmental disturbances, waves (1st order and resistance increase in waves), wind (steady wind) and current, was presented in previous reports. Note that the relation between obtained optimal weights of the performance index for the combined environmental disturbances is about the same as considering wind only.

The wind action can be separated into two components, the first representing the action of a steady wind and the second representing the action of turbulence and of gust. Only the steady wind component was used. The Davenport model of the wind vertical variation was used to describe the steady wind component, and its square value integrated along the vessel's projected area to obtain the disturbance estimation.

4. Performance Index

In the second half of the 60's and the beginning of the 70's the first works analysing the increase of the propulsion resistance, while sailing in a straight course, were published. The pioneer paper on the subject was by Nomoto⁶⁾, who determined from the Laws of Newton, a transfer function for the autopilot, which related the heading deviation with the rudder angle, or in other words, what the necessary rudder angle would be for a desired heading angle deviation. Nomoto also pointed toward the possible propulsion increase while sailing in a straight course.

Koyama⁷⁾ proposed the minimization of the propul-

Fig. 2 Histogram of the Analysed Bibliography

sive losses through an adequate choice of the P.I.D. controller gains. These gains should be chosen in order to minimize a quadratic performance index of the average heading deviation and the rudder angle. Koyama suggested a composition of this performance index for a Mariner class vessel.

Norrbin⁸⁾ showed that the performance index suggested by Koyama was related to the propulsion resistance increase, giving a physical meaning for the suggested cost function. Norrbin suggested a value forty times smaller for a Mariner class ship.

In 1973, with the high increase in the petroleum price, any subeject associated with energy conservation received attention. The automatic pilotage was one of the alternatives for fuel conservation. At that time, different researchers studied this problem and this phase reached its apex at the first, and even until now, the only SSSAC-Symposium on Ship Steering Automatic Control in Genova-Italy 1980. After this congress, there was work continuity, but with less intensity, as shown in Fig. 2.

A basic aspect common to all proposed autopilots is the formulation of the performance index. Since the autopilot has the goal of minimizing this index, it is essential for the autopilot's good performance that the performance index is a trustworthy indicator regarding fuel consumption, but on this there is no consensus.

The proposed performance indices can be roughly organized into two main parties :

- heavily weigh the rudder movement (Koyama party) ;
- lightly weigh the rudder movement (Norrbin party).

5. The Performance Index Mapping

Considering the computational resources, it was decided to adopt a similar approach to the problem as the one of Tiano⁹⁾ and Amerongen^{10&11)}. Instead of entering into propulsion resistance considerations to determine the performance index, the performance index was mapped regarding the objective of fuel consumption minimization as shown in Fig. 3. This means that first a performance index is chosen, then the corre-

Fig.3 Performance Index Mapping Regarding Consumption

sponding optimal controller is calculated. Finally, a voyage is simulated and the fuel consumption is calculated.

In order to use this procedure, it is necessary to define precisely the autopilot purpose. The purpose of the automatic pilot is defined as :

Maximize the vessel's speed component in the desired heading direction and minimize the vessel's transversal deviation from the desired trajectory.

It is considered that the transversal deviation $|\Delta\epsilon(Yog)|$ from the desired trajectory at the final time

Table 2 Turning Circle Parameters - Mariner

rud. ang. (°)	advan. at 90° (m)	trans. at 90° (m)	max. advan. (m)	tact. diam. (m)	time for h. 90° (sec)	time for h. 180° (sec)	max. trans. (m)	steady tn. rad (m)	steady dr. ang (°)	fin. spd. knot
5	1077.4	728.7	1080.0	1532.7	196.0	365.0	1535.1	737.3	4.7	13.2
-5	1077.4	728.7	1080.0	1532.7	196.0	365.0	1535.1	737.3	-4.7	13.2
10	773.1	504.1	776.2	1067.2	141.2	263.6	1070.2	485.4	6.3	12.1
-10	773.1	504.1	776.2	1067.2	141.2	263.6	1070.2	485.4	-6.3	12.1
15	640.4	401.0	644.0	850.8	117.0	217.9	854.0	361.0	7.6	11.0
-15	640.4	401.0	644.0	850.8	117.0	217.9	854.0	361.0	-7.6	11.0
20	563.5	339.1	567.2	850.8	103.0	217.9	720.3	282.4	8.7	10.1
-20	563.5	339.1	567.2	850.8	103.0	217.9	720.3	282.4	-8.7	10.1
25	512.9	296.3	516.7	622.9	93.8	172.3	626.4	226.2	9.7	9.2
-25	512.9	296.3	516.7	622.9	93.8	172.3	626.4	226.2	-9.7	9.2
30	477.2	264.4	481.2	552.0	87.4	159.1	555.6	183.7	10.6	8.3
-30	477.2	264.4	481.2	552.0	87.4	159.1	555.6	183.7	-10.6	8.3
35	451.2	240.1	455.2	496.3	82.8	149.5	499.7	150.2	11.5	7.4
-35	451.2	240.1	455.2	496.3	82.8	149.5	499.7	150.2	-11.5	7.4

instant T is sailed as in an ideal sail. The speed component in the desired heading direction and its mean value is given by :

$$V_p(t) = [v(t)^2 + u(t)^2]^{1/2} * \cos[\psi(t) - \psi_{\text{desired}} - \beta(t)] \quad (7)$$

$$V_{pm} = 1/T \int_0^T V_p(t) dt \quad (8)$$

The evaluator becomes :

$$A_v = 1/T \{ \int_0^T V_p(t) dt - |\Delta\epsilon(Yog)| \} \quad (9)$$

6. Simulation Results

There is no better confirmation of a theory than practice. However, the costs involved in an analysis through full-scale tests are extremely high, so that it is usual to take intermediate steps in order to reduce costs. In this paper, the solution implementation was done through computer simulations and, consequently, the results are limited by the utilized models and approximations. The results can be divided into three main parts :

- system dynamics ;

Fig. 4 Wind Force and Moment - Mariner

Fig. 5 Wind Force and Moment - Eppo Osaka

- regulator operation ;
- performance index mapping.

In the results, the wind velocity corresponds to a referential fixed to the earth and with an axis in the same direction of the original straight course.

6.1 System Dynamics

A few intermediate results are shown as follows :

- Table 2. Mariner turning circle simulation ;
- Fig. 4. & 5. wind force for the Mariner & Eppo Osaka ;

6.2 Regulator Operation

- Fig. 6. Mariner sailing in a straight course under a steady bow of 15 m/s wind, blowing 60° degrees star-board, receives a command to change heading of 60° degrees.

6.3 Performance Index Mapping

- Fig. 7. the obtained performance index. In this figure, the weighting coefficients of the state vector are (Q_{11} , Q_{22} , Q_{33} , Q_{44}) corresponding respectively to the state vector (v , r , δ , ψ) and the weighting coefficient of

Fig. 6 Change of Heading in Wind - Mariner

the control vector is (R_{55}) corresponding to the control input (δc).

7. Comparison with Other Performance Indices

The main divergence exists between the performance indices suggested by Koyama⁷⁾ and by Norrbin⁸⁾, which has the same form given by :

$$J = \psi^2 + \lambda * \delta^2 \quad (10)$$

where ψ is the vessel heading ;

δ is the rudder angle ;

λ is a constant.

Koyama suggested $\lambda=8$ for the Mariner, while Norrbin suggested a value of $\lambda=0.084$ for full form vessels and $\lambda=0.14$ for slender vessels.

An equivalent value for λ obtained from the performance index mapping is given by the relation between R_{55}/Q_{44} , shown in Fig. 8. The maps indicate that for the external disturbances with a small approaching angle, R_{55}/Q_{44} is very high, or in other words, the commanded rudder angle should be constrained, since the external disturbances do not deviate the vessel strongly from the straight course. As the approaching angle increases, the smaller R_{55}/Q_{44} becomes. This

Fig. 7 The Variation of the Optimal Coefficients with the Disturbance Angle

Fig.8 Relation between Rudder and Heading Coefficient

Table 3 A Few Values of R55/Q44

Approaching Angle(deg)	Mariner (R55/Q44)	Esso Osaka (R55/Q44)
0°	100.	100.
30°	12.	3.
90°	0.07	0.1

Table 4 A Few Values of $\lambda 1$ and $\lambda 2$

	Clarke		Amerongen		The Maps			
	Tanker Const. Rpm	Container Const. Thrust	Tanker	Cargo Ship	Esso Osaka Const. Rpm	Osaka Const. Thrust	Mariner Const. Rpm	Mariner Const. Thrust
$\lambda 1$	71,048	335.2	15,000	1,600	16,901	169,010	4,097	40,970
$\lambda 2$	4.218	0.128	8.	6.	100.	0.1	100.	0.07

happens faster in the case of the Esso Osaka (unstable vessel). A few values of R55/Q44 are given in Table. 3. Note however that a direct comparison between λ and R55/Q44 is inadequate, since for the adopted LQ regulator, there are other states-input other than ψ, δ .

In this paper, the state variables considered are (v, r, δ, ψ) and the input is (δ_c) , and all the state variables and

the input are considered measurable. However, there is no appropriate commercial piece of equipment with an adequate precision for measuring the sway speed. All the authors adopted an approximation for the sway velocity, which depends on the fact that the vessel seems to turn about a fixed pivot point near the bow :

$$v = k * r \quad (11)$$

where v is the sway speed ;

r is the yaw rate ;

k is approximately $0.4 * l / pp$ (ship's length).

Adopting the above mentioned approximation for the maps obtained, a rough comparison can be made with the results obtained by Clarke¹²⁾ and Amerongen^{10&11} (Table 4.) :

$$J = \psi^2 + \lambda 1 * r^2 + \lambda 2 * \delta^2 \quad (12)$$

Fig.9 Deterioration with the Sway Speed Approximation

Fig.10 Deterioration for a P.I.D. Controller

Fig.11 Deterioration for a P.D. Controller

It is interesting to verify the effect of the sway speed approximation on the fuel consumption. This was implemented for an external disturbance composed of only wind at 15 m/s and the results are shown in Fig. 9. for the Esso Osaka. For the Mariner, the approximation does not significantly deteriorate the fuel consumption evaluator. In fact for a full-form type vessel, the coefficient k of equation (11) is more likely given by $0.5 * l_{pp}$, which can be obtained from the turning test.

Another comparison is shown in Fig. 10. between the LQ Controller and a PID Controller for an external disturbance composed of wind at 15 m/s. In the case of the Mariner, the PID Controller has an outstanding actuation without any practical deterioration of the fuel consumption evaluator. But for the Esso Osaka, the deterioration is strong. In fact, a PD Controller had a much better performance regarding fuel consumption, as shown in Fig. 11.

8. Closed-Loop Eigenvalues

Considering the linear system given in equation (1) and the feedback component of the control law given in equation (3), the closed-loop system becomes :

$$\dot{x}(t) = (A - B * R^{-1} * B^T * P) * x(t) \quad (13)$$

The solution of (13) is given by :

$$x(t) = e^{(A - B * R^{-1} * B^T * P) * t} * x(0) \quad (14)$$

While the components corresponding to the faraway poles will vanish comparatively fast, the components corresponding to the closer poles will have the lasting effects. The simulation results lead to the conclusion that the weight corresponding to the yaw angle (Q_{44})

and commanded rudder angle (R_{55}) are the key weights for the design of an optimal regulator, since they are the weights whose variation strongly influence the closer poles position.

The comparison of the dominant closed-loop eigenvalues in the range of the significant eigenvalues, for an LQ controller with the performance index in the form given in equation (10), and with the performance index given in equation (2) with weights of Fig. 7. is presented in Fig. 12. An interesting aspect of this figure is the proximity of the complex poles for the directional stable and unstable vessels for external disturbances approaching the vessel from a small angles and for higher angles, in fact for these angles the poles almost perfectly match.

9. Directives for Design

As mentioned in chapter 5, the performance index mapping strategy was derived from kinematics considerations. In order to set directives for design, an understanding of the results is a prerequisite, and a good way to obtain correct answers is by making appropriate questions. In the case on study, the appropriate questions may be :

- Why the performance index should be changed according to the external forces?
- Should the dominant closed-loop poles be the same for both stable and unstable vessels?

9.1 Why

The purpose of the automatic pilot was defined in chapter 5 as :

Maximize the vessel's speed component in the desired heading direction and minimize the vessel's transver-

Fig. 12 Poles Placement

sal deviation from the desired trajectory.

It was noted during the simulations, that the transversal deviation from the desired trajectory had no significant effect whatsoever. This leads to the analysis of only the vessel surge non-linear equation given by :

$$(m - X_{\dot{u}}) * \dot{u} = f1(u, v, r, \delta) \quad (15)$$

where for the Mariner :

$$\begin{aligned} f1(u, v, r, \delta) &= X^* + X_u * \Delta u + 1/2 * X_{uu} * \Delta u^2 \\ &+ 1/6 * X_{uuu} * \Delta u^3 + 1/2 * X_{uv} * v^2 \\ &+ (1/2 * X_{rr} + m * x_v) * r^2 + 1/2 * x_{\delta\delta} * \delta^2 \\ &+ 1/2 * X_{vuu} * v^2 * \Delta u + 1/2 * X_{rru} * r^2 * \Delta u \\ &+ 1/2 * X_{\delta\delta u} * \delta^2 * \Delta u + (X_{vr} + m) * v * r \\ &+ X_{v\delta} * v * \delta + X_{r\delta} * r * \delta + X_{vru} * v * r * \Delta u \\ &+ X_{v\delta u} * v * \delta * \Delta u + X_{r\delta u} * r * \delta * \Delta u \quad (16) \end{aligned}$$

where $\Delta u = u - u_1$ and u_1 is the ship's nominal speed.

Whenever the vessel has a propulsion installation with a governor that keeps the propeller revolution

approximately constant, the equation for $f1$ can be expressed through a more convenient form :

$$\begin{aligned} f1(u, v, r, \delta, \eta) &= X^* + X_u * u^2 + X_{u\eta} * u * \eta + X_{\eta\eta} * \eta^2 \\ &+ X_{vv} * v^2 + X_{rr} * r^2 + X_{\delta\delta} * \delta^2 \\ &+ (m + X_{vr}) * v * r + X_{v\eta} * v^2 * \eta \\ &+ X_{\delta\eta} * \delta^2 * \eta \quad (17) \end{aligned}$$

where η is the propeller's revolution number.

Let's analyse the situation of the vessel sailing with a bow wind of 15° and with a stern wind of 165° . A basic aspect regards the generally adopted approximation for the sway velocity and shown in equation (11). While this approximation is valid for example in a turning-circle, it is not appropriate for a vessel performing small-amplitude oscillations. This approximation jeopardizes the proposed performance indices.

A comparison of the values of Table 5. indicates that for the Mariner at 1.2 time units, the dominant aspect is varying with the external disturbance approaching angle. In other words, added resistance due to sway is dominant for small bow approaching angles, while rudder-drag is dominant for higher approaching angles. Regarding added resistance in the Esso Osaka case, it can be seen in Table 6. that for a small approaching angle of the external disturbances, rudder-drag is the dominant aspect ; while for higher angles the coupled sway-yawing becomes the dominant aspect. Consequently, the optimal performance index varies with the external disturbances predominant approaching angle.

It was suggested in Fig. 8. that for the Mariner, the obtained optimal performance index does not change with the magnitude of the external disturbances. This is the case for a wind range at which the vessel's safety are not hazarded. However, note that the LQ regulator adopted is continuous and unconstrained. For a discrete-time non-linear controller, the magnitude independent condition is inaccurate.

9.2 Should

The optimal closed loop eigenvalues placement for directional stable and unstable vessels can be divided into three regions regarding approaching angles, as illustrated in Fig. 12. by the 1° , 30° and 120° approaching angles. In other words, an equal placement region corresponding to small approaching angles for which the vessel's deviating effects are small, a transition region

Table 5 Components in (v, r, δ) for the Mariner

values in percentage (%) of the total		
t = 1.2 units		
component	15 deg.	165 deg.
$1/2 * X_{vv} * v^2$	-153.50	-12.65
$(1/2 * X_{rr} + m * x_v) * r^2$	-20.94	-1.44
$1/2 * X_{\delta\delta} * \delta^2$	-84.44	-67.55
$(X_{vr} + m) * v * r$	122.55	-9.22
$X_{v\delta} * v * \delta$	36.33	-9.14

Table 6 Components in (v, r, δ) for the Esso Osaka

values in percentage (%) of the total		
t = 1.2 units $\eta = 1$		
component	15 deg.	165 deg.
$X_{vv} * v^2$	11.42	6.57
$X_{rr} * r^2$	3.78	5.16
$X_{\delta\delta} * \delta^2$	-5.15	-26.03
$(m + X_{vr}) * v * r$	85.00	-80.00
$X_{v\eta} * v^2 * \eta$	6.85	3.94
$X_{\delta\delta} * \eta * \delta^2 * \eta$	-1.90	-9.64

Table 7 Work Model Open - Loop Poles and Zeros

	Cargo Ship	Cargo Ship	Tanker	Ore-Oil		Tanker		Tanker
	Mariner	Esso Osaka	190,000 ton	Ballast	Full	Ballast	Full	Esso Osaka
p1	-6.9965	-6.8294	-12.3434	-13.0221	-12.1915	-14.3532	-13.1467	-29.2476
p2	-0.05997	-0.36379	0.09542	0.32563	0.15131	0.34662	0.05814	0.13704
p3	-2.3520	-3.2478	-2.5621	-2.2622	-2.2846	-2.6205	-2.2477	-2.1201
p4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
zero-transfer function between the rudder angle and the heading								
z1	-1.1233	-1.4000	-1.0709	-0.9535	-0.8582	-0.9285	-0.6694	-1.08786

that varies according to the vessel's characteristics and an equal placement region corresponding to higher approaching angles. Is this result unexpected?

Let's recall the known result⁴⁾ valid for a single-input-single-output (SISO) type regulator with the performance index given by equation (2), in which $Q=1$ and $R=\rho$. The n -closed-loop characteristic values asymptotically approach the open-loop n -poles $\pi_i(-\pi_i)$, if the open-loop eigenvalue has real negative part (real positive part) for $\rho \rightarrow \infty$. The closed-loop characteristic eigenvalues asymptotically approach the open-loop p -zeros $v_i(-v_i)$, if the open-loop zero has real negative part (real positive part) for $\rho \rightarrow 0$. The other $n-p$ closed-loop characteristic eigenvalues approach a Butterworth configuration of order $(n-p)$.

In the single-input-multi-output case (SIMO), the determination of the closed-loop eigenvalues is not as simple as for the SISO case. However, for $\rho \rightarrow \infty$ the same results applies. This implies in view of the dominant open-loop pole listed in Table 7. and $R55$ in FIG. 7. that for a small approaching angle, the dominant poles for six vessels in eight different loading conditions are approximately the same.

The same is valid for the dominant closed-loop poles when the external disturbances approach the vessel with an open angle. In this case $R55$ should be small, close to zero (0), as shown in Fig. 7. for the Mariner and Esso Osaka. The dominant closed-loop poles for six-vessels for $\rho \rightarrow 0$ are shown in Fig. 13. All the vessels have the same characteristics, a Butterworth configuration of 1st order and three finite eigenvalues with similar distribution, as sketched in Fig. 14.

This leads to the formulation of the following variation law for the dominant closed-loop eigenvalues loca-

Fig. 13 Closed-Loop Poles Location - Illustration

tion :

$$x \pm iy = -0.05 - 1.08 * cship * e^{\pm i * 0.41} \quad (18)$$

where $x \pm iy$ are the dominant complex poles :

$$cship = \theta \quad \text{for } \theta \leq 0.78 ;$$

$$cship = 0.78 \quad \text{for } \theta > 0.78 ;$$

θ is the external disturbances approaching angle.

Note however, that this variation law was set as a directive for design, aiming at fuel consumption, and its fuzzy aspect must be carefully understood. It was shown in Fig. 13., two main paths for the root-locus. These paths can not be classified according to the vessel characteristics, since in both paths directionally stable and unstable vessels are presented. More than this, the root-locus of the same vessel with different loading

Fig. 14 Root-Locus for Six Vessels

conditions switches from one path to the other, according to the value of its open-loop poles. Also, the poles location variation is not linear with the disturbance encounter angle, neither the limit value for the coefficient *cship* in equation (18) is the same for different vessels.

10. Conclusions

Let's summarize a few conclusions from previous reports^(10,3) and present research stage by answering the primary questions :

Can Autopilots save fuel ?

The results indicate non-significant improvement for a directional stable vessel (Mariner), but significant improvement was obtained for a directional unstable vessel (Esso Osfaka). This improvement is about 1.6% for the Esso Osaka.

Is the vessel's sway speed available ?

No commercial piece of equipment is available at the moment. Considering the approximation as given by equation (11), this does not represent any handicap for the Mariner, but it is a handicap of something around of 0.55% for the Esso Osaka. While it is not very likely that an adequate commercial sensor will be available in the market, the sway speed estimate can be improved in order to minimize this handicap.

Can any directive useful for design be established ?

A general directive for the system shaping can be established for the dominant closed-loop eigenvalues location and it is described in equation (18). This directive aims at fuel consumption and disregards the location of the weak poles of the system, though it can be applied to any conventional type of vessel.

The concluding note regards the present stage of this research which consists in the design of an adaptive controller for the automatic placement of the dominant closed-loop poles according to the suggested shaping directive.

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