A study on correlation of vortex cavitation noise of propeller measured in model experiments and full scale

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Summary

A correlation of noise radiated from sheet or bubble cavitation occurring on propeller blade surface is well established. On the other hand, the correlation is not clear on noise radiated from vortex cavitation occurring at a distance from propeller blade. In the present paper, cavitation patterns and noise characteristics of the propeller accompanying only the vortex cavitation were investigated by model tests in a cavitation tunnel and full-scale measurements in order to know the correlation of noise radiated from vortex cavitation of propeller. Comparisons of full-scale cavitation pattern with model scale showed that vortex cavitation depends on not only cavitation number but also Reynolds number. Predicted noise level from model tests using scaling formulas of cavitation noise agreed well with full-scale measurements in the case where vortex cavitation patterns in model and full scale are similar. Cavitation number may vary at Reynolds number to the 0.15 power to keep similarity of pattern of vortex cavitation.

1. Introduction

Propeller cavitation is known as one of sources of inboard as well as outboard noise of ships. The inboard noise causes crew discomfort. The outboard noise disturbs underwater acoustic measurements. Therefore the prediction of cavitation noise is important in the design stage of a ship to improve noise characteristics. Methods to predict underwater noise of propeller in full scale may be of mainly three different types, as follows:

1) Empirical formulas based on full-scale noise data
2) Transformation of model noise to full scale using scaling formulas of cavitation noise
3) Theoretical estimation of the cavitation development and corresponding radiation of noise

The validity of the method of type 1) is often restricted to configurations similar to those on which the method is based. The method of type 3) is mainly used to estimate the amplitude of blade frequency component and has a problem to cover a wide frequency band. At present, the method type 2) is considered to be most reliable to predict full-scale noise. The correlation of propeller cavitation noise between model and full scale is an important problem in the method 2).

Propeller cavitation is divided into two types.

a) Sheet or bubble cavitation attached to propeller blade surface
b) Vortex cavitation occurring at a distance from propeller blade

As for the cavitation of type a), it is considered that the correlation of propeller cavitation noise is well established. On the other hand, the correlation is not clear on the cavitation of type b). In the present paper, observations and noise measurements of vortex cavitation of propeller were made in model tests in a cavitation tunnel and in full-scale measurements to know the correlation of vortex cavitation noise.

2. Scaling of Cavitation Noise

2.1 Scaling Formulas of Cavitation Noise

The scaling formulas applied in this paper are based on the theory of radial motion of an inviscid and incompressible fluid outside a spherical cavity, since it is considered that cavitation consists of many cavitation bubbles and noise is emitted from growth and collapse of such bubbles. Two formulas on frequency f and power spectrum density G are required, since cavitation noise are shown as spectrum levels in frequency domain. Levkovskii introduced scaling formulas from dimensional analysis.

The formula for frequency scaling is obtained using the collapse time \( \tau \) of a spherical cavity filled with vapor. The basic equation for incompressible liquid motion adjacent to a bubble is known as the Rayleigh equation:

\[
\ddot{R} + \frac{3}{2} (\dot{R})^2 = \frac{\Delta P}{\rho}
\]

(1)

where \( \Delta P = P_t - P_c \)
$P_0$: static pressure in reference point
$P_v$: vapor pressure of water
$R$: radius of a spherical cavitation bubble
$\rho$: density of water

Now, $P_0$ is supposed to be much larger than $P_v$ so that the bubble will collapse. Rewriting Eq. (1), we find equivalently

$$\frac{d}{dt} \left( R^2 \frac{dR}{dt} \right) = \frac{\Delta P}{\rho} \frac{d}{dt} \left[ \frac{2}{3} R^3 \right] \tag{2}$$

Assuming the initial condition $R=0$ and $R=R_w$ at $t=0$ we set following equation:

$$\left( \frac{dR}{dt} \right)^2 = \frac{\Delta P}{\rho} \frac{2}{3} \frac{R_w^3}{R^3} - 1 \tag{3}$$

The integration of Eq. (3) gives the radius-time relationship as:

$$t = \left[ \frac{3\rho}{2\Delta P} \right]^{0.5} \int_{R_s}^{R_w} \frac{R^{1.5} \Delta R}{R_s^{0.5}} \tag{4}$$

If the integrand of Eq. (4) is integrated over the interval $0 \leq R \leq R_w$, the time for complete collapse is given by:

$$\tau_c = 0.915 \frac{R_s}{D} \left( \frac{\rho}{\Delta P} \right)^{0.5} \tag{5}$$

Using $\tau_c$, the following expression for frequency scaling is obtained:

$$f_s = \left( \frac{\rho}{\Delta P} \right)^{0.5} f_m \tag{6}$$

where suffixes $m$ and $s$ denote the values in model and full scale, respectively.

The formula for the scaling of the power spectrum is introduced as follows. The pressure spectrum is defined as:

$$S = \frac{1}{2\pi} \int_{-\infty}^{\infty} p e^{i\omega dt} \tag{7}$$

the radiated acoustic energy spectrum:

$$G = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} p e^{i\omega dt} \tag{8}$$

and, the power spectral density:

$$G = \Delta P \frac{d}{dt} \left[ \frac{2}{3} R^3 \right] \tag{9}$$

For a monopole source, the pressure is given by:

$$p = \frac{\rho V}{4\pi r} \tag{10}$$

where $V$: volume of source
$r$: distance from source point to field point

In case of the spherical cavity bubble, $V$ can be approximated as follows:

$$V = \frac{4}{3} \pi R^3 \cos \left( \frac{\pi t}{2\tau_c} \right) \tag{11}$$

Then, Eq. (10) is rewritten as:

$$p = \frac{\rho}{4\pi r} \frac{1}{2} \left( \frac{\pi}{\tau_c} \right) R^3 \cos \left( \frac{\pi t}{2\tau_c} \right) \tag{12}$$

By substitution of Eq. (12) into Eq. (9), we obtain:

$$G \approx \frac{1}{2\tau_c} \int_{0}^{\tau_c} p e^{i\omega dt} \approx \frac{1}{2\tau_c} \frac{\rho}{4\pi r} \frac{R^3}{2} \approx \frac{\rho}{4\pi r} \frac{R^3}{2} \frac{\Delta P^{0.5}}{\rho^{0.5}} \tag{13}$$

Thus, the following expression for the power spectrum scaling is obtained.

$$G_s \approx G_m \left( \frac{\rho_s}{\rho_m} \right)^{1.5} \left( \frac{\Delta P_s}{\rho_s} \right)^{1.5} \tag{14}$$

In case of marine propeller, cavitation number is generally defined as follows:

$$\sigma = \frac{1}{2\rho_\pi D^2} \tag{15}$$

where $D$: propeller diameter
$n$: rate of revolutions of propeller shaft

The difference between $\rho_s$ and $\rho_m$ may be negligible. Assuming that radius of a cavitation bubble is proportional to diameter of a propeller, Eq. (6) and (14) are transformed to following:

$$f_s = \left( \frac{n_m}{n_s} \right)^{0.5} f_m \tag{16}$$

$$G_s = \left( \frac{\rho_s}{\rho_m} \right)^{1.5} \left( \frac{D_s}{D_m} \right)^{1.5} \frac{n_s}{n_m} \tag{17}$$

In applying Eq. (16) and (17), it is necessary that cavitation patterns in model and full scale are similar.

### 2.2 Scale Effects on Cavitation

As mentioned in the introduction, propeller cavitation is divided into two types i.e. sheet or bubble cavitation attached to propeller blade surface and vortex cavitation occurring at a distance from propeller blade. For the former type, cavitation extents on propeller blade are similar between in model and full scale at equal cavitation number. For well developed cavitation, this assumption may be reasonable and Eq. (16) and (17) are reduced to the following equations:

$$f_s = \left( \frac{n_m}{n_s} \right)^{0.5} f_m \tag{18}$$

$$G_s = \left( \frac{\rho_s}{\rho_m} \right)^{1.5} \left( \frac{D_s}{D_m} \right)^{1.5} \frac{n_s}{n_m} \tag{19}$$

It is well-known that the estimated noise levels from model tests using Eq. (18) and (19) show good agreement with those of full scale measurements.

On the other hand, it is known that the inception of vortex cavitation depends on not only cavitation number but also Reynolds number. McCormick assumed that the incipient cavitation number is equal to the minimum pressure coefficient of a Rankine combined vortex so that:

$$\sigma = \text{constant} \left( \frac{1}{aU} \right)^2 \tag{20}$$

Where $\sigma_i$: incipient cavitation number
$a$: vortex core radius
$U$: free-stream velocity
$\Gamma$: strength of vortex

In case of hydrofoil, the strength of vortex can be expressed as:

$$\Gamma = \text{constant} \ C_l \ \bar{C} \tag{21}$$

where $C_l$: lift coefficient
$\bar{C}$: chord length

Assuming that the core radius is proportional to the boundary layer thickness as suggested by McCormick’s experiments, $a$ is written as:

$$a \approx \frac{\delta}{R_{e}^{-0.8}} \tag{22}$$

where $\delta$: boundary layer thickness
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Reynolds number $= \frac{UC}{\nu}$

$v$: kinematic viscosity

$n$: exponent of $Re$ in expression for $\delta$

Employing Eq. (21) and (22) in Eq. (20) yields:

$$k=2n$$

The value of $k$ in Eq. (23) is 0.4 for a turbulent boundary layer, since the boundary layer thickness varies at the Reynolds number to the $-1/5$ power. The desinent cavitation number for tip vortex cavitation on the rectangular hydrofoils are given as a function of a Reynolds number in Fig. 1. The cavitation number in Fig. 1 varies at the Reynolds number to the 0.35 power. This fact indicates that vortex cavitation is not always similar at equal cavitation number. Therefore it may be necessary that cavitation number in model test is different from that in full scale in order to keep similarity of pattern of vortex cavitation between in model and full scale. Then Eq. (16) and (17) should be used to predict full-scale noise for the case of vortex cavitation, since these equations include cavitation number as a parameter.

3. Model and Full Scale Measurements

In order to know the correlation of vortex cavitation noise of propeller, it is necessary to study the case that only the vortex cavitation occurs. In this study, such propeller and operating conditions were selected to investigate cavitation patterns and noise characteristics.

### 3.1 Principal Particulars of Ship and Propeller

The principal particulars of the ship, which has two controllable pitch propellers, are given in Table 1. The ship uses acoustic instruments at the cruising speed and the propeller was required to reduce underwater noise at the cruising speed. Therefore, much care were taken to reduce the propeller cavitation. Propeller diameter and revolutions were selected so as to reduce propeller loading and hence to increase the cavitation inception speed. High skew and tip unloading were introduced to minimize cavitation. Principal particulars and geometry of the propeller are given in Table 2 and Fig. 2.

### 3.2 Model Tests

Model tests were carried out in the cavitation tunnel of the Nagasaki Experimental Tank, MHI. Cavitation patterns were observed with a stroboscope and recorded on photographs. Radiated noise was measured using a hydrophone set in the cavitation tunnel as shown in Fig. 3. Output signals from the hydrophone were analyzed by an FFT analyzer.

The operating conditions of the propeller were defined by pitch ratio, cavitation number $\sigma_n$ and thrust coefficient $K_t$ as:

$$K_t = \frac{T}{\rho n^2 D^4}$$

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Table 1 Principal particulars of ship

| Length (PP) | 90.0 m |
| Breadth (MLD) | 16.2 m |
| Depth (MLD) | 8.9 m |
| Draught | 6.0 m |
| Main Engine Output | 3,800 PSx2sets |
| Number of Propeller Revolutions | 148 rpm |

Fig. 1 Tip vortex cavitation data for NACA0015 hydrofoils (From Billet and Holl [16])

Table 2 Principal particulars of propeller

| Diameter | Full Scale | 3.70 m |
| Pitch Ratio at 0.7R | 1.295 (variable) |
| Expanded Area Ratio | 0.602 |
| Boss Ratio | 0.265 |
| Skew Angle | 39.2 deg. |
| Number of Blades | 4 |
where $T$: thrust of propeller

The measurements were conducted in the wake field of a shaft-and-bracket dummy as shown in Fig. 3. The relative air content at atmospheric pressure was kept about 30% during model tests.

No cavitation appeared on the blade surface of the propeller and only the vortex cavitation was observed on the face side of the propeller as shown in Fig. 4 where the pitch ratio was reduced from the designed value.

Since only vortex cavitation occurred in model tests, cavitation patterns and radiated noise characteristics of the propeller were investigated changing cavitation number widely, expecting to correlate cavitation number at similar patterns of vortex cavitation in full scale. The full-scale source levels were estimated from noise data measured in the cavitation tunnel by using Eq. (16) and (17). Assuming that point sources were uniformly distributed on the circumference of the propeller, the distance $r$ in Eq. (17) was calculated by

$$\frac{1}{r^2} = \frac{1}{2\pi} \int_{\theta_1}^{\theta_2} \frac{d\theta}{r'^2}$$

(25)

where $r'$: distance from point source on the circumference of the propeller to field point

The results are shown in Fig. 5 and Fig. 6, where pitch ratio was 0.871. With decreasing cavitation number, the extents of cavitation increase and also noise levels increase.

3.3 Full Scale Measurements

Full scale measurements were carried out at Suoshunda, where the water depth was about 25m. The arrangements of observation windows are shown in Fig. 7 and Fig. 8. A block diagram of observation system is shown in Fig. 9. In Fig. 7, windows A and E were used for TV camera and windows B and D were used for stroboscopic lighting. The back side of the propeller blade was observed for the port propeller and face side was observed for the starboard propeller. The stroboscope was flushed synchronized with propeller revolution. The cavitation pattern at the moment was stored in a video memory and recorded by a video tape recorder. Using the phase angle controller, the cavitation pattern was observed at an arbitrary position of blade angle. Radiated noise was measured using hydrophone sets, which were mounted on the hull plate above the propeller. The positions of hydrophones are shown in Fig. 7 and Fig. 8. A block diagram for noise measurements is shown in Fig. 10. The signals received by the
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Fig. 5 Vortex cavitation patterns observed in model tests (\(\rho = 0.871\))

Fig. 6 Full-scale noise spectra estimated from model tests

Fig. 7 Arrangement of observation windows and hydrophones

Fig. 8 Arrangement of observation windows and hydrophones (view from stern)

Fig. 9 Arrangement for propeller observation system in full scale

Fig. 10 Arrangement for measuring underwater noise in full scale
hydrophone were recorded and analyzed by an FFT analyzer. The measured sound levels were transformed to source levels. The distance from source was calculated by Eq. (25).

No cavitation appeared on the blade of the propeller and only the vortex cavitation was observed on the face side of the propeller, where the pitch ratio was reduced from the designed value. The pattern of vortex cavitation observed in full scale measurements is shown in Fig. 11 and Fig. 12. The radiated noises are shown in Fig. 13. The noise levels measured at port side are nearly equal to those at starboard side.

4. Discussion on the Correlation of Vortex Cavitation Noise

From the comparison between Fig. 5 and Fig. 12, it was found that cavitation patterns in full scale were similar to those in model scale when cavitation number in the model test was between 2.25 and 2.5. Further, cavitation patterns in model scale at \( \alpha_n = 2.25 \) were much similar to those in full scale than at \( \alpha_n = 2.5 \), so the noise data measured at \( \alpha_n = 2.25 \) were used to compare with those measured in full scale. The noise levels estimated from the model tests at \( \alpha_n = 2.25 \), using Eq. (16) and (17), are compared with the full scale measurements in Fig. 14, Fig. 15 and Fig. 16. The estimated noise levels are nearly equal to those of the full scale measurements.

Cavitation numbers are plotted versus Reynolds numbers in Fig. 17. Reynolds number is defined as follows:

\[
Re = (0.9\pi n D) \cdot \frac{C_{0.88}}{\nu}
\]  

(26)

Assuming that the relation of Eq. (23) is kept, the relationship between cavitation number and Reynolds number is expressed as follows:

\[
\frac{\sigma_{ns}}{\sigma_{nm}} = \left[ \frac{Re_s}{Re_m} \right]^k
\]  

(27)

The value of \( k \) in Eq. (27) corresponds to 0.15 in the present study. It is small as compared with the case of the inception of tip vortex cavitation.

Because of the shortage of this kind of data, it is not certain to use generally the value of \( k \) as 0.15, which is found in the present study, for the determination of corresponding cavitation number in model test. Rather it can be considered that the value of \( k \) is varied according to cavitation number in full scale, i.e. \( k \)
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becomes 0.35~0.4 for high cavitation number near the inception and $k$ becomes smaller for low cavitation number. Further study and collection of data at various cavitation numbers both in model and full scale are necessary.

5. Concluding Remarks

In order to know the correlation of vortex cavitation noise measured in model experiments and full scale, cavitation patterns and noise characteristics of a propeller were investigated by model tests in a cavitation tunnel and full-scale measurements. The results are summarized as follows.

(1) Comparisons of full-scale cavitation pattern with model scale showed that vortex cavitation depended on not only cavitation number but also Reynolds number.

(2) Predicted noise data from model tests using scaling formulas of cavitation noise agreed well with full-scale measurements in the case where patterns of tip vortex cavitation in model and full scale were similar.

(3) Assuming that cavitation number varies at Reynolds number to the $k$ power, $k$ would be 0.15 to keep the similarity of pattern of vortex cavitation.

Though it is not certain to adopt the relationship found in the present study as a general one, the present result gives a guide for the determination of cavitation number in model scale where the vortex cavitation is studied. Needless to say, a continuous effort is necessary to investigate more into vortex cavitation experimentally and theoretically for both model and full scale.

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References


5) Lovik, A.: Scaling of Propeller Cavitation
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