Local Approach to Notch Depth Dependence of CTOD Results

by Claudio Ruggieri*, Fumiyoshi Minami*, Member
Masao Toyoda*, Member Yukito Hagiwara**, Member
Takehiro Inoue**

Summary

The effect of the notch depth on the critical CTOD value at cleavage fracture initiation was analysed by the local approach. Three-point bend CTOD specimens with different notch depth extracted from a high strength steel were tested at the lower shelf temperature. The critical CTOD values showed large dependence on the notch depth. Three-dimensional elastic-plastic finite element analyses were performed to investigate the crack tip stress fields for all specimen geometries. The numerical results showed that shallow and deep notch specimens exhibit very different near tip stress fields at the same CTOD level, which leads to the specimen geometry dependence of the fracture behaviour. The analysis of the notch depth dependence of CTOD results was conducted on the basis of two formulations of the local approach. In the first formulation, cleavage fracture was assumed to be controlled by the maximum principal stress. In the second formulation, the stress tensor ahead of the crack tip was fully described in terms of all principal stresses and cleavage fracture was assumed to be controlled by an equivalent stress for mode I loading. It was shown that both formulations of the local approach lead to geometry independent parameters, especially when the stress triaxiality effect is taken into account. Prediction of the notch depth dependence of CTOD results based on the local approach presented good agreement with the experimental data, particularly when stable crack growth prior to cleavage was not significant.

1. Introduction

The assessment of structural integrity against brittle fracture generally relies upon fracture toughness values measured from simple test specimens, which are assumed to be a material property independent of geometry. Assessment of cracked components under arbitrary loading, however, may not be fully resolved by this approach since notched specimens and real cracks may present very different near tip stress fields, especially in the transition region. At the lower shelf temperature, cleavage fracture initiation is a stress controlled phenomenon and the fracture stress depends largely upon geometric factors governing the crack tip conditions.

An alternative procedure for assessing the fracture mechanics behaviour of a notched body has recently been developed in terms of the local approach, where the local stress field is combined with a local criterion of crack instability. In the local approach to the cleavage fracture, the weakest link model is coupled with the non-uniform stress field ahead of the crack tip to yield a local parameter controlling crack instability, namely the Weibull stress. The probability distribution of the Weibull stress follows a Weibull distribution with two parameters which are assumed to be independent of the specimen thickness as well as of the crack shape and crack size.

In this work, the effect of notch depth on critical CTOD values obtained from three-point bend specimens with different \(a/W\) ratios was analysed by the local approach. Three-dimensional elastic-plastic finite element analyses were performed to investigate the crack tip stress field in each specimen geometry. The Weibull stress was computed for two formulations of the effective stress characterizing the stress tensor around the crack tip. In the first formulation, cleavage fracture is assumed to be controlled by a local tensile stress to the crack plane and the effective stress was taken as the maximum principal stress. In the second formulation, the stress tensor ahead of the crack tip was fully described in terms of all principal stresses and cleavage was assumed to be controlled by an equivalent stress for mode I loading. The local approach to notch depth dependence of CTOD results was shown to lead to improved correlations of the CTOD at fracture with \(a/W\) ratios.

2. Local Approach to Cleavage Fracture

Cleavage fracture of ferritic steels has long been recognized as being controlled by the unstable propagation of microcracks which are nucleated in the fracture
process zone ahead of a notch\(^3\). According to this mechanism, the scatter in fracture stress and fracture toughness values is attributed to the random nature of these microcracks. Probabilistic analyses of fracture has generally been conducted on the basis of the weakest link model\(^2\), where the critical event in the fracture process is the nucleation and unstable propagation of a critical flaw in a uniformly stressed volume of the material. This theory has been widely applied to describe the statistical behaviour of brittle materials, but it generally fails to treat stress gradients that arise from arbitrary loading.

Thus a general approach for the probabilistic analysis of fracture under complex loading conditions has been developed on the basis of the local approach\(^3\). For a notched body, the stress gradient in the vicinity of the crack tip is taken into account by dividing the volume \(V_f\) of the fracture process zone into small volume elements. If the stress controlling fracture is related to an effective stress \(\sigma_{eff}\) in each volume element\(^5\), then the failure probability \(P_\theta\) for a notched body under an arbitrary stress state \(\alpha\) is shown to be

\[
P_\theta = 1 - \exp \left[ -\frac{1}{V_0} \int_{V_f} \left( \frac{\sigma_{eff}}{\sigma_u} \right)^m \, dV_f \right] \tag{1}
\]

where \(V_0\) is a unit volume. Defining the Weibull stress \(\sigma_w\) as

\[
\sigma_w = \left[ \frac{1}{V_0} \int_{V_f} \sigma_{eff}^m \, dV_f \right]^{1/m} \tag{2}
\]

then Eq. (1) is rewritten in the form

\[
P_\theta = 1 - \exp \left( - \left( \frac{\sigma_w}{\sigma_u} \right)^m \right) \tag{3}
\]

where the parameters \(m\) and \(\sigma_u\) should be material property independent of specimen size as well as of notch depth and notch shape.

If cleavage fracture is assumed to be controlled only by the maximum principal stress \(\sigma_1\), which is taken as the effective stress \(\sigma_{eff}\), then the Weibull stress \(\sigma_w\) is evaluated by

\[
\sigma_w = \left[ \frac{1}{V_0} \int_{V_f} \sigma_1^m \, dV_f \right]^{1/m} \tag{4}
\]

Under multiaxial stress states, the effective stress \(\sigma_{eff}\) can be derived by considering a random orientation of circular planar microcracks in a small volume \(V\) subjected to all three principal stresses \((\sigma_1, \sigma_2, \sigma_3)\), as presented in Fig. 1. If the incidence of microcracks in the volume \(V\) follows a Poisson process\(^7\) and fracture occurs when the microcrack size \(a\) reaches a critical value \(a_0\) in a uniformly stressed volume element \(\delta V\), then the failure probability \(\delta P\) is given by

\[
\delta P = \delta V \int_{a_0}^\infty g(a) \, da \tag{5}
\]

where \(g(a)\) is the density of microcracks in volume \(V_0\).

Following the extreme value distribution of the largest flaw in the volume element and assuming that the microcrack density distribution behaves like a Cauchy-type function, the integral over the crack size \(a\) can be readily evaluated\(^9\). Taking into account the position of the microcrack in the volume \(V\) referred to the coor-

3. Notch Depth Dependence of CTOD Results

3.1 Testing Procedures

Fracture toughness data were obtained from three-point bend CTOD specimens with different notch depth extracted from a 15 mm thick, high strength steel\(^10\). Table 1 shows the chemical composition and the mechanical properties of the steel used. Experiments were conducted at \(-120^\circ C\) on different \(a/W\) ratios, which were designed to be \(a/W = 0.5\), \(a/W = 0.3\) and \(a/W = 0.1\). Fig. 2 illustrates the specimen geometry used in the experiments. All specimens are 30 mm wide and
Local Approach to Notch Depth Dependence of CTOD Results

Table 1 Chemical composition and mechanical properties at room temperature of the steel used.

<table>
<thead>
<tr>
<th>Chemical Composition (mass %)</th>
<th>Mechanical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Si</td>
</tr>
<tr>
<td>0.10</td>
<td>0.27</td>
</tr>
</tbody>
</table>

\(\sigma_y\): 0.2 \% proof stress
\(\sigma_T\): Tensile strength
\(\epsilon_r\): Uniform elongation
YR: Yield to tensile ratio

Fig. 2 Three-point bend CTOD specimens with different notch depth.

Fig. 3 Plastic rotational factor \(r_p\) for different \(a/W\) ratios obtained by finite element analysis.

Fig. 4 presents the effect of the \(a/W\) ratio on the critical CTOD value at cleavage fracture initiation. In this work, \(\delta_c\) indicates cleavage fracture with stable crack growth less or equal to 0.1 mm, and \(\delta_d\) indicates cleavage fracture preceded by stable crack growth exceeding 0.1 mm. All specimens with \(a/W=0.1\) developed significant crack extension prior to cleavage fracture and provided much larger values of the critical CTOD. Specimens with \(a/W=0.5\) exhibited completely brittle behaviour with negligible crack extension.
Fig. 4 Effect of $a/W$ ratio on the critical CTOD value at cleavage fracture initiation.

Fig. 5 Weibull distribution of critical CTOD values at cleavage fracture initiation.

Table 2 Weibull parameters for critical CTOD distributions obtained by CTOD specimens with different $a/W$ ratios.

<table>
<thead>
<tr>
<th>Weibull Parameter</th>
<th>$a/W = 0.5$</th>
<th>$a/W = 0.3$</th>
<th>$a/W = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape ($m_s$)</td>
<td>2.75</td>
<td>1.70</td>
<td>2.10</td>
</tr>
<tr>
<td>Scale ($b_s$), mm</td>
<td>0.08</td>
<td>0.18</td>
<td>1.14</td>
</tr>
</tbody>
</table>

$F(\delta_k) = 1 - \exp \left[ - \left( \frac{\delta_k}{b_s} \right)^{m_s} \right]$, $\delta_k = \delta_e, \delta_u$

Fig. 5 shows the cumulative distribution of the experimental data on a Weibull plot and table 2 gives the Weibull parameters estimated by the maximum likelihood method\(^1\). The Weibull shape parameter $m_s$ for each $a/W$ ratio is similar to the theoretical value of 2 for cleavage fracture\(^1\). However, the scale parameter $b_s$ strongly depends on the $a/W$ ratio, which leads to large differences in the failure probability values at the same CTOD level, especially for specimens with $a/W = 0.1$.

4. Finite Element Analysis

Three-dimensional finite element analyses of the CTOD specimens were performed by a general purpose nonlinear code\(^1\) using an elastic-plastic material model with isotropic hardening and von Mises yield criterion. The material model is based on a piecewise linear approximation of the true stress-true strain curve presented in Fig. 6. Because of symmetry, only one quarter of the specimen was modelled by employing 8-node isoparametric elements with 8 Gaussian points. The finite element model for the specimens with $a/W = 0.5$ is presented in Fig. 7. Very similar mesh densities are used for the specimens with $a/W = 0.3$ and $a/W = 0.1$. The core of the crack tip region is common for all three meshes. The minimum size of the crack tip element is
Local Approach to Notch Depth Dependence of CTOD Results

The crack tip stress fields were investigated to correlate the fracture behaviour of CTOD specimens with different $a/W$ ratios. Fig. 8 provides the distribution of the nondimensional normal stresses ($\sigma_{xx}/\sigma_Y$, $\sigma_{yy}/\sigma_Y$, $\sigma_{zz}/\sigma_Y$) ahead of the crack tip at the CTOD level $\delta=0.3$ mm. Fig. 9 presents the nondimensional normal stress to the crack plane ($\sigma_{xx}/\sigma_Y$) along the thickness direction, also at $\delta=0.3$ mm. Fig. 10 gives the distribution of stress triaxiality ahead of the crack tip and in the center of the specimen. In this investigation, the stress triaxiality was defined as the ratio of the maximum principal stress $\sigma_i$ to the von Mises equivalent stress $\sigma_e$.

The analytical results demonstrated that shallow and deep notch specimens have quite different near tip stress fields at the same CTOD level. Furthermore, the stresses are significantly above the uniaxial yield stress, which indicates a high degree of stress triaxiality near the crack tip, especially in the center of the specimen. The increase in critical CTOD values observed in the experiments can then be accounted for in terms of relief of the stress triaxiality ahead of the crack tip. When the triaxial stress state is relaxed, the near tip stress intensity is reduced and a critical fracture stress can be reached only at a greater deformation level. Thus the use of the critical CTOD value as a fracture toughness parameter may no longer be valid and the analysis of the fracture behaviour requires a procedure based on the local approach.

Fig. 8 Distribution of crack tip normal stresses in the center of the specimen.

Fig. 9 Distribution of normal stress to the crack plane along the thickness direction.

Fig. 10 Distribution of crack tip stress triaxiality in the center of the specimen.
5. Application of the Local Approach to CTOD Results

The experimental results were analysed by the local approach to take into account the geometry dependence of the critical CTOD values and differences in the near tip stress fields for the CTOD specimens. The Weibull stress $\sigma_w$ and the Weibull parameters $m$ and $\sigma_u$ were calculated according to an iterative procedure5), where $\sigma_w$ is evaluated from the stress distribution within the yielded region ahead of the crack tip and $m$ and $\sigma_u$ are estimated on the basis of $\sigma_w$-values obtained from the CTOD at cleavage fracture. Both formulations of the Weibull stress, given by Eqs. (4) and (6), were employed. In the triaxial stress state, $\sigma_w$ was calculated through a Gaussian quadrature\(^\text{15}\).

Fig. 11 gives the cumulative distribution of the Weibull stress for each $a/W$ ratio and both formulations of $\sigma_w$. Table 3 compares the Weibull parameters $m$ and $\sigma_u$ estimated by the maximum likelihood method as well as the 90% confidence interval for each parameter\(^\text{12}\). All specimen geometries show very similar Weibull shape ($m$) and scale ($\sigma_u$) parameters, irrespective of the $a/W$ ratio. The results are in good agreement with the local approach theory and the parameters obtained can be regarded as a material property independent of the notch depth $a$. Furthermore, both formulations of $\sigma_w$ provided almost the same Weibull parameters, which indicates that the assumption of cleavage fracture being controlled by a local tensile stress still holds for the triaxial stress state in the specimens.

As an application of the local approach, the notch depth dependence of critical CTOD values was predicted for the shallow notch specimens. Fig. 12 gives the predicted scatter bands for CTOD specimens with $a/W=0.1$ and $a/W=0.3$ obtained from the correlation with the Weibull stress distribution for the specimens with $a/W=0.5$. The Weibull parameters were evaluated from the $\sigma_w$-formulation for the triaxial stress state. The prediction of the notch depth dependence for specimens with $a/W=0.3$ agrees well with experimental data. In case of the specimens with $a/W=0.1$, the prediction of notch depth dependence resulted in a conservative prob-

![Graph](image)

(a) Maximum principal stress formulation.

![Graph](image)

(b) Triaxial stress state formulation.

Fig. 11 Cumulative distribution of the Weibull stress for CTOD specimens with different $a/W$ ratios.

<table>
<thead>
<tr>
<th>Weibull Parameter</th>
<th>Maximum Principal Stress</th>
<th>Triaxial Stress State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a/W=0.5$</td>
<td>$a/W=0.3$</td>
</tr>
<tr>
<td><strong>Shape ($m$)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(11.4,28.5)</td>
<td>20.7</td>
<td>22.7</td>
</tr>
<tr>
<td>(12.0,31.5)</td>
<td>(11.4,28.5)</td>
<td>(12.0,31.5)</td>
</tr>
<tr>
<td><strong>Scale ($\sigma_u$, MPa)</strong></td>
<td>2260</td>
<td>2314</td>
</tr>
<tr>
<td>(2183,2326)</td>
<td>(2238,2380)</td>
<td>(2368,2526)</td>
</tr>
</tbody>
</table>

$$F(\sigma_w) = 1 - \exp \left[ - \left( \frac{\sigma_w}{\sigma_u} \right)^m \right]$$
ability distribution, especially for larger critical CTOD values. This behaviour can be mostly attributed to the occurrence of considerable crack growth before the onset of cleavage fracture, which may not necessarily be described by a local stress-based criterion.

6. Concluding Remarks

The present work focused on the fracture mechanics assessment of three-point bend CTOD specimens with different notch depth based on the local approach. Test of the specimens was conducted at the lower shelf temperature and the experimental results exhibited large dependence of the critical CTOD at cleavage initiation on the notch depth. A three-dimensional elastic-plastic finite element analysis presented large differences in the near tip stress fields for shallow and deep notch specimens at the same CTOD level, which showed that the use of the critical CTOD value as a fracture toughness parameter may no longer be valid.

The local approach to the cleavage fracture behaviour of the CTOD specimens was based on two different formulations of the Weibull stress $\sigma_w$. It was shown that both formulations provided geometry independent parameters, especially when the triaxiality effect was taken into account. Prediction of the notch depth dependence of CTOD results based on the local approach presented good agreement with experimental data, particularly when stable crack growth prior to cleavage was not significant.

References