On Buckling Accepted Design of Ship Structures Utilizing High Tensile Steels

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Summary

Among the trends of revolution for ship structures, the use of high tensile steels in many parts of the structures has been tried by many organizations concerned. With high tensile steels, ship structures will be generally thinner than those with conventional mild steel, taking account of higher yield strength of the material. This, in turn, will lead to lower buckling strength. This fact may suggest to accept buckling of such structures in order to utilize the advantage of high yield strength of the material.

Allowing buckling in ship structures will bring out several problems. This paper discusses some of these problems, namely: the maximum stress, ultimate strength and fatigue strength under regular and random loads. A design philosophy on buckling accepted design is proposed. Design criteria, design methods and related design graphs are presented in connection with the new design philosophy.

1. Introduction

Among the trends of revolution in the shipbuilding industry, high tensile steel is considered for use in many parts of ship structures. Lighter scantlings are expected utilizing effectively the high yield stress of this material. Such lighter scantlings may be lower in buckling strength than those of ship structures built using conventional mild steel. From practical viewpoint against improvement of buckling strength, reducing stiffener spacing in proportion to the high yield stress is not an efficient way. This suggests an alternate way that buckling should be accepted in ship structures if the high yield strength of the material is aimed to be utilized.

Buckling in ship structures may be classified into column buckling, stiffened panel buckling and buckling of primary supporting members. Column buckling (flexural, torsional or lateral) leads to collapse of the column with ultimate strength which is almost equal to the buckling strength. No benefits may be obtained by using higher strength steels.

Buckling of stiffened plates may take place in several different modes.

a) Overall buckling: In this mode of buckling, plate panels and stiffeners comprising a stiffened plate bounded by four primary supporting members, such as bulkheads or strong girders and stringers, buckle together.

b) Stiffeners lateral buckling: This mode of buckling usually leads to overall buckling and collapse of the stiffened plate. Such practice as avoiding this mode of buckling is usually provided.

c) Local buckling of plate panels between stiffeners: Plate panels may buckle but still continue to carry substantial further load after buckling as long as its edges continues to be effectively supported by the stiffeners. This mode of buckling may be allowed to exploit the high yield stress of the material.

Primary supporting members, such as deck and bottom girders may buckle in three different modes.

a) Lateral buckling: The carrying capacity of a girder is mainly governed by this mode of buckling and nothing can be gained by allowing this mode of buckling.

b) Torsional buckling of the flange: This usually leads to lateral buckling and should not be allowed.

c) Web buckling: In stiffened web plates, only plate panels between stiffeners may be allowed to buckle.

From the above discussion, it may be seen that only effectively supported plate panels may be allowed to buckle in order to exploit the advantage of high tensile steels. These plate panels, however, may constitute about 80% of the ship hull weight and substantial economic benefits may be gained by reducing the thickness of these plate panels.

Allowing buckling in ship structures will bring out several problems. Three subjects related to rectangular plate panels subjected to uniaxial compression are discussed in this paper, namely, the maximum stress,
ultimate strength and fatigue strength under regular and random loads.

A design philosophy on buckling accepted design is proposed, and design criteria, methods and graphs are presented in connection with these subjects.

2. Consequences of plate buckling in ship structures

Let's consider a flat stiffened plate, as shown in Fig. 1, free of any imperfections such as initial deflection and/or welding residual stresses, and simply supported at its edges. It is assumed that the plate thickness is such that it will buckle in an early loading stage, and the stiffeners be such that they will not buckle or collapse before the plate panels have reached their ultimate strength. The edges are assumed to be kept straight but free to move in the plane of the plate.

For simplicity of discussion, the panel is supposed to be subjected to an increasing uniaxial inplane compression in the direction of the stiffeners. The arguments to be mentioned here should be valid, however, for other combined loading conditions. As the load increases starting from zero, a linear (uniform) stress distribution develops across the breadth. When the load reaches the critical load, plate panels buckle, and buckling deflection is induced in the following patterns characterized as:

a) each half buckling wave tends to have a length close to stiffener spacing,

b) buckling deflection of adjacent plates is produced usually in opposite directions as shown in Fig. 1.

This phenomena resembles buckling of simply supported plate panels, except the restraint provided by the torsional stiffness of the stiffeners, and the webs of the primary supporting members around the stiffened plate.

With further loading, buckling deflection increases. This leads to a more complicated stress distribution as shown in Fig. 2. Four important consequences of this stress distribution are:

1) maximum (locally maximum) stresses higher than the nominal (average) stress are developed. These may occur at the edges or at the central portion of each half buckling wave on the concaved surface.

2) ultimate strength becomes smaller than the fully plastic strength which the plate could have attained if buckling did not occur. The smaller the ratio $\sigma_{cr}/\sigma_0$ is, the smaller is $P_u/P_p$ where,

- $\sigma_{cr}$ = critical (buckling) stress of the plate panels,
- $\sigma_0$ = yield stress of the material,
- $P_u$ = plate ultimate strength,
- $P_p$ = plate fully plastic strength.

3) complicated 3-dimensional stress distribution is developed in the vicinity of welds between the stiffeners and panels. This is due to plate buckling being restrained by the torsional stiffness of the stiffeners.

4) Tangential stiffness of the buckled plate is reduced to about 0.5 of its original value.

The first of these consequences should be discussed with respect to the allowable maximum stress design criterion which is used with conventional design loads. The second is related to safety evaluation in extreme loading conditions. The third is related to fatigue strength and the fourth is concerned with the stiffness of ship hull in response to different loading conditions.

In the following sections, the first three consequences are discussed.

3. Design philosophy

Traditionally ship structures are designed based on rules developed from the past experience. Although the concept of design by analysis has been popular for quite some time, it has been applied only partially in some ship hull designs such as LNG carriers. To the knowledge of the authors, there is only one design which is fully carried out according to this concept. In this
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design, selected factors of safety, allowable stresses, safety criteria etc., are based on the past experience. This is natural because it is only rational to have utilized the experience accumulated during hundreds of years of shipbuilding and operation. Although the authors are in favour of the idea of design by analysis, until some absolute design criteria for all possible loading conditions could be established, design should be carried out “in comparison with present ships with satisfactory performances” or “in reference to conventional design rules as an average of the present practice”.

In ship structures, however, fatigue strength is substantially influenced by many factors such as (1) the type of ship (2) types and locations of individual members even in the same ship and (3) the kind of used steels. Therefore, the design for fatigue strength needs absolute values of criteria and accurate evaluation of actual fluctuating stresses based on experimental studies and experience which have been gained in the design of offshore structures.

For the use of high tensile steels for plate panels in ship hulls, taking account of this discussion and considering the consequences of plate buckling mentioned in the preceding section, the following criteria should be added to the conventional ones, if the buckling accepted design is applied to plate panels in ship hulls.

1. Maximum stress criterion: Maximum stress anywhere in the plating or the attached stiffeners should not exceed the yield stress divided by a factor of safety which is equal to that used with ship hulls built with conventional mild steel. i.e.

\[ \sigma_{max} \leq \sigma_{y} / \alpha \]

where, \( \sigma_{max} \) =maximum stress in the buckled plate or attached stiffeners,

\( \alpha \) =a suitable factor of safety based on present experience.

2. Ultimate strength: Ultimate strength of a high tensile steel plate panel should be equal to or greater than that of a corresponding conventional mild steel plate i.e.

\[ P_{uts} \geq P_{us} \]

where, \( P_{uts} \) =ultimate strength of a high tensile steel plate panel,

\( P_{us} \) =ultimate strength of a corresponding conventional steel plate panel.

A corresponding conventional steel plate panel is defined as a conventional steel plate designed to carry the same design load according to the maximum stress criterion in 1. above.

3. Fatigue strength: Alternating random stresses are to be limited to the values corresponding to the required fatigue life. In other words, the calculated fatigue life should be more than or equal to the required fatigue life (in ship structures, fatigue life is usually taken as \( 10^6 \) cycles) i.e.

\[ N_{c} \geq N_{t} \]

where, \( N_{c} \) =calculated fatigue life,

\( N_{t} \) =required fatigue life.

These criteria will be discussed in details in sections 4, 5 and 6.

4. Design against maximum stress

The same stiffened plate as shown in Fig. 1 is considered, assuming the attached stiffeners will not fail before the plate panels reach their ultimate strength. The buckling strength of the plate panels may be expressed as follows.

\[ \sigma_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)}(1/b)^2 \]  

where, \( E \) is the Young's modulus and, \( \nu \) is the Poisson's ratio for long plates \( k \) may be taken equal to 4.

With loads higher than the buckling load, the stress distribution becomes as shown in Fig. 2. Maximum membrane stress \( \sigma_{max} \) (compression) in \( x \) direction occurs along the longitudinal edges of the plates while the minimum membrane stress \( \sigma_{min} \) (tension) in \( y \) direction occurs in the middle of each half buckling wave. If the aspect ratio of one half buckling wave is assumed to be 1.0, \( \sigma_{max} \) and \( \sigma_{min} \) may be expressed as follows.

\[ \sigma_{max} = 2\sigma_{av} - \sigma_{cr} \]
\[ \sigma_{min} = -\sigma_{av} + \sigma_{cr} \]  

On the concave surface of the panel on the central portion of each half buckling wave, bending stress is superimposed on the membrane stress. However, bending stress is not considered in this study for two reasons.

1. For thin plates, which are the main interest of this study, this surface stress is smaller than the edge membrane stress.

2. Even for thicker plates this surface stress is not a direct cause of collapse.

Since the stiffeners are assumed not to fail before the plate panel does, the stress in the stiffeners will always be smaller than the maximum equivalent stress in the buckled plate with respect to the yield condition. Only the maximum membrane equivalent stress of the plate at the edges is considered here.

Maximum equivalent stress occurs where \( \sigma_{x} \) is maximum (in compression) and \( \sigma_{y} \) is minimum (in tension), i.e. in the middle of individual half buckling waves along the longitudinal edges and may be expressed as follows.

\[ \sigma_{eq} = \sigma_{2max} + \sigma_{2min} - \sigma_{max} \sigma_{min} + 3\sigma_{av} \]  

Substituting Eqs. (2) and (3) into Eq. (4) and imposing the condition that \( \sigma_{eq} \) be equal to the allowable stress \( \sigma_{all} \), a design limit may be obtained as follows.

\[ \sigma_{av} = 7\sigma_{av} - 9\sigma_{av} \sigma_{cr} + 3\sigma_{cr} \]  

Solving this equation for \( \sigma_{av} \), choosing the correct solution and dividing by \( \sigma_{all} \),

\[ \frac{\sigma_{av}}{\sigma_{all}} = \frac{1}{14} \left[ 9\sigma_{cr} - \left( -3 + 28 \sigma_{cr} \right)^{1/2} \right] \]  

where, \( N_{c} \) =calculated fatigue life,

\( N_{t} \) =required fatigue life.

These criteria will be discussed in details in sections 4, 5 and 6.
Substituting Eq. (1) into Eq. (6), a relation between the highest allowable average stress $\sigma_{x_{av}}$ and $b/t$ may be obtained as,

$$\sigma_{x_{av}} = \frac{1}{14}\left[\frac{9c}{\beta_0^2} - \left(-3\left(\frac{c}{\beta_0}\right)^2 + 28\right)\right] \tag{7}$$

where, $\beta_0 = b/(\sigma_{ut}/E)^{1/2}$

$$c = k\pi^2/[12(1-\nu^2)]$$

Equation (7) may be plotted as indicated in Fig. 3.

Equation (7) or Fig. 3 may be used to evaluate the allowable average stress $\sigma_{x_{av}}$ to be applied on a plate when $b/t$, $E$ (Young's modulus) and the allowable maximum stress, $\sigma_{ut}$, for the specified material are known.

Here, several cases of the panel shown in Fig. 1 are considered. The standard case is a conventional mild steel plate with $\sigma_{ut}=15$ kgf/mm² ($\sigma_{ut}=30$ kgf/mm², the factor of safety $f_s=2$). In the other cases, the plate is replaced by various high tensile steel plates. Using Eq. (7), Fig. 4 shows the maximum allowable average stress for plates with different thicknesses made of high tensile steel with allowable stress equal to 25 kgf/mm² ($\sigma_{ut}=50$ kgf/mm², the factor of safety $f_s=2$) plotted against the stiffener spacing. Figure 5 shows the maximum allowable load per unit breadth ($\sigma_{av}t$) of these plates also plotted against the stiffener spacing. These figures indicate the possible reduction of plate thickness which may be achieved by adopting such a high tensile steel. For example, with a fixed frame spacing equal to 900 mm, reduction of plate thickness by 2 mm may be achieved while maintaining the same load as that carried by the conventional mild steel plate 14 mm in
thickness. With a frame spacing of 700 mm, a high
tensile steel plate 10.5 mm thick may be used to carry
the same load as that carried by the original plate.
Similar relationships for $\sigma_{00} = 40$ kgf/mm$^2$ ($\sigma_0 = 80$
kgf/mm$^2$, $f_s = 2$) are shown in Figs. 6 and 7.

It is to be noted here that plates 8.4 mm thick with
stiffener spacing of 460 mm, and 5.25 mm thick with
stiffener spacing of 225 mm may be used in cases of $\sigma_{00} = 25$ kgf/mm$^2$ and 40 kgf/mm$^2$ respectively, if buckling
is to be prevented, while carrying the same load as the
original steel plate of 14 mm in thickness.

5. Design against ultimate strength

For the stiffened plate in Fig. 1, the ultimate strength
after plate panels have buckled, may be accurately
estimated by assuming that the ultimate strength state
is reached when the yield condition (Von Mises' condi-
tion in this paper) is satisfied in the middle of a half
buckling wave along the longitudinal edges. As
mentioned in section 4, at such a location, $\sigma_x$ is maxi-
mum (compression) and $\sigma_y$ is minimum (tension),
which may still be expressed by Eqs. (2) and (3), and
the Von Mises yield condition may be expressed as
follows.

$$\sigma^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y$$  \hspace{1cm} (8)

Substituting Eqs. (1), (2) and (3) into Eq. (8) and
noting that the average stress $\sigma_{av}$ in this case equals to
the ultimate average stress $\sigma_{uu}$, an equation similar to
Eq. (7) may be obtained in which $\sigma_0$ replaces $\sigma_{uu}$ and
$\sigma_{uu}$ replaces $\sigma_{av}$.

$$\sigma_{uu} = \left\{ \frac{9c}{\beta^2} + \left[ -3\left( \frac{c}{\beta} \right)^2 + 28 \right] \right\}^{1/2}$$  \hspace{1cm} (9)

where, $\beta = b/t(\sigma_0/E)^{1/2}$.

Equation (9) may be represented by the same curve in
Fig. 3.

Writing Eq. (9) for a conventional mild steel panel
and a high tensile steel panel, using subscripts ss and
HTS for these steels,

$$\sigma_{uu}\text{ss} = \left\{ \frac{9c}{\beta^2} + \left[ -3\left( \frac{c}{\beta} \right)^2 + 28 \right] \right\}^{1/2}$$  \hspace{1cm} (10)

where, $\beta = (b/t)(\sigma_0/E)^{1/2}$.

$$\sigma_{uu}\text{HTS} = \left\{ \frac{9c}{\beta^2} + \left[ -3\left( \frac{c}{\beta} \right)^2 + 28 \right] \right\}^{1/2}$$  \hspace{1cm} (11)

Dividing Eq. (11) by Eq. (10), the $\sigma_{uu}\text{HTS}/\sigma_{uu}\text{ss}$ may be
expressed as follows

$$\frac{\sigma_{uu}\text{HTS}}{\sigma_{uu}\text{ss}} = \frac{(t/b)_{\text{ss}}}{(t/b)_{\text{HTS}}} \left[ 4 + \left( \frac{b_0}{c} \right)^2 \right]^{1/2}$$  \hspace{1cm} (12)

The condition for equal ultimate strength ($N_{uu}$) of
both plates may be expressed as follows,

$$N_{uu}\text{HTS} = \frac{\sigma_{uu}\text{HTS}}{\sigma_{uu}\text{ss}}$$  \hspace{1cm} (13)

Writing Eq. (9) for a conventional mild steel panel
and a high tensile steel panel, using subscripts ss and
HTS for these steels,

$$\sigma_{uu}\text{ss} = \left\{ \frac{9c}{\beta^2} + \left[ -3\left( \frac{c}{\beta} \right)^2 + 28 \right] \right\}^{1/2}$$  \hspace{1cm} (14)

Equation (14) gives the relationship between the
thickness of a conventional mild steel plate and that of
a high tensile steel plate for the same ultimate strength.
The design concept against ultimate strength is
applied to the same panel of Fig. 1 using high tensile
steel. Figure 8 represents the average ultimate stress $\sigma_{uu}$
of plates of high tensile steel with a yield stress of 50
kgf/mm$^2$ with respect to stiffener spacing for different
thicknesses. The average ultimate stress $\sigma_{uu}$ of the 14

![Fig. 7 Relationships of maximum allowable load to stiffener space](image)

![Fig. 8 Relationships of ultimate average stress to stiffener space](image)
A HTS plate (σ₀=50 kgf/mm²) of 12 mm thickness has an ultimate strength nearly equal to the original 14 mm steel plate for all frame spacing between 700 and 1000 mm and it has higher ultimate strength for smaller frame spacing.

Similar relationship for σ₀=80 kgf/mm² are shown in Figs. 10 and 11. A HTS plate of 9 mm thickness has an ultimate strength equal to that of the original steel plate at stiffener spacing equal to 1000 mm. While a 10 mm thick plate is necessary to carry the same load as that of the 14 mm conventional mild plate if b=700 mm.

6. Design against fatigue

Fatigue strength at a point of a structure is generally measured by the magnitude of the cyclic stress range S and the necessary number of cycles N (fatigue life) before a crack is produced at this point. Two approaches are available to assess the fatigue strength of structures, which are the S-N curve approach and the fracture mechanics approach. The S-N curve approach is simpler and more frequently used in the conventional design stage. In this paper, only this approach is applied.

6.1 Response to cyclic longitudinal in-plane loading

Let the panel shown in Fig. 1 be considered again. When the panel is subjected to a cyclic in-plane load in the form of uniform shortening and extension in x-direction (the direction of the stiffeners), the acting stress range, Sₓ at a point may be expressed as follows

\[ Sₓ = \sigma_r - \sigma_t \] (15)

where, \( \sigma_r \) is the peak compressive (positive) stress at this point and \( \sigma_t \) is the peak tensile (negative) stress at the same point.

Then, the mean stress may be expressed as

\[ \sigma_m = (\sigma_r + \sigma_t)/2 \] (16)
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where, $\sigma_{av}$ is positive when it is compressive.

These stresses are illustrated in Fig. 12, in the case of regular cyclic loading.

In the case where the peak compressive stress acting on the stiffened plate is smaller than the uniaxial buckling stress $\sigma_{cr}$ of the plate panel and the plate panel are assumed to be free from initial imperfection, the stress range, $S_x$, will be the same at any point in the plate. In this case, no stress in $y$-direction is produced since the longitudinal edges are free to move in the plane of plate.

On the other hand, if the peak compressive stress is higher than $\sigma_{cr}$, the plate panels between the stiffeners will buckle. As mentioned before, the stress distribution is no longer uniform and becomes as shown in Fig. 2 with the peak maximum compressive membrane stress $\sigma_{x,\text{max}}$ in $x$-direction is induced along the longitudinal edges of the plates (along the stiffeners) and is expressed in terms of $\sigma_{av}$ (the peak average compressive stress) as given by Eq. (2). Notations “maximum” and “average” are used with respect to the plate breadth $b$, while terms “mean” and “peak” are used with respect to the elapse of time as indicated in Figs. 2 and 12. In this case, the acting stress range at a point on a longitudinal edge of a plate panel may be expressed by the following equation,

$$S_{x,\text{max}} = \sigma_{x,\text{max}} - \sigma_t = 2\sigma_{av} - \sigma_{cr} - \sigma_t$$  \hspace{1cm} (17)

and the mean stress may be given by

$$\sigma_{x,\text{m}} = (\sigma_{x,\text{max}} + \sigma_t)/2 = (2\sigma_{av} - \sigma_{cr} + \sigma_t)/2$$  \hspace{1cm} (18)

In $y$-direction, the minimum (tension) membrane stress $\sigma_{y,\text{min}}$ occurs in the middle of each half buckling wave and is given by Eq. (3). At the stiffeners, restraining moments are produced by the stiffeners resisting the rotation of the plate around its longitudinal edges as shown in Fig. 13. These moments are small and usually neglected when the buckling and post-buckling behaviors of the plate panels are dealt. However these moments cause high local stress $\sigma_{yb}$ in $y$-direction on the plate surface in the vicinity of the toe of the weld connecting the plate to the stiffener. This stress acts together with $\sigma_{y,\text{min}}$ and is maximum at points such as A and B which are in the middle of each half buckling wave as indicated in Fig. 13. Although analytical evaluation of this stress is difficult, it may be expressed in the following form,

$$\sigma_{yb} = -\Psi(\sigma_{av} - \sigma_{cr})/2$$  \hspace{1cm} (19)

The function $\Psi$ depends on the plate dimensions, stiffener torsional stiffness and the size of the weld. The values of $\sigma_{y,\text{m}}$ and $\Psi$ may be evaluated in each particular case using a numerical method such as a finite element method.

In summary, in the post-buckling stage, local cyclic stresses at points $A$ and $B$ are produced in two directions:

1. The stress range in $x$-direction is referred to as the longitudinal stress range $S_x$ and is given by Eq. (17).

2. The stress range in $y$-direction is referred to as the transverse stress range $S_y$ and is given by consideration of Eqs. (3) and (19). These two local cyclic stresses are illustrated in Fig. 14.

Since these cyclic stresses in $x$ and $y$ directions occur
at the same position, some interaction may be expected. Experimental evaluation of S-N curves using specimens with representative geometries and loading conditions including two dimensional stress state is necessary for an accurate evaluation of fatigue behavior. In absence of such S-N curves, however, these two-dimensional cyclic stresses are treated separately in this study neglecting any interactions.

6. 2 Fatigue due to random cyclic loading

In this section, the consequences of local buckling on fatigue behavior under random wave loading are considered. The Miner rule is used to evaluate damage factors, adopting the assumptions generally accepted. Necessary modifications are made in order to take account of buckling consequences.

6. 2. 1 Transfer function

Generally, a transfer function may be defined as the ratio of the stress response to the wave height. That is

\[ T = \frac{S}{H} \]  

where, \( T \) is the transfer function, \( S \) is the stress range and \( H \) is the wave height.

\( T \) is usually assumed to be constant, that is, a linear relationship between \( H \) and \( S \) is assumed as shown by a solid line in Fig. 15.

For the stiffened plate under consideration, the dimensions of the plate panel are assumed as such it will buckle at a wave height \( H_{cr} \) and the corresponding stress range is equal to \( S_{cr} \).

\[ S_{cr} = S_{cr} - \sigma_{cr} \]  

For wave heights smaller than \( H_{cr} \), the stress range in \( x \)-direction, \( S_x \), will follow Eq. (20), where \( S_x \) is given by Eq. (15)

\[ T_x = \frac{S_x}{H} = S_{cr}/H_{cr} \]  

Equation (22) assumes a linear relationship between \( S_x \) and \( H \) for \( 0 \leq S_x \leq S_{cr} \).

For wave heights larger than \( H_{cr} \), the average stress range in \( x \)-direction, \( S_{xav} \), will follow Eq. (20) and

\[ S_{xav} = S_{xav} - \sigma_t \]  

However, the stress range in \( x \)-direction at points A and B, \( S_{xav} \), as given by Eq. (17), will be higher than \( S_{xav} \). For this post-buckling range, a transfer function \( T_x' \) (\( =S_{xav}/H \)) may be expressed as follows, based on the response at a wave with a probability of exceedance of \( 10^{-8} \).

\[ T_x' = \left[ (\sigma(H - H_{cr}) + \sigma_{cr}) - \sigma_{cr}(H/H^*) \right]/H \]  

where, \( \alpha = (\sigma_{xav} - \sigma_{cr})/(H^* - H_{cr}) \)

\( \ast \) refers to values at probability of exceedance of \( 10^{-8} \).

Equation (24) assumes a linear relationship between \( S_{xav} \) and \( H \) for the range of loading between the buckling and the point of \( 10^{-8} \) probability of exceedance, as shown by the dashed line in Fig. 15. It is to be noted that \( \sigma_{xav} \) and \( S_{xav} \) after buckling are corresponding to \( \sigma_t \) and \( S_t \) before buckling.

In \( y \)-direction, the stress range \( S_y \) for wave heights larger than \( H_{cr} \) is given by consideration of Eqs. (3) and (19)

\[ S_y = \gamma S_{xav} \]  

As mentioned before \( \gamma \) may be evaluated from a non-linear finite element analysis. More directly, \( S_y \) at a wave of a probability of exceedance of \( 10^{-8} (S^y) \) may be evaluated by such a finite element analysis. Based on the response at a wave with a probability of exceedance of \( 10^{-8} \), a transfer function \( T_y \) may be expressed as follows

\[ T_y = S_y/H = S_y^y(H - H_{cr})/(H^* - H_{cr})H \]  

Equation (26) assumes a linear relationship between \( S_y \) and \( H \) for the range of loading between the buckling and the point of \( 10^{-8} \) probability of exceedance, as shown by the dash-dot line in Fig. 15.

6. 2. 2 The probability density function

Generally, the Weibull or the exponential probability density function is used to represent the stress response based on a linear transfer function. In this paper the exponential probability density function is adopted.

\[ P(S) = (1/\lambda)e^{-S/\lambda} \]  

where, \( P(S) \) is the exponential probability density function, \( S \) is the nominal stress range and \( \lambda \) is the scale parameter of this function.

The probability that the stress range \( S^* \) is reached or exceeded during the total numbers of cycles \( N^* (=10^8) \) is given as follows

\[ P(S^*) = \int_{S^*}^{\infty} P(S)dS = 1/N^* \]  

From which, \( \lambda \) may be evaluated as follows

\[ \lambda = S^*/ln N^* \quad \text{in case of } H^* < H_{cr} \]  

\[ \lambda = S_{xav}^*/ln N^* \quad \text{in case of } H^* > H_{cr} \]

Equation (28) is represented by the solid lines in Figs. 16-a and 16-b.

Let the dimensions of the plate panels be such that buckling occurs at a certain stress range, \( S_{cr} \). For stress ranges larger than \( S_{cr} \), the probability density distribution of the average stress range \( S_{xav} \) as defined by Eq.
will follow Eq. (27). However, $S_{x_{\text{max}}}$ (given by Eq. (17)) will be larger than $S_{x_{\text{av}}}$ and may be represented by the dashed line in Figs. 16-a and 16-b. The probability of exceedance of any $S_{x_{\text{max}}}$ at any wave height is equal to the probability of exceedance of $S_{x_{\text{av}}}$ (which produces this $S_{x_{\text{max}}}$) at this wave height.

In $y$ direction, $S_y$ is given by Eq. (25) and represented by the dash-dot line in Figs. 16-a and 16-b. Here, also the probability of exceedance of any $S_y$ at any wave height is equal to the probability of exceedance of $S_{x_{\text{av}}}$ (which produces this $S_y$) at this wave height.

### 6.2.3 Miner’s damage factor

Miner’s damage factor $D$ may be expressed as follows,

$$D = \sum \frac{dn}{N}$$

where, $dn$ is the number of cycles at a stress range $S$ and $N$ is the number of cycles at failure under this stress range $S$.

$dn$ may be expressed as follows

$$dn = (N^* / \lambda) e^{-s/S} dS$$

while $N$ may be expressed using an $S$-$N$ curve as follows

$$N = CS^*_y$$

where, $C$ is the value of $N$ at the intersection of the $S$-$N$ curve with the log $N$ axis as shown in Fig. 17, $m$ is the inverse negative slope of the $S$-$N$ curve, and, $S_y$ is the actual stress range acting at a point, under a nominal stress range equal to $S$.

Substituting Eqs. (32) and (33) into Eq. (31), $D$ may be expressed as follows,

$$D = N^* / C \int_0^{S_{x_{\text{av}}}} (S^*_y / \lambda) e^{-s/S} dS$$

$$+ \int_{S_{x_{\text{av}}}}^{S_{x_{\text{max}}}} (S^*_y / \lambda) e^{-s/S} dS$$

First, the damage factor $D_x$ due to the longitudinal stress range at point A or B is considered. Since the stress is represented by a bi-linear function (Eq. (22) for $0 \leq S_x \leq S_{x_{\text{cr}}}$ and Eq. (24) for $S_{x_{\text{cr}}} \leq S_x \leq S_{x_{\text{av}}}$ i.e. $S_{x_{\text{cr}}} \leq S_{x_{\text{max}}} \leq S_{x_{\text{av}}}$). Equation (34) may be written for the longitudinal stress range as follows,

$$D_x = N^* / C \int_0^{S_{x_{\text{av}}}} (S_{x_{\text{av}}}^* / \lambda) e^{-s/S} dS$$

Equation (35) may be rearranged and written as follows

$$D_x = N^* / C (\lambda' / (1 + m) + q)$$

where, $\Gamma$ is the Gamma function, and,

$$q = \int_{S_{x_{\text{cr}}}^{S_{x_{\text{av}}}}} [(S_x^* - S_{x_{\text{cr}}}) / \lambda] e^{-s/S} dS$$

$q$ may be evaluated by numerical integration. Equation (17) may be used to evaluate $S_{x_{\text{max}}}$. The upper limit of integration may be taken equal to $S_{x_{\text{av}}}$ without any appreciable error.

The damage factor $D_y$ of the transverse stress range $S_y$ may be similarly evaluated from Eq. (34). Since the value of $S_y$ is equal to zero in the pre-buckling range, the integration is taken between the point of buckling and infinity.

$$D_y = N^* / C \int_{S_{x_{\text{cr}}}^{S_{x_{\text{av}}}}} (S_y^* / \lambda) e^{-s/S} dS$$

$S_y$ may be expressed as follows

$$S_y = S_{x_{\text{av}}} / (S_{x_{\text{av}}}/S_{x_{\text{cr}}}) = \phi(S_{x_{\text{av}}}/S_{x_{\text{cr}}})$$

$D_y$ may then be obtained as,

$$D_y = N^* / C \int_{S_{x_{\text{cr}}}^{S_{x_{\text{av}}}}} \phi^*(S_{x_{\text{av}}}/S_{x_{\text{cr}}}) e^{-s/S} dS$$

The integration in Eq. (40) may be performed numerically. Similarly, the upper limit of integration may be taken equal to $S_{x_{\text{av}}}$ without any appreciable error.
6.3 Design procedure and numerical results

Equation (36) and (40) may be used to produce design graphs. Due to many variables involved in these two equations, however, too many design graphs would be necessary. It may be more convenient to use these equations directly to evaluate damage factors and compare them with allowable values (usually \( \leq 1.0 \)). A procedure to evaluate the damage factors \( D_x \) and \( D_y \) in the \( x \)- and \( y \)-directions respectively is outlined in Fig. 18 and may be summarized as follows.

1. Calculate the longitudinal compressive and the tensile stresses \( \sigma_{c*} \) and \( \sigma_{t*} \) acting on the considered panel under a load with a probability of exceedance of \( 10^{-8} \). Then, evaluate the stress range under this load
   \[ S_2 = \sigma_{c*} - \sigma_{t*} \]  

2. Compare \( \sigma_{c*} \) with \( \sigma_{c_{cr}} \), the buckling stress of plates between stiffeners. If \( \sigma_{c*} \leq \sigma_{c_{cr}} \), no buckling is expected, and the fatigue life may be checked, if necessary, in the conventional way.

3. In the case where \( \sigma_{c*} > \sigma_{c_{cr}} \), buckling is expected. \( \sigma_{t_{cr}} \), the tensile stress induced by the same wave height as \( \sigma_{c*} \) may be evaluated as follows assuming a linear transfer function.
   \[ \sigma_{t_{cr}} = \frac{\sigma_{c_{cr}}}{H} \]  

4. \( S_{cr} \) appearing in Eqs. (37) and (40) is evaluated as given in Eq. (21).

5. Evaluate \( q \) using Eq. (37) by numerical integration (using Simpson's rule for example).

6. Evaluate \( D_x \) using Eq. (36).

7. Perform a nonlinear finite element analysis to evaluate \( S_{yx}^* \), the stress in \( y \)-direction at the weld toe in the middle of half buckling waves, under a load with a probability of exceedance of \( 10^{-8} \). An example of such an analysis may be found in Ref. 3).

8. Evaluate \( \Phi \), that is
   \[ \Phi = \frac{S_{yx}^*}{(S_{cr}^* - \sigma_{c_{cr}})} \]

9. Evaluate \( D_y \) using Eq. (40) by numerical integrations.

10. Check \( D_x \) and \( D_y \) against their allowable values and modify the structure and reanalyse if necessary.

Figure 19 shows the damage factor due to the longitudinal stress range for different plate thicknesses and frame spaces. In this figure, the load per unit breadth of plates \( S^*t \) is kept constant. \( S^* \) of a plate 14 mm thick is taken equal to 20 kgf/mm². \( S-N \) curve of class \( D \) from DNV is adopted with \( C = 10^{12.8} \) and \( m = 3 \). It may be seen that the damage factor for each plate thickness does not change substantially (in this range of thickness and frame space) with the change of frame spaces. This indicates that buckling effect on fatigue damage in this range of thickness and frame space is not appreciable. On the other hand, the damage factor increases substantially as the plate thickness decreases. This is due to the higher stress range \( S^* \) developed in thinner plates in order to carry the same loads as thicker ones.

In Fig. 20, the damage factor due to the transverse stress range is plotted for different plate thicknesses and frame spaces. Here also, \( (S^*t) \) is kept constant, and
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$S^*$ is taken equal to 20 kgf/mm² for a plate 14 mm thick. $S$-N curve of class F from DNV is used with $C = 10^{1.8}$ and $m = 3$. $\sigma^*$ is calculated using a value of $\Psi = 6$ (see Eq. (19)) which is evaluated from a practical case. It may be seen that damage factors for plate thicknesses more than 8 mm become negligibly small and those thinner plates (where $S_{xcr}$ is small) become larger.

When a different $S$-N curve with $C = 10^{1.8}$ and $m = 6$ is adopted, damage factors calculated under similar assumption are plotted in Figs. 21 and 22.

Figures 23 and 24 show the damage factor for different plate thicknesses and different loads per unit breadth of plates using DNV's curves D and F respectively. Appreciable increase of damage factors for the transverse stress range $S_y$ may be observed with thin plates in the range of thickness being smaller than 5 or 6 mm.

The allowable average stresses are calculated based on the three criteria described in this paper, and they are plotted against the plate slenderness ratio $\lambda$ in Fig. 25. Ultimate compressive strength and allowable maximum stress criteria are to be used to compare high tensile steel panels with conventional mild steel ones.

The allowable average compressive stresses are calculated with respect to the fatigue criteria (damage factor $D_f = 1$) under a longitudinal stress range $S_x$ assumed as being $\sigma^* = -\sigma^*$, using $S$-N curves which are furnished for $m = 3$ and $S_x^*/\sigma_0 = 0.5, 0.8$ and 1.2. Denoting $S_x$ as the allowable stress range of the material corresponding to a fatigue life of $10^6$ cycles, $S_x^*$ is expressed as

$$S_x^* = S_x \ln N^*/[\Gamma(1 + m)]^{1/m} = S_x \xi$$

$\xi = 10.13$ when $m = 3$

In constructing this graph, the effect of the interaction between the longitudinal and the transverse stresses is not taken into account. This effect needs to be clarified by further experimental and theoretical
studies.

It may be seen also from this figure that buckling has only a little effect on fatigue for plates with slenderness ratio up to 6 or 7.

It is to be noted that effects of initial deflection of plates and welding residual stresses in the stress range above buckling are not included in the above results. These will lead to some increase of the damage factors. These effects will be investigated and the results be reported in future publication.

7. Conclusions

In this paper, three subjects associated with buckling accepted design applied to ship structures are discussed. Namely, maximum stress, ultimate strength and fatigue under regular and random loads. A design philosophy is proposed. Design criteria, design methods, design graphs and relevant equations are presented in connection with these subjects. Numerical examples are also presented.

From these graphs and numerical results, the following conclusions may be drawn.

1. The use of high tensile steels may make it possible to reduce plate thickness substantially, satisfying the current design and safety criteria such as maximum allowable stress and ultimate strength of plates.

2. Buckling may have only negligible effects on fatigue strength of perfectly flat rectangular plates being thicker than 8 mm. Effects of initial deflection and welding residual stresses on fatigue need to be investigated to check the validity of this conclusion with actual ship plates.

In this study, the effect of the interaction of the longitudinal and the transverse stress ranges is not taken into account. This effect should be clarified by further experimental and theoretical studies, since this effect may affect the conclusion mentioned above.

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