Numerical Study of the Flow and Thrust Produced by a Pitching 2D Hydrofoil

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Summary

Unsteady flow field around a rigid 2D NACA 0012 foil performing pitching oscillations is studied numerically, changing the reduced frequency and pivot point location. Special attention is payed to the ability of thrust production.

Viscous flow simulation is based on solving the full 2D laminar incompressible Navier–Stokes equations in vorticity-stream function formulation, defined in moving noninertial frame of reference. An implicit factored finite-difference numerical algorithm is used. Performed numerical tests for simulation indicate that, the simulated flow is of sufficient accuracy to merit a physical analysis. The computed flow pattern is compared with the flow visualized experimentally and both results seem to agree satisfactorily.

It was proved that, for large products of reduced frequency and distance to the pivot point location, the pitching hydrofoil is able to produce thrust but of low efficiency. However, the comparison with available results based on nonviscous flow investigations shows that the thrust and efficiency in these cases is highly overestimated. It is connected with the strong vortical structures near the leading edge. The same combination of parameters but in the case of pivot point located in front of the foil, is found more efficient. The flow pattern analysis shows that in this case the dominant leading edge vortex is weaker and it is washed downstream easier.

1. Introduction

During the past several decades, the study of the unsteady viscous flow phenomena around an oscillating airfoil and its dynamic performance has been one of the main topics in the theoretical, computational and experimental unsteady fluid dynamics. This remarkable interest has been inspired not only by attempts to prevent or utilize the problems of dynamic stall and flutter phenomena, but to understand and simulate high efficient flying and swimming propulsion.

Till now, starting with the pioneering works of Lighthill (1960) and Wu (1961) (further developed in and ), unsteady hydrodynamics of swimming have been studied theoretically and numerically mainly by means of ideal fluid model. The hydrodynamics of the old fashion "Ro" and "Scull" boats was investigated by Azuma et al. Numerical models were limited to unsteady lifting line and lifting surface models incorporating distributed singularities, vortex lattice or panel approaches. An interesting full-scale experimental study on "dolfin-style fin ship" is presented in. Accomplished numerical and experimental investigations of ship propulsion based on a rigid and flexible foil oscillations are published in. However, their method was also discrete vortex method. During the oscillating motion, the resultant angle of attack to the hydrofoil reaches almost 90 [deg] depending on the condition. The flow separation must occur, which can not be simulated well by such inviscid flow model.

The present paper is treating the unsteady viscous flow and force produced by a rigid, pitching, 2D hydrofoil. In this study, the position of the axis of "pitching" is extended out of the foil domain. The main objectives could be summarized as follows: (1) Investigation of the unsteady viscous flow phenomena around a pitching hydrofoil; (2) Parametric investigations of the effects of reduced frequency of pitching and pivot point location on the flow pattern and unsteady hydrodynamic loads.

The numerical algorithm proposed by Mehta (1977) for dynamic stall study of a pitching 2D airfoil is adopted and extended for the objectives of the present study.

2. Basic Equations and Numerical Method

This paper treats a hydrofoil pitching around a pivot point located in a uniform flow. Such formulation requires the introduction of a noninertial rotating coordinate system. The Navier–Stokes equations for an
incompressible fluid in an inertial or fixed coordinate system are:

\[ \frac{Dq}{Dt} = -\vec{v} \cdot \vec{q} + \frac{1}{Re} \vec{P} \cdot \vec{q}, \]  

(1)

where subscript \( I \) refers to inertial frame of reference.

Reynolds number is \( Re = U \cdot c / \nu \)

c - chord length (chosen as characteristic length \( L \))

\( \vec{u} \) - free stream velocity

\( \vec{P} \) - pressure

The velocity \( \vec{q} \) relative to the rotating coordinate system of a point located by the vector \( \vec{r} \) is given by \( \vec{q} = \vec{u} + \Omega \times \vec{r} \) where \( \Omega \) is the angular velocity of the pitching motion.

In the noninertial rotating coordinate system momentum equation (1) is

\[ \frac{\partial \vec{q}}{\partial t} + \frac{1}{2} \vec{v} \times (\vec{q} \times (\vec{v} \times \vec{q})) + \frac{\partial \vec{q}}{\partial t} \times \vec{r} \]

\[ = -\vec{P} + \frac{1}{Re} \vec{P} \cdot \vec{q} \]  

(2)

This equation in the computational domain \((r, \theta)\) in terms of the vorticity \( \omega \) is

\[ H^2 \omega = \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \omega \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( r \frac{\partial}{\partial \theta} \omega \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \omega \right) \]

(3)

with the stream function \( \Psi \) defined by

\[ r^2 \frac{\partial \Psi}{\partial r} + r \frac{\partial}{\partial \theta} \left( r \frac{\partial \Psi}{\partial \theta} \right) = -H^2 r^2 \omega \]

(4)

where

\[ H^2 = \left( \frac{\partial x}{\partial r} \frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta} \frac{\partial y}{\partial r} \right) / r \]

(5)

and \( J(\omega, \Psi, r, \theta) \) is the Jacobian. The coordinates \( x, y \) are connected with the moving frame of reference fixed with the foil, with origin at the leading edge (Fig. 1).

In order to generate the computational domain, the classical Joukowski transformation is used. On the surface, the constraint of no slip is applied to define the surface vorticity. The flow at the far downstream boundary is governed with first order differential relations obtained from the tangential Navier-Stokes equation by dropping the viscous term. Such condition is more accurate than zero-extrapolation type boundary conditions. For the upstream condition, free stream velocity is assumed. Both vorticity and stream function eqs. are subjected to periodicity in \( \theta \)-direction.

The surface pressure distribution is obtained by integrating the tangential component of Navier-Stokes eq. More detailed description could be found in [14]. To obtain pressure distribution in the whole computational domain, the corresponding Poisson equation have to be solved.

Vorticity equations is solved by an implicit factored method. The truncation error is \( O((\Delta r)^2 + (\Delta \theta)^4 + (\Delta t)^2) \), where \( \Delta r, \Delta \theta \) and \( \Delta t \) are space and time increments respectively. The Fourier transform method is used to solve the stream function equation by direct approach. The vorticity and stream function equations are solved sequentially. The step is repeated until the convergence of surface vorticity is reached.

3. Motion of the Hydrofoil and Computational Parameters

The motion of "pitching" is schematically presented in Fig. 1. All of the computed cases being presented here concern symmetric oscillations around zero mean incidence, except some validatory computations.

The motion of the hydrofoil is as follows:

1. Impulsive start at zero incidence till steady-state solution is obtained (nondimensional time \( t^* = tU \cdot c / \nu \) \( \geq 10 \));

2. Upstroke pitching till maximum incidence angle is reached (defined here as half cycle, \( t^* < t^* \leq t^* \));

3. Pitching oscillations at zero main incidence and amplitude \( \alpha_a \) (one or more cycles, \( t^* > t^* \)).

The used notations are: \( r_p \) - the distance from the leading edge to the pivot point. Negative value of \( r_p \) means that the pivot point is located ahead of the leading edge; \( a(t) \) - instantaneous angle of attack; \( \alpha_a \) denotes the maximum amplitude and is fixed to the 15 [deg]. The described pitching motion is illustrated in Fig.1.

A summary of all computed cases according to the different values of normalized pivot axis location \( r_p = r_p / c \) and reduced frequency \( k = fc/2U_w \) is provided in Table 1.

The main computational and physical parameters during the present numerical study are as follows:

- Hydrofoil section - NACA 0012;
- Chord Reynolds number \( Re_c = U \cdot c / \nu = 5.0 \times 10^5 \);
- Grid Resolution:
  - number of points in \( \theta \)-direction - 130 (132 with overlapping stations)
  - number of points in \( r \)-direction - 84;

Fig. 1 Pitcing motion of a hydrofoil.
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Table 1 Completeness of the study according to the parametric variations.

<table>
<thead>
<tr>
<th>Reduced Freq. k</th>
<th>Normalised Pivot Point Location r_p*</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>[0.04421, 0.0001382]</td>
</tr>
<tr>
<td>0.1</td>
<td>[0.0, 0]</td>
</tr>
<tr>
<td>0.5</td>
<td>[0, 0]</td>
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<tr>
<td>1.0</td>
<td>[0, 0]</td>
</tr>
<tr>
<td>1.5</td>
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<tr>
<td>2.5</td>
<td>[0, 0]</td>
</tr>
<tr>
<td>3.0</td>
<td>[0, 0]</td>
</tr>
</tbody>
</table>

- Time Increments $\Delta t* \in [0.04421, 0.0001382]$
- Computational Domain with diameter 15 chord lengths $c$
- Average Number of Iteration $N_{iter} = 50$
- Reduced frequency $k = \delta c/2U \in [0.1, 3.0]$
- Normalized pivot point location $r_p^* = r_p/c \in [-10.0, 10.0]$
- Amplitude of pitching $\alpha_a = 15.0 [\text{deg}]$ (fixed).

4. Numerical Test and Comparison with Experimental Flow Visualization

The effect of the time increment on the flow pattern and hydrodynamic loads was studied for several cases. Fig. 2 a) and b) show the history of drag coefficient $C_d(t)$, coefficient $C_a(t) = C_d/\sqrt{C_l^2 + C_d^2}$ where $C_l$ denotes the lift coefficient, instantaneous angle of attack $\alpha(t)$ and effective incidence $\alpha_{eff}$ which is defined in section 5. Both results, which are calculated by the same grid resolution (66 x 42) with different time increment, are qualitatively similar and the mean value of the plotted quantities is almost the same. The oscillations of the second case, where the time increment is five times smaller, are with lower magnitude. The limit of the time increment depends on the reduced frequency $k$ and pivot point location $r_p$ or on the product $p^* = 2kr_p^*$, where $r_p^*$ represents the normalized pivot point location. During the present computation the time increment was carefully chosen.

A comparative calculations for two grid systems were performed (66 x 42 and 130 x 84). Comparing drag coefficients for these cases—Fig. 2a) and c)—although the time evolution is similar, the quantitative discrepancies between mean values are about 10%. The truncation error of present method is $O(\Delta r)^2 + (\Delta \theta)^2 + (\Delta r)^2$, so that the maximum solution error of the second grid (130 x 84) could be estimated. A comparison of computed streamlines in a moving frame of reference for both cases is shown in Fig. 3a) and b) respectively. Here and further, the direction of foil motion is indicated by the signs $\circlearrowleft$, $\circlearrowright$, $\circlearrowleft$ and $\circlearrowright$ denoting downstroke, upstroke, the change from downstroke to the upstroke motion or in opposite respectively. We can conclude that the flow simulation in both computational cases is phenomenologically identical except some local differences near the leading and trailing edges of the foil. Although, further grid refinement is desirable, the second grid system (130 x 84) seems to be a reasonable compromise of computational time and accuracy.

In order to compare the computational results with the real flow phenomena, an experimental flow visualization of the pitching motion of a hydrofoil was performed. Pitching oscillation was performed by an intelligent actuator. Experiment was conducted at the towing tank basin of Hiroshima University. For visualization aluminum powder was scattered on the free surface of the water. The aspect ratio of the foil is 2 (chord length of 11 cm and span of 22 cm) was employed. The foil is not exactly NACA 0012, but has similar section. The Reynolds number during the experimental runs was 5000 as in the case of computational flow simulation.

The data were taken after the foil had been perfor-
med several cycles of oscillations. The comparison is made between the particle pass (for the time of exposition) and the contour lines of the stream function in the inertial frame of reference. Since the flow is unsteady, however, these images are almost the same if there is no rapid flow changes.

In Fig. 4, the experimental result a) is compared with the result of numerical flow simulation b) for the case of reduced frequency $k=2.0$ and pivot point $r_p=5.0$ chord lengths. Additionally, the streamlines in noninertial frame c) are presented. Experimental flow visualization and computations have been performed for a fixed Reynolds number of $5 \times 10^3$ and amplitude of pitching $\alpha_0=15.0$ [deg]. It is well seen that qualitatively the both flows are almost identical. All of the main flow structures predicted numerically are documented by the experiment. The only difference seems to be the intensity of the leading edge vortex which is a little bit larger in case of experimental data.

The case of twice higher reduced frequency ($k=2.0$, pivot point $r_p=5.0$ chord lengths) is shown in Fig. 5. The white areas in the numerical results are the regions with higher velocity than the maximum and minimum contour values. They are representing the strong vorticity structures well observed in the experiment. The numerical simulation for this case also coincides quantitatively with the experimental results.

5. Results and Discussions

5.1 Simplified Analysis of Thrust Produced by a Pitching Hydrofoil

The mechanism of thrust production by means of a pitching hydrofoil is simply analyzed in this section. Schematically it is explained in Fig. 6. The total force $R$ is a vector sum of a lift force $L$ and a drag force $D$. The direction of the lift force is always perpendicular to the oncoming flow vector, when the direction of darg force coincides with it. If we decompose the total force to the vector with direction equivalent to the direction of the free stream and to the lateral force, the sign of the first component will be important for the drag or thrust production. In Fig. 6 are shown the free stream velocity vectors $U_w$. The foil is performing pitching oscillations with pivot point located after the trailing edge. According to the foil motion, the effective oncoming flow which is defined here as the relative motion of the fluid with respect of the foil will alter sinusoidally. For the fixed moment which is considered, the foil perform upstroke pitching motion with pivot point $r_p$ and frequency $f$. The effective oncoming flow at this moment is presented by the vectors $U_x$ and $U_y$. The $x$-component is a sum of the free stream velocity and $x$-component of the velocity according to the foil motion with opposite sign. The $y$-component depends on the foil motion only. The sum of these vectors define the vector of effective oncoming flow $U$. Obviously, the thrust production will depends on the foil inclination and effective angle of attack which is defined as the angle between the velocity vector of the effective flow with respect to the chord inclination-instantaneous angle of attack $\alpha$. If this angle has the direction showed in the figure, the lift force will have direction (as is shown) opposite to the foil motion (negative) and its component in $x$-direction will produce thrust. If the angle is less than $a$ the lift force will coincide with the direction of rotation and its $x$-component will resist foil motion (drag). The conclusion is that this alternative depends on foil motion only. If the foil pitching is slow, thrust cannot be obtained and in opposite, when rotational motion is fast enough, the thrust is available. Such very simple analysis is valuable because we can estimate the zones of thrust and drag production.

5.2 Flow Simulation

Fig. 7 shows the time history of drag ($C_D$), lift ($C_L$) and moment ($C_M$) coefficients including time variations of instantaneous angle of attack $\alpha$, for the case of reduced frequency of 0.5 and pivot point location at 0.1 (close to the leading edge of the foil). Performed four cycles of oscillation show that, no thrust is produced. The forces are periodic except the first cycle, where the influence of initial upstroke motion is expected. The periodic forces have same maximum values at same position and almost perfectly the same behavior. It means that the flow in this case is fully developed.
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Fig. 4 Flow visualization for a pitching hydrofoil with amplitude $\alpha_s=15.0$ [deg], $Re=5\times10^3$, reduced frequency $k=1.0$, pivot point $r_p=5.0c$. For computations $\alpha=7.39$ [deg], $t^*=0.528$.

Fig. 5 Flow visualization for a pitching hydrofoil with amplitude $\alpha_s=15.0$ [deg], $Re=5\times10^3$, reduced frequency $k=2.0$, pivot point $r_p=5.0c$. For computations $\alpha=4.82$ [deg], $t^*=0.311$. 
The simplified steady state considerations, presented in the previous sections, show that we can expect a large propulsive force when $p^* = k\alpha_p$ is large. Figs. 8 and 9 show the time history of $CD$ in case of $\alpha_p = -0.5$ and $0.5$ with different reduced frequency $k$. The coefficient $C_{\alpha}$ is defined as the ratio $CD/\sqrt{C_L^2 + C_D^2}$, and $\alpha_{eff}$ is defined as effective incidence angle. $CT_{max}$ is defined equal to the maximum value of $CD$. In this case, during the foil pitching, a negative drag (thrust) is obtained. In general and qualitatively, the thrust and the zones of thrust production are depending on the foil inclination and realized effective incidence angle (i.e., agree with the simple steady state predictions in the previous section). In case of $r^*_p = -5.0$, the negative drag (thrust) region is larger than the positive drag region. This means that the mean value per a cycle will be propulsive. Whereas in the case $r^*_p = -5.0$, the negative drag region is smaller than the positive drag region. The same tendency is shown in case of $k = 1.0$. The efficiency $\eta$ of $r^*_p = -5.0$ is higher than that of $r^*_p = 5.0$. This means the case of pitching axis located ahead of the hydrofoil is more effective than the case of the axis located aft of the hydrofoil. Referring the results in the case Fig. 9 (where the efficiency is lower than the previous one) we can assume that it has a maximum value between $k = 1.0$ and $k = 2.0$.

Figs. 10 and 11 show the streamlines around the hydrofoil in a moving frame. The main flow pattern is summarized as follows.

As the pitching motion begins, the formation of the leading edge bubble starts on the lower surface of the hydrofoil (Fig. 10b, 11b). Further in time, as the separation at the leading edge develops, the leading edge vortex grows in size and as a whole structure is convected along the lower surface (Fig. 10d, 11d). The primary leading edge vortex is slowly propagated downstream. This generalized picture is more or less valid for a wide range of reduced frequencies and pivot point locations.

The main differences of the flow between $r^*_p = -5.0$ and 5.0 are the wake of the hydrofoil, the secondary leading edge vortex and the strength of the primary leading edge vortex. The secondary leading edge vortex is shown in case of $r^*_p = 5.0$ (Fig. 11f, g). This secondary vortex will reduce the thrust producing force. As the distances from the pivot point to the leading and trailing edge differ by one chord length, but they have the same angular velocity, the effective oncoming flow will differ with the product of the angular velocity and chord length. So that in case of $r^*_p = -5.0$, the angular velocity of the trailing edge due to the pitching motion is larger than that of $r^*_p = 5.0$. In addition, the trailing edge vortex in the case of $r^*_p = -5.0$ becomes stronger compared with that of $r^*_p = 5.0$.

In case $r^*_p = -5.0$, the strong trailing edge vortex deforms the wake flow. The wake deformation produces a “stagnant region” near the trailing edge (Fig. 10e).

Figs. 12 and 13 show the pressure distribution. The secondary vortex, which is observed in the case $r^*_p = 5.0$, $a = 8.37 \, [\text{deg}]$ (Fig. 11b) affects the pressure on the hydrofoil. The pressure on the lower side at $a = 8.37 \, [\text{deg}]$ is almost positive due to the stagnant flow of the secondary vortex and this positive pressure results in drag force. Concerning the stagnant region due to the wake deformation (Fig. 10d, e), the pressure at the corresponding $a = -9, -14 \, [\text{deg}]$ becomes more moderate.

6. Conclusions

The unsteady flow around a pitching rigid 2D NACA 0012 airfoil has been studied numerically with parametric conditions. The following conclusions could be made.

- The performed validatory computations indicate
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that the present numerical study is of sufficient accuracy to merit a physical analysis.

- In case of large products of reduced frequency and pivot point location, the strong and stable vorticity structures formed as result of the flow separation near the leading edge of the foil, could dominate the flow features and further flow evolution.

- The pressure distribution depends on the surface vorticity and is strongly affected by the dominant vortical structures and their convection. This pre-
determines the consequent changes in the induced loads.

- The case of pitching axis located ahead of the hydrofoil leading edge is more effective from the propulsive point of view.

7. Acknowledgments

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References


5) Wu, T. Y., “Hydromechanics of swimming pro-
Fig. 10 Streamlines in a moving frame around a NACA 0012 during its pitching oscillations in the range $\alpha = [-15, 15]$ [deg], $Re = 5 \times 10^3$, reduced frequency $k = 1.0$, pivot point $r_p = -5.0c$. 

a) $\alpha = 12.8[deg] \psi, t^* = 0.276$

b) $\alpha = 6.7[deg] \psi, t^* = 0.553$

c) $\alpha = -1.3[deg] \psi, t^* = 0.829$

d) $\alpha = -9.0[deg] \psi, t^* = 1.105$

e) $\alpha = -14.0[deg] \psi, t^* = 1.382$

f) $\alpha = -15.0[deg] \psi, t^* = 1.569$

g) $\alpha = -12.8[deg] \psi, t^* = 1.846$

h) $\alpha = -6.8[deg] \psi, t^* = 2.122$
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Fig. 11  Streamlines in a moving frame around a NACA 0012 during its pitching oscillations in the range $\alpha = [-15, 15] \deg$, $Re=5 \times 10^3$, reduced frequency $k=1.0$, pivot point $r_p=5.0c$. 

- a) $\alpha = -12.80 \deg$, $t^* = 1.84$
- b) $\alpha = -6.7 \deg$, $t^* = 2.11$
- c) $\alpha = 1.3 \deg$, $t^* = 2.37$
- d) $\alpha = 9.0 \deg$, $t^* = 2.76$
- e) $\alpha = 14.0 \deg$, $t^* = 3.03$
- f) $\alpha = 15.0 \deg$, $t^* = 3.22$
- g) $\alpha = 13.6 \deg$, $t^* = 3.43$
- h) $\alpha = 8.37 \deg$, $t^* = 3.71$


