Optimization of Ship Structures Based on Plastic Design

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Summary

In this paper a procedure for the optimization of ship structures is developed based on plastic design of ship structures consisting of the transverse and longitudinal members using the Sequential Unconstrained Minimization Technique (SUMT). Using the developed method, the optimization of a segregated ballast tanker is carried out considering the weight of one hold length as the objective function. Midship section scantlings are chosen as the design variables. Constraints are considered with respect to plastic collapse of hull girder under longitudinal bending, of structural members subjected to axial force, shear force and bending moment, and of platings subjected to lateral pressure. Constraints are also set in buckling strength of platings under inplane load and in design variables such as minimum plate thickness. The variations of maximum bending moments and shear forces due to changes in design variables are calculated based on Taylor series expansion method so as to reduce the computation time of iterative calculation for function minimization in the SUMT procedure. The tanker structure got optimized and the results show that the proposed method based on plastic design is an effective tool in the optimization of ship structures. A comparison of the proposed method with the optimization based on elastic design is also made and the results are presented.

Nomenclature

a : length of the rigid part at end A of a beam AB
 A : sectional area of a beam
 A_f : sectional area of a flange
 A_p : sectional area of a plate
 A_w : sectional area of a web
 b : length of the rigid part at end B of a beam AB
 E : modulus of elasticity
 G : shear modulus
 I : moment of inertia
 l : length of the non-rigid part of a beam AB
 M : maximum bending moment
 M_0 : bending moment satisfying fully plastic interaction relationship
 M_0^H : M_0 for ship hull
 $M(x_0)$: maximum bending moment of the initial ship
 q : water pressure acting on a plate
 q_u : allowable pressure to prevent plastic collapse
 Q : shear force
 Q_0 : shear force satisfying fully plastic interaction relationship
 Q_0^H : Q_0 for ship hull
 w : deflection
 x_{0i} : i th design variable for the initial ship

Z_p : plastic section modulus
 α : safety factor
 σ_{c0} : allowable buckling stress
 σ_{cr} : critical buckling stress
 σ_L : allowable axial stress in longl. members
 σ_y : yield stress
 ϕ : slope

1. Introduction

The optimum design of a ship structure is aimed at reducing the hull weight and/or construction cost by optimizing the scantlings and spacings of structural members without sacrificing the functional and strength requirements.

The optimum design of ship structures based on elastic structural analysis had been extensively carried out¹⁻⁴⁾ using the Sequential Unconstrained Minimization Technique (SUMT), a nonlinear optimization method. However, the optimization based on elastic design results in a margin on the safety factor which leaves further room for optimization, and the concept of plastic design is of help in overcoming this. Recently, sensitivity analysis of elastoplastic structural problems has been investigated⁵⁻⁸⁾ and optimization incorporating elastoplastic sensitivity analysis is of increasing interest. However, in case of ship structures, which are inherently complex, the optimization based on elastoplastic analysis may be difficult and may require more computation time than that based on elastic analysis.

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One method to overcome these problems of massive computation is to initially perform elastic structural analysis to find out the internal forces in transverse (deck, side and bottom frames) and longitudinal (deck and bottom girders) members and then to constrain these with the respective fully plastic conditions in the optimization process.

In the present paper, such a procedure for the optimization of ship structures is developed using the SUMT method incorporating Taylor's approximation with the first and second order sensitivities based on the results of three dimensional structural analysis. Applying the proposed method, the structural optimization of a segregated ballast tanker is carried out. One hold length between two adjacent transverse bulkheads is optimized taking hull weight as the objective function to be minimized with necessary constraints. The wave loads and allowable buckling stresses for the analyses are based on NK Rule. A comparison of the proposed method with the optimization based on elastic design is also made.

2. Structural Optimization

2.1 SUMT Method

A nonlinear optimization problem can be expressed as

- determine a vector $\mathbf{X}$ which
- minimizes $F=f(\mathbf{X})$
- subject to $g_i(\mathbf{X}) \geq 0, \quad i=1, 2, 3, \dots, m.$

Here, $\mathbf{X} = [x_1, x_2, \dots, x_n]^T$ represents the n -dimensional column vector. The vector $\mathbf{X}$ consists of design variables x_i which may be represented by the dimensions of a structure such as sizes of frames, girders, plate thicknesses, frame spacings, etc.. The function $f(\mathbf{X})$ is the objective function to be minimized which, in this paper, is taken as the hull weight. $g_i(\mathbf{X})$ are the constraints.

In the present paper, the Sequential Unconstrained Minimization Technique (SUMT) is used for the optimization of the objective function. The SUMT method converts a constrained function into an unconstrained function and then finds out its minimum. This is very effective especially when the objective and constraint functions are very complex. Moreover, the optimization algorithm of SUMT is simple and can be executed using computers of relatively smaller size.

In order to minimize the constrained objective function $f(\mathbf{X})$, the following unconstrained function $F(\mathbf{X}, r_k)$ is defined in the SUMT method.

$$F(\mathbf{X}, r_k) = f(\mathbf{X}) + r_k \sum_{i=1}^m \left(\frac{1}{g_i(\mathbf{X})} \right) \quad (1)$$

As a starting point of the optimization process, $\mathbf{X} = \mathbf{X}^{(0)}$ such that $g_i(\mathbf{X}^{(0)}) > 0$ for all i is initially assumed. Then r_1 is set such that $F(\mathbf{X}^{(0)}, r_1)/f(\mathbf{X}^{(0)})$ is approximately equal to 1.15-1.2. Proceeding from $\mathbf{X}^{(0)}$, $\mathbf{X} = \mathbf{X}^{(1)}$ that minimizes $F(\mathbf{X}, r_1)$ is determined. Now, starting from $\mathbf{X}^{(1)}$, $\mathbf{X} = \mathbf{X}^{(2)}$ is determined that minimizes $F(\mathbf{X}, r_2)$. Repeating this procedure, $\mathbf{X} = \mathbf{X}^{(k)}$ ($k=1, 2, 3, \dots$) that

minimizes $F(\mathbf{X}, r_k)$ is calculated. In this iterative calculation, r_k is set equal to $r_{k-1}/C1$ but for $k=1$, with the value of $C1$ as 10. The optimum value $\mathbf{X}^{(k)}$ can be obtained when r_k fully decreases to zero. In the present paper it is assumed that the procedure converges to the optimum when the following condition

$$\left| 1 - \frac{f(\mathbf{X}^{(k)})}{f(\mathbf{X}^{(k-1)})} \right| \leq \epsilon \quad (2)$$

is satisfied. In Eq. (2), $f(\mathbf{X}^{(k)})$ and $f(\mathbf{X}^{(k-1)})$ are the objective functions in Eq. (1) minimized at the k th and $(k-1)$ th optimization processes, respectively. In the numerical calculation, ϵ is taken as 10^{-6} . The optimization flow chart is shown in Appendix I.

2.2 Objective Function and Design Variables

In the optimum design of ship structures, the objective function can be the hull weight and/or the cost of hull construction. In this paper, the hull weight of one hold length is taken as the objective function. In the evaluation of the objective function, all the structural members are included except lightening holes, collar plates and small tripping brackets.

It is required to reduce the number of design variables to a minimum in order to save the computation time. After careful consideration, nineteen parameters are chosen as the design variables. The first thirteen and the last five of them are shown in Fig. 1 and the fourteenth parameter is the total sectional area of the side shell and the longitudinal bulkheads with attached longitudinal stiffeners. The width of the side ballast tank (web depth of side transverse) is not considered as a design variable because it is a parameter mainly of the volume of the side ballast tank.

2.3 Constraints

The constraints considered for the analyses are detailed as follows.

(1) Fully plastic condition of ship hull :

The bending moment and shear force in a hull cross

Fig. 1 Midship section of the SBT

section should not violate the fully plastic condition, subjected to a safety factor.

$$g(x) = M_T^H - a_H M_T \geq 0 \quad (3)$$

$$g(x) = Q_T^H - a_H Q_T \geq 0 \quad (4)$$

where M_T and Q_T are the maximum bending moment and shear force that may act on the ship hull with an exceeding cumulative probability of 10^{-8} , and is evaluated as the sum of the respective wave and stillwater values.

The fully plastic interaction relationship of bending moment to shear force is derived assuming fully plastic stress distribution in the ship hull cross section.

(2) Fully plastic condition of structural members :

The bending moments, shear forces and axial forces in transverses and longitudinal girders should not violate the respective fully plastic conditions, subjected to a safety factor.

$$g(x) = M_0 - a_T M \geq 0 \quad (5)$$

$$g(x) = Q_0 - a_T Q \geq 0 \quad (6)$$

The constraint given by Eq. (5) is considered for the deck, side and bottom transverses as well as for both the primary and secondary longitudinal members of deck and bottom. The constraint given by Eq. (6) is considered for the deck, side and bottom transverses.

The fully plastic interaction relationships of bending moment to shear force and axial force are derived assuming fully plastic stress distribution in the cross section of the respective structural members.

(3) Plastic collapse strength of panels :

The water pressure acting on local panels should not exceed the collapse pressure to prevent their plastic collapse, subjected to a safety factor.

$$g(x) = q_u - a_q q \geq 0 \quad (7)$$

This constraint is considered for panels in bottom shell and deck plate.

The collapse pressure is evaluated considering plastic mechanism under lateral pressure.

(4) Buckling strength :

The stresses in local panels should not exceed the allowable buckling stresses.

$$g(x) = \sigma_{cr} - \sigma_{co} \geq 0 \quad (8)$$

The elastoplastic buckling constraint is considered for the deck and bottom longitudinals, and for panels in bottom shell and deck plate.

To evaluate elastoplastic buckling stress, Johnson's formula is applied with elastic buckling stress and yield stress.

(5) Minimum values of design variables :

In order to effectively optimize the design variables, it is necessary to set limits on them. Here, lower limits $x_{i,min}$ are set on the design variables x_i , and these limits are given in percentage of the initial values of the design variables.

$$GX_i = x_i - x_{i,min} \geq 0, \quad i = 1, 2, \dots, n. \quad (9)$$

3. Structural Analysis

3.1 Application of Transfer Matrix Method

Three dimensional structural analysis is carried out

Fig. 2 Mathematical model of one hold length

using the Transfer Matrix Method. The main members considered for the analysis are the deck and bottom members, both transverse and longitudinal. One hold length, the mathematical model of which is shown in Fig. 2, is considered for analysis. The structure has n frames and thus $n-1$ segments, and one segment length corresponds to one frame spacing. The forward and aft ends of the hold bounded by the transverse bulkheads are assumed to be fixed.

The boundary conditions at the aft and forward bulkheads are given by Eqs. (10) and (11), respectively.

$$V_i^L = R_L \cdot A \quad (10)$$

$$R_R V_n^R = 0 \quad (11)$$

The transferences of state vectors from left end to right end in a segment i and from left to right at a frame point i are done using Eqs. (12) and (13), respectively.

$$V_i^R = F_i V_i^L \quad (12)$$

$$V_i^L = P_i V_{i-1}^R \quad (13)$$

From Eqs. (10), (11), (12) and (13), the following relationship is derived.

$$R_R \cdot F_n \cdot P_n \cdot F_{n-1} \dots P_3 \cdot F_2 \cdot P_2 \cdot F_1 \cdot R_L \cdot A = 0$$

This on simplification yields

$$\begin{bmatrix} a_{1,3} & a_{1,4} & a_{1,7} & a_{1,8} \\ a_{2,3} & a_{2,4} & a_{2,7} & a_{2,8} \\ a_{5,3} & a_{5,4} & a_{5,7} & a_{5,8} \\ a_{6,3} & a_{6,4} & a_{6,7} & a_{6,8} \end{bmatrix} \begin{bmatrix} Q_B \\ M_B \\ Q_D \\ M_D \end{bmatrix}_1 = \begin{bmatrix} -a_{1,9} \\ -a_{2,9} \\ -a_{5,9} \\ -a_{6,9} \end{bmatrix} \quad (14)$$

Solving Eq. (14), Q_B , M_B , Q_D and M_D are derived and thus the initial vector A . Now using Eqs. (10), (12) and (13), the state vectors V_i^L and V_i^R at each frame point i are obtained, and thus the deflections, slopes, shear forces, bending moments and in turn the stresses at each frame point.

The field matrix F_i of the segment i is derived from the slope-deflection equations as well as from the shear force and bending moment equations for the corresponding longitudinal girders. The point matrix P_i of the frame point i is derived considering the conditions of continuity of deflections and slopes, and the equilibrium of forces and moments in the longitudinal girders at the frame point i .

The state vectors V_i^L and V_n^R , the initial vector A , the end matrices R_L and R_R , the field matrix F_i and the

point matrix P_i are given in Appendix II.

3.2 Application of Taylor Series Expansion Method

The maximum bending moments and shear forces for the initial ship are calculated by the direct three dimensional analysis mentioned above. However, to save computation time, the subsequent maximum bending moments at each iteration step when design variables are changed are calculated based on Taylor series expansion with sensitivities up to second order using Eq. (15).

$$M_{\max} = M(x_0) + \sum_{i=1}^n \frac{\partial M}{\partial x_i} (x_i - x_{0i}) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 M}{\partial x_i \partial x_j} (x_i - x_{0i})(x_j - x_{0j}) \quad (15)$$

The derivatives in the above equation are numerically evaluated and are given as follows.

$$\begin{aligned} \frac{\partial M(x_1, x_2, \dots)}{\partial x_1} &= \{M(x_1 + \Delta x_1, x_2, \dots) \\ &\quad - M(x_1 - \Delta x_1, x_2, \dots)\} / (2\Delta x_1) \\ \frac{\partial^2 M(x_1, x_2, \dots)}{\partial x_1 \partial x_2} &= \{M(x_1 + \Delta x_1, x_2 + \Delta x_2, \dots) \\ &\quad - M(x_1 + \Delta x_1, x_2 - \Delta x_2, \dots) \\ &\quad - M(x_1 - \Delta x_1, x_2 + \Delta x_2, \dots) \\ &\quad + M(x_1 - \Delta x_1, x_2 - \Delta x_2, \dots)\} \\ &\quad / (4 \Delta x_1 \Delta x_2) \\ \frac{\partial^2 M(x_1, x_2, \dots)}{\partial x_1^2} &= \{M(x_1 + \Delta x_1, x_2, \dots) \\ &\quad - 2M(x_1, x_2, \dots) \\ &\quad + M(x_1 - \Delta x_1, x_2, \dots)\} / (\Delta x_1)^2 \end{aligned}$$

Δx_i = increment in $x_i = 0.05x_i$

The subsequent maximum shear forces at each iteration step are also similarly calculated.

3.3 Constraints Due to Fully Plastic Interactions Between Internal Forces

In order to calculate the fully plastic interaction relationships between internal forces, the members are categorized into two groups as follows :

- 1) Members subjected to axial force and bending moment
 - Deck and bottom center girders
 - Deck and bottom longitudinals
- 2) Members subjected to shear force and bending moment
 - Transverse ring members (deck, side and bottom)
 - Ship hull

The yield function Γ of a beam-bar subjected to axial force and bending moment can be written as

$$\begin{aligned} \Gamma &= m - \frac{1}{Z_p} (a_1 + b_1 n + c_1 n^2) = 0 \\ &\text{for } -1 \leq n < -1 + \frac{2A_f}{A} \\ \Gamma &= m - \frac{1}{Z_p} (a_2 + b_2 n + c_2 n^2) = 0 \\ &\text{for } -1 + \frac{2A_f}{A} \leq n < 1 - \frac{2A_p}{A} \end{aligned}$$

$$\begin{aligned} \Gamma &= m - \frac{1}{Z_p} (a_3 + b_3 n + c_3 n^2) = 0 \\ &\text{for } -1 - \frac{2A_p}{A} \leq n \leq 1 \end{aligned} \quad (16)$$

where

$$A = A_f + A_w + A_p$$

$m = M/M_p$ is the ratio of the bending moment to the fully plastic bending moment, and $n = P/P_p$ is the ratio of the axial force to the fully plastic axial force. $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3$ and c_3 are functions of the scantlings of a member.

The yield function Γ of a beam subjected to shear force and bending moment can be written as

$$\begin{aligned} \Gamma &= m - \frac{1}{Z_p} (a_4 + b_4 v + c_4 v^2) = 0 \\ &\text{for } A_f \geq A_p + A_w \\ &\text{or} \\ &A_f \geq A_p \text{ and } |f| \geq FV \\ \Gamma &= m - \frac{1}{Z_p} (a_5 + b_5 v + c_5 v^2) = 0 \quad \text{for } |f| \leq FV \\ \Gamma &= m - \frac{1}{Z_p} (a_6 + b_6 v + c_6 v^2) = 0 \\ &\text{for } A_p \geq A_f + A_w \\ &\text{or} \\ &A_f \leq A_p \text{ and } |f| \geq FV \end{aligned} \quad (17)$$

where

$$\begin{aligned} FV &= \sqrt{1 - [(A_p - A_f)/A_w]^2} \\ v &= \sqrt{1 - f^2} \end{aligned}$$

and $f = F/F_p$ is the ratio of the shear force to the fully plastic shear force. $a_4, b_4, c_4, a_5, b_5, c_5, a_6, b_6$ and c_6 are functions of the scantlings of a member.

The typical fully plastic interaction relationships represented by Eqs. (16) and (17) are plotted in Fig. 3 (a) and (b), respectively. From the structural analysis for the initial ship, the ratios n/m and f/m are evaluated as

$$n/m = C_n \quad (18)$$

$$f/m = C_f \quad (19)$$

As the intersecting point of Eqs. (16) and (18), m_0 is obtained satisfying the fully plastic interaction relationship, and in turn the corresponding maximum bending moment M_0 as follows.

$$M_0 = m_0 M_p \quad (20)$$

From this, the bending moment constraint due to fully plastic condition is calculated using Eq. (5). In the same manner, the maximum bending moment and shear force for constraints based on Eqs. (17) and (19) are derived.

4. Optimization of a Segregated Ballast Tanker

4.1 Calculation Procedure

Using the method detailed above, the optimization of one hold structure of a segregated ballast tanker (SBT) is carried out. The main particulars of the SBT are Length B. P. = 224.00 m ; Breadth (mld.) = 32.20 m

Fig. 3 Fully plastic interaction curves

Depth (mld.) = 19.40 m ; Draught = 12.17 m
 Dead Weight = 61,000 t ; Block Coeff. = 0.832

One hold length bounded by two transverse bulkheads is taken for optimization, considering the hull weight as the objective function. The hold configuration is shown in Fig. 4, and the midship section in Fig. 1. For analysis, the structural members in the midship section are simplified to beam-bars with rigid portion at both ends as shown in Fig. 5. The length of the rigid portion is evaluated based on the concept of span point⁹⁾. The

Fig. 4 configuration of one hold length

Fig. 5 Idealized midship section

optimization is carried out for ten cases (A1 to A10) by changing r_1 from 10^3 to 10^6 and X_{min} from 90% to 50% of x_i , one at a time, keeping other parameters constant. The basic parameters of the various cases are shown in Table 1.

The optimization is carried out based on wave loads presented in NK Rule¹⁰⁾. Both ballast and full load conditions are considered, which are indicated in Fig. 6. The allowable stresses based on NK Rule are shown in Table 2. The safety factor for both transverse and longitudinal members is taken as 1.5 and that for hull section is taken as 1. The bending moment, shear force and bending stress distributions are calculated for the initial ship in both ballast and full load conditions. The maximum value of the bending moments in the respective members are used in the analyses. The sensitivities of the maximum bending moments with respect to the design variables are required to effectively optimize the structure with minimum computation cost, and are calculated for the initial ship. The value of m_0 in Eq. (20) for various structural members are shown in Table 3 for both ballast and full load conditions.

4.2 Optimization Results

4.2.1 Standard Ship

In the case of the standard ship, the number of frame spaces (N) is five in a hold length and the optimization is carried out with $N=5$. Since the effect of shear force is small, the constraint given by Eq. (6) is not consid-

Table 1. Basic parameters of various cases

Cases	N	X_{min}	r_1	Tr.Fr.Sp. (mm)	Iteration number	Weight (tons)
A1	5	90 %	5.0E4	4520.0	26912	919.42
A2	5	80 %	5.0E4	4520.0	17951	860.28
A3	5	70 %	5.0E4	4520.0	19369	781.74
A4	5	60 %	5.0E4	4520.0	16313	734.05
A5	5	50 %	5.0E4	4520.0	14301	719.23
A6	5	70 %	1.0E3	4520.0	15233	785.74
A7	5	70 %	1.0E4	4520.0	13423	784.16
A8	5	70 %	5.0E4	4520.0	19369	781.74
A9	5	70 %	1.0E5	4520.0	17347	804.39
A10	5	70 %	1.0E6	4520.0	15333	807.89

Table 2. Allowable stresses (Plastic design)

Notation	Stress (kgf/mm ²)
σ_L	18.0
σ_y	28.0
σ_{co}	20.0

Fig. 6 Loading on the ship hull

Table 3. Values of m_0 for structural members

Structural Member	m_0	
	Ballast	Full load
Deck Trans.	0.8365	0.8121
Side Trans.	0.9738	0.9738
Bottom Trans.	0.8575	0.8575
Deck Lgl Gdr.	0.1217	0.1419
Btm. Lgl Gdr.	0.3011	0.1005
Deck Lgl.	0.1324	0.1324
Bottom Lgl.	0.2894	0.0834
Hull Section	0.9453	0.9453

ered in this case and hence the design parameters x_{15} to x_{19} are also not considered as design variables. The maximum bending moment in a hull cross section (M_T) is calculated based on⁽¹⁾.

In cases A1 to A5, X_{min} is varied from 90% to 50% in order to check the accuracy of the Taylor's approximation used in the bending moment calculation by evaluating the change in moments in primary structural members. It is defined as

$$\% \text{ change in moment} = \frac{(M_{dir} - M_{apr})}{M_{dir}} \times 100$$

where M_{dir} and M_{apr} are the maximum bending moments obtained by direct calculation and by using Taylor's approximation respectively for the optimized ship.

It has been found from Table 4 that the percentage change in moment increases as X_{min} decreases. The allowable change is set as 10% which is assumed to provide sufficient accuracy. This necessitated lower limits in the design variables which are incorporated as constraints GX1 to GX14. To make the optimization effective, the lower limit has to be a minimum within the allowable change in moment of 10%. Thus case A3

Table 4. Percentage change in moments for N=5

CHANGE IN MOMENTS IN %	Notation	Initial Values	VALUES FOR CASES				
			A1	A2	A3	A4	A5
BALLAST							
Deck Trans	CM1	0.00	0.00	0.35	1.53	3.08	2.54
Side Trans	CM2	0.00	0.36	0.81	1.14	2.06	-0.03
Bottom Trans	CM3	0.00	0.18	0.09	0.26	1.97	0.13
Deck lgl Gr	CM4	0.00	-3.48	-6.40	9.50	-12.01	-11.16
Bottom lgl Gr	CM5	0.00	-0.89	-1.74	-2.82	-7.86	-0.04
FULL LOAD							
Deck Trans	CM6	0.00	-1.25	-4.61	7.21	-8.93	-10.17
Side Trans	CM7	0.00	0.01	0.01	0.02	0.01	0.07
Bottom Trans	CM8	0.00	0.09	-0.09	0.08	1.65	-0.47
Deck lgl Gr	CM9	0.00	-1.93	-4.19	-6.86	-9.96	13.82
Bottom lgl Gr	CM10	0.00	-0.78	-1.53	-2.46	-5.33	-0.43

Table 5. Optimized values of design variables for N=5 (Plastic design)

DESIGN VARIABLES	Notation (unit)	Initial Values	OPTIMIZED VALUES FOR CASES				
			A6	A7	A8	A9	A10
B Tr Fl Width	x_1 (cm)	50.00	39.38	36.29	52.38	61.38	56.38
B Tr Wb Depth	x_2 (cm)	420.00	416.85	415.66	317.36	304.92	319.98
D Tr Fl Width	x_3 (cm)	25.00	17.50	17.51	17.51	17.50	17.50
D Tr Wb Depth	x_4 (cm)	179.00	125.30	125.33	125.30	125.30	125.30
Lgl Frame Spacing	x_5 (mm)	900.00	773.89	771.00	778.08	768.52	813.13
R Lgl Wb Depth	x_6 (cm)	69.50	59.43	59.28	56.51	50.51	52.46
D Lgl Wb Depth	x_7 (cm)	35.00	24.50	24.50	24.50	24.50	24.50
R Lgl Gr Fl Width	x_8 (cm)	100.00	89.65	102.27	99.18	71.30	71.61
R Lgl Gr Wb Depth	x_9 (cm)	700.00	694.84	669.17	519.83	522.40	490.96
D Lgl Gr Fl Width	x_{10} (cm)	35.00	24.51	24.52	24.50	24.50	24.55
D Lgl Gr Wb Depth	x_{11} (cm)	399.00	279.32	279.41	279.32	279.32	279.74
R Shell Thickness	x_{12} (mm)	21.00	17.87	17.84	20.75	25.75	26.21
D Plate Thickness	x_{13} (mm)	22.00	15.40	15.40	15.40	15.40	15.40
Side Shell Area	x_{14} (cm ²)	19621.	13735.	13735.	13735.	13735.	13735.
X_{min} (% of x)	PC	-	70%	70%	70%	70%	70%
r_1	-	-	1.0E3	1.0E4	5.0E4	1.0E5	1.0E6
Total no. of ltr.	-	-	15233	13423	19369	17347	15533
Weight	W(Tons)	980.58	785.74	784.16	781.74	804.39	807.89
Function value	-	-	785.74	784.16	781.74	804.39	807.89

is chosen and the corresponding $X_{min}=70\%$ is taken as the basis for other calculations. Now with X_{min} set at 70%, the optimization is carried out by varying r_1 from 10^3 to 10^6 , which corresponds to cases A6 to A10. The optimized variables for these cases are shown in Table 5, from which the minimum weight is found to be 781.74 tons and it corresponds to case A8. This is the optimum design.

Figure 7 shows the change in hull weight as the optimization progresses for various r_1 values, the cases being A6 to A10. It goes without saying that there are a number of local minima for a nonlinear function depending on the starting value of r_1 , and this necessitated the different cases by varying r_1 . A few sample distributions of design variables and constraints in moments in ballast condition are shown in Fig. 8(a)-(i) against the iteration number, for case A8. The different stages r_1 to r_{10} are also shown. It is clear from these figures that the calculations need to be done only up to r_5 for sufficient accuracy. However, the calculations are done up to r_{10} as a check on accuracy.

In order to verify the effectiveness of the present method of optimization, it is necessary to check the effectiveness of the constraints. From Fig. 8(a)-(i), for case A8, the following conclusions can be made.

- for deck transverse GX3, GX4 and GX13 are effective
- for side transverse GX14 is effective

Fig. 7 Hull weight vs. iteration number for various cases for $N=5$

- for bottom trans. $GB3$ is effective
- for deck girder $GX13, GX10$ and $GX11$ are effective
- for deck longl. $GX7$ and $GX13$ are effective
- for bottom longl. $GB7$ is effective

Since $GB3$ and $GB7$ are effective for bottom transverse and bottom longitudinal respectively, and since transverses, longitudinals and girders interact each other, it is evident that $GB3$ and $GB7$ are effective for bottom girder also. Thus the assumed constraints are effective for all the primary and secondary members considered for the analysis.

A few constraints other than those mentioned above are not getting effective. This is due to the fact that in the optimization process when any one or more of the related constraints has become effective, the related member is optimized and this in turn prevents the other constraints related to this member from getting further reduced. However, these constraints are necessary in the optimization process as it is not sure which of these constraints will affect the objective function most.

For lower r_1 values (cases A6 and A7), the moment constraint related to bottom longitudinal girder is effective whereas, for higher r_1 values (cases A8, A9 and A10), the moment constraint related to bottom

Fig. 8 Distribution of variable and constraint values for $N=5$

transverse is effective. This shows that, depending on the value of r_1 , the constraints that get effective vary. This also necessitates different cases by varying the value of r_1 .

A comparison of the Taylor series expansion method and the direct method was made for case A8. The optimized values of design variables using these methods are almost the same, the hull weights being 781.74 and 780.66 tons for the former and latter cases, respectively. The CPU time taken for the Taylor series expansion method is 1.123 min. against 2.593 min. for the direct method. It may be said that the proposed method using the Taylor series expansion results in a saving of 57% in CPU time.

4.2.2 Effect of Number of Frame Spaces on Hull Weight

In order to determine the effect of variation in the number of frame spaces (N) on the hull weight, optimization is carried out by varying N from 4 to 7. It should be noted that the transverse frame spacing varies with variation in N as the hold length is kept as a constant. For each value of N , r_1 is varied from 10^3 to 10^6 and the design with the minimum weight is chosen as the optimum design, as in the case of the standard ship.

For these cases, the lower limits on the design variables are not considered as $X_{\min}$ cannot be set but for the case of $N=5$ and thus $X_{\min}$ is taken as 0%. Because of this, the Taylor series expansion method cannot be used to evaluate the maximum bending moments and shear forces at each iteration step as the percentage changes in moments may exceed the allowable limit. Hence, direct three dimensional analysis is used to calculate the maximum bending moments and shear forces at each iteration step. The factor of safety is taken as 1.0 for all the structural members.

Based on sample calculations it has been found that the web thicknesses of deck and bottom longitudinal girders (x_{18} and x_{19} , respectively) are not getting reduced and hence they are not considered as design variables for this analysis. The value of Q_7 in Eq. (4) is assumed to be $\Delta/10$, where Δ is the displacement of the ship.

From the optimization results of these cases, it has been found that :

- 1) Both the constraints given by Eqs. (3) and (4) are effective for the hull girder.
- 2) The constraint given by Eq. (5) is effective for the bottom transverse and the deck longitudinal girder.
- 3) The constraint given by Eq. (6) is effective for the side transverse.

Since transverses, longitudinals and girders interact each other, as detailed in Ch. 4.2.1, the assumed constraints are effective for all the primary and secondary members considered for analysis.

The total hull weight (W), the weight of longitudinal members (W_L) and the weight of transverse members (W_T) of one hold length are plotted against the number of frame spaces in Fig. 9. It is clear from this figure that,

as N increases, the hull weight increases. The weight of transverse members also increases with increase in N , as the number of transverse frames increases. However, the weight of longitudinal members slightly decreases as N increases the reason for which is as follows. The hull strength requirement is in excess in the case of plastic design. As N increases the span length of the longitudinal members decreases and hence their scantlings (those which affect the hull strength) decrease so as to enable the local strength to get constrained. This results in the slight decrease in the weight of longitudinal members with increase in N .

4.3 Comparison of Optimization Results Based on Plastic Design and Elastic Design

In order to compare the proposed method with the optimization based on elastic design, optimization is carried out on the same standard ship based on elastic design using the SUMT method. The constraints considered for this analysis are as follows.

- 1) The maximum shear stress in a member should not exceed the allowable shear stress, subjected to a safety factor. This constraint is considered for the deck, side and bottom transverse members.
- 2) The maximum bending stress in a member should not exceed the allowable bending stress, subjected to a safety factor. This constraint is considered for the deck, side and bottom transverse members as well as for both the primary and secondary longitudinal members of deck and bottom.
- 3) The maximum bending stress in a local panel should not exceed the allowable stress, subjected to a safety factor. This constraint is considered for panels in bottom shell and deck plate.
- 4) The buckling stress in a local panel should not exceed the allowable buckling stress. The elastoplastic buckling constraint is considered for panels in bottom shell and deck plate.
- 5) The maximum bending stress and shear stress in a hull girder should not exceed the allowable bending stress and shear stress, respectively, subjected to a safety factor.

The above mentioned constraints in items 3) and 4)

Fig. 9 Relationship between hull weight and number of frame spaces N (Plastic Design)

are evaluated in the same way as that for plastic analysis. The maximum bending moments and shear forces are calculated by direct three dimensional structural analysis. The allowable stresses are based on NK Rule, which are shown in Table 6. All other parameters and assumptions remain the same as that for the case of plastic design. The number of frame spaces is varied from 4 to 7 and for each of these cases r_1 is varied from 10^3 to 10^6 and the optimum design is chosen, as in the case of plastic design.

From the optimization results, it has been found that :

1) The shear stress constraint is effective for the deck, side and bottom transverses as well as for the hull girder.

2) The bending stress constraint is effective for the deck transverse and the hull girder.

Thus, the assumed constraints are effective for all the members considered for the analysis.

The total hull weight and the weights of longitudinal and transverse members of one hold length are plotted against the number of frame spaces in Fig. 10. It is clear from this figure that, as N increases, the hull weight increases. The weight of transverse members also increases as in the case of plastic design. However, the weight of longitudinal members remains almost the same the reason for which is as follows. Since the hull girder scantling requirement is more in the case of elastic design, the hull strength depends on the scantlings of the longitudinal members. As N increases, the

local strength requirement of longitudinal members is in excess as their span length decreases. However, the scantlings of the longitudinal members cannot get reduced because of the constraint in hull strength as the hull strength depends on them. This keeps the weight of longitudinal members almost the same with increase in N .

It is worth noticing that the optimization results of both plastic and elastic designs are based on the respective design conditions assumed for the analyses. As these design conditions change, the optimization results may also change.

In the case of elastic design, the transverses and longitudinal girders are affected by the constraints in bending stresses and shear stresses considered separately. On the other hand, in the case of plastic design, these members are affected by the constraints due to the fully plastic interactions between bending moments and shear forces. In the optimization of ship structures, it is reasonable to take the fully plastic interaction relationships (plastic design) rather than considering the bending moments and shear forces separately (elastic design), as the latter case may result in an overestimate of the design scantlings.

5. Conclusions

A procedure for the optimization of ship structures based on plastic design is developed using the Sequential Unconstrained Minimization Technique (SUMT) method. This procedure is applied in the optimization of a hold structure of a segregated ballast tanker taking one hold length bounded by two transverse bulkheads for analysis. Hull weight is taken as the objective function to be minimized.

The number of frame spaces N in a hold length is five for the standard ship. Optimization is carried out for $N = 5$ considering various cases by varying the lower limit on variables $X_{\min}$ from 90% to 50% of the initial values and the coefficient r_1 from 10^3 to 10^6 , one at a time. The effect of N on the hull weight is computed by varying N from 4 to 7. Since the hold length is kept as a constant, the transverse frame spacing varies with variation in N . A comparison of the proposed method with the optimization based on elastic design is also made. The results were analyzed and the following conclusions were arrived at.

- (1) The proposed method is effective in the optimization of ship structures as is confirmed taking the case of the optimization of a hold structure of a segregated ballast tanker. The hull weight of the hold length analyzed has got reduced.
- (2) In order to reduce the computation time for structural analyses in the optimization process, the variations of maximum bending moments and shear forces in structural members due to change in design variables are calculated using Taylor series expansion method. In this case, a

Table 6. Allowable stresses (Elastic design)

Structural Member	Bending Stress (kgf/mm ²)	Shear Stress (kgf/mm ²)
Transverses	18.0	8.5
Longl. Girders	10.0	-
Secondary Longls.	10.0	-
Deck and Btm. plates	20.0	-
Hull	20.0	10.0

Fig. 10 Relationship between hull weight and number of frame spaces N (Elastic Design)

lower limit has to be set on the design variables in order to effectively optimize a ship structure using the proposed method. In the present analyses the lower limit is set at 70% of the respective initial values of the initial ship so that the changes in moments are within the allowable limit.

- (3) The proposed method using the Taylor series expansion has resulted in a saving of 57% in CPU time compared to the direct method.
- (4) As the number of frame spaces increases, the total hull weight and the weight of transverse members increases. The weight of longitudinal members slightly decreases in the case of plastic design and it remains almost the same in the case of elastic design.
- (5) In setting up the constraints, the bending moments and shear forces considered separately (in the case of elastic design) may result in an overestimate of the design variables, and therefore it is more reasonable to consider their fully plastic interaction relationships (in the case of plastic design), in the optimization of ship structures.

The numerical computations in this paper were carried out using HITAC M-680H of the Information Processing Center of Hiroshima University.

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Appendix I : Optimization Flow Chart

Appendix II

The state vectors at frame point 1 to the left V_1^L and at frame point n to the right V_n^R are

$$V_1^L = V_n^R = \begin{bmatrix} w_B = 0 \\ \phi_B = 0 \\ Q_B \\ M_B \\ w_D = 0 \\ \phi_D = 0 \\ Q_D \\ M_D \\ 1 \end{bmatrix}$$

Subscripts B and D represents bottom and deck respectively.

The initial matrix, $A = \begin{bmatrix} Q_B \\ M_B \\ Q_D \\ M_D \\ 1 \end{bmatrix}$

The aft end matrix, $R_i =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward end matrix,

$$R_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The field matrix of the segment i ,

$$F_i = \begin{bmatrix} 1 & a+l+b & F1 & F2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & F3 & F4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a+l+b & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a+l+b & F5 & F6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & F7 & F8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a+l+b & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$F1 = (-l/EI)[la/2 + l^2/6 + ab + lb/2] + l/(GA)$$

$$F2 = (-l/EI)[b + l/2]$$

$$F3 = (-l/EI)[a + l/2]$$

$$F4 = -l/EI$$

$F1, F2, F3$ and $F4$ are for the bottom member and $F5, F6, F7$ and $F8$ are similar values for the deck member.

The point matrix at the frame point i ,

$$P_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -R_B^B & 0 & 1 & 0 & -R_B^D & 0 & 0 & 0 & -R_B \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -R_D^B & 0 & 0 & 0 & -R_D^D & 0 & 1 & 0 & R_D \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where R_B and R_D are the vertical reaction forces acting on bottom and deck center girders respectively at frame point i under full load condition. R_B^B and R_B^D are similar values with unit deflection at bottom girder. Similarly, R_D^B and R_D^D are with unit deflection at deck girder.