A New Plate Buckling Design Formula (2nd Report)  
—On the Plasticity Correction—

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Summary

In the previous paper, a new buckling design formula for simply supported plate panels subjected to combined in-plane and lateral loads was suggested. The effect of welding residual stress was also included. For the plasticity correction, the well-known Johnson-Ostenfeld formula was used.

In the present study, more advanced formula of the plasticity correction is proposed taking account of edge condition effects. The applicability of the proposed formula is demonstrated by comparing with the present and the conventional result.

1. Introduction

Rectangular plate elements are the primary member in ship structures. The strength of the whole structure is very much dependent on that of the individual plate elements. For the preliminary design of ship structures, the plate buckling strength is then one of the most important criteria.

For thin plates, the buckling usually occurs in the elastic range. Theoretical and accurate design formulas of simply supported plate panels subjected to combined loads are available in this case. For thick and/or clamped plates, however, buckling may take place in the elasto-plastic range. It is very difficult to estimate the elasto-plastic buckling strength of plate panels using the analytical approach, and in usual the numerical methods such as the finite element method is practical. For this problem, one of the most useful methods is to correct the elastic design formula using the plasticity correction factor with the assumption that the buckling interaction relationship in the elastic range is available even in the elasto-plastic range. The corrected formula is then employed in the elasto-plastic as well as the elastic range.

Plate buckling design codes of classification society have employed the well-known Johnson-Ostenfeld parabola to make a plasticity correction. Using this formula, however, too conservative design results are occasionally provided. Also the edges of actual plate panels may have some restraints with regard to the rotational movement and are different from simply supported boundary condition. Under the assumption of simply supported edge condition, too conservative design results are also obtained when the strong stiffening members which restrict the rotational movement of plates are attached. In order to produce more rational design results, therefore, the effect of more accurate plasticity correction and real edge condition should be accounted for.

In the previous paper, the authors suggested a new buckling design formula of plate panels subjected to combined in-plane and lateral loads, in which the plate edges are assumed to be simply supported and the Johnson-Ostenfeld formula is employed for the plasticity correction.

In the present study, more advanced formula to make a plasticity correction is suggested taking account of edge condition effects. The applicability of the proposed formula is demonstrated by comparing with the present and the conventional result.

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2. The Elasto-Plastic Large Deflection FE Analysis

2.1 Method
To estimate the elasto-plastic buckling strength of plate panels, the elasto-plastic large deflection finite element method is employed. As shown in Fig. 1, four-noded rectangular plate element is used.

The characteristics of the finite element used here is as follows:

1) It is a rectangular, plane-shell element which has only four corner nodal points having five degrees of freedom at each nodal point.

2) It includes the geometric nonlinear effect due to both in-plane and out-of-plane large deformation of the element.

3) The expanded yielding zone through the plate thickness is condensed into plastic nodes inserted in the nodal point based on the concept of the plastic node method. As a result, the elasto-plastic stiffness equation of the element can be obtained by the simple matrix calculation without performing complicated numerical integration if once the elastic stiffness matrix is given.

2.2 Estimation/Definition of the Critical Buckling Strength
In the present analysis, a small initial deflection ($W_0 = 0.001t, t$: plate thickness) is always assumed in the plate panel. In this case, the lateral deflection of the plate gradually increases with increase in the applied load and occasionally the clear buckling point is not shown. Therefore, the so-called "P-W2 method" ($P$: load, $W$: deflection) shown in Fig. 2 is used to estimate or define the critical buckling point. As indicated later, the accuracy of this method is checked for the case of thin plates which buckle in the elastic range and have theoretical value of the critical buckling strength.

2.3 Parametric Study
To derive a new plasticity correction formula, a parametric study for square plates under each load component of axial compression or edge shear is performed varying the edge condition and the plate thickness. It is assumed that the effect of different aspect ratios is similar as that of square plates. Table 1 and 2 represent the summary of the parametric study, for uniaxial compression and edge shear, respectively.

For all cases, the following properties are used:

- plate length, $a = 500$ mm
- plate breadth, $b = 500$ mm
- Young's modulus, $E = 21,000$ kg/mm²
- Poisson's ratio, $\nu = 0.3$
- yield stress, $\sigma_0 = 24.0$ kg/mm²

Three kinds of the plate edge condition shown in Fig. 3 are considered:

- all edges simply supported
- two edges simply supported and two edges clamped
- all edges clamped

Also five or six kinds of the slenderness ratio ($\lambda = b/t\sqrt{E/\sigma_0}$) are considered for each kind of the plate edge condition as indicated in Table 1 and 2. Fig. 4 shows modelling and mesh size of the finite elements used in the present analysis.

3. A New Formula for the Plasticity Correction
Based on the numerical results obtained through parametric study, relationship between the critical buckling stress and the Euler's elastic buckling stress for uniaxial compression and edge shear is shown in Fig. 5.a and 5.b, respectively. First of all, it is found that the estimated critical buckling stress in the elastic range is in agreement with the Euler's buckling one. This means that the definition accuracy of the critical buckling strength described in section 2.2 is reliable.

Now, we derive a new plasticity correction formula. From Fig. 5, it is observed that the Johnson-Ostenfeld parabola is estimating the critical buckling stress at too...
### Table 1: Summary of the critical buckling stress analysis for square plates in axial compression

<table>
<thead>
<tr>
<th>Edge Cond.</th>
<th>$\lambda$</th>
<th>$K_E$</th>
<th>$\sigma_E$</th>
<th>$\sigma_{cr}$</th>
<th>$\sigma_{cr}/\sigma_o$</th>
<th>$\sigma_{cr}/\sigma_o$</th>
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**Note:** $1) \lambda = \frac{b}{\sqrt{E\omega}}$

### Table 2: Summary of the critical buckling stress analysis for square plates in edge shear

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<th>Edge Cond.</th>
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<th>$\tau_{cr}$</th>
<th>$\tau_{cr}/\tau_o$</th>
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**Note:** $1) \lambda = \frac{b}{\sqrt{E\omega}}$

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Fig. 3  Ideal plate edge conditions considered in the present study

Fig. 4  Modelling and mesh size of the finite elements according to edge conditions
low level. As the new formula covering all kinds of the plate edge condition, therefore, the following equation which has the same expression for both axial compression and edge shear is derived.

\[ \sigma_{cr} = \sigma_E \quad \text{for } 0.5 \leq \sigma_E/\sigma_0 < 1.9 \]
\[ \tau_{cr} = \tau_E \quad \text{for } 0.5 \leq \tau_E/\tau_0 < 1.9 \]

(1.a)

(1.b)

where

- \( \sigma_{cr} \) = the critical buckling stress in uniaxial compression = \( \sigma_x \) or \( \sigma_y \) according to \( x \) or \( y \) direction, respectively
- \( \sigma_E \) = the Euler's elastic buckling stress in uniaxial compression = \( \sigma_{Ex} \) or \( \sigma_{Ey} \) according to \( x \) or \( y \) direction, respectively
- \( \tau_{cr} \) = the critical buckling stress in edge shear = \( \tau_x \) or \( \tau_y \) according to \( x \) or \( y \) direction, respectively
- \( \tau_E \) = the Euler's elastic buckling stress
- \( \sigma_0 \) = the yield stress
- \( \tau_0 = \sigma_0/\sqrt{3} \)

It is clear from Fig.5 that the proposed formula provides more advanced results than the Johnson-Ostenfeld-parabola. Therefore, if the Euler's elastic buckling stress is known then the critical buckling stress can be estimated from Eq. (1) without the nonlinear numerical analysis.

Here, the elastic buckling strength of plate panels can be obtained by employing the theoretical approach, and many researchers have proposed empirical formulas or design charts for predicting the elastic buckling strength of plate panels under uniaxial compression or edge shear for various kinds of the edge condition. It is preferable for designers to have the simple formulas at the preliminary design stage.

In this regard, existing design charts predicting the elastic buckling strength are approximated by the simple formulas in the present study. The dotted line in Fig.6 shows the existing theoretical solutions of the elastic buckling coefficient with variance in the aspect ratio and the edge condition. The solid line indicates the approximate formulas derived in the present study (see Appendix).

If the elastic buckling coefficients are known, then the elastic buckling stress of plate panels can be calculated by:

\[ \sigma_E = k_x \pi^2 \frac{D}{b^4} \cdot R_q \]
\[ \tau_E = k_y \pi^2 \frac{D}{b^4} \]

(2.a)

(2.b)

where

- \( k_x \) = the elastic buckling coefficient in compression = \( k_x \) or \( k_y \) according to \( x \) or \( y \) direction, respectively
- \( k_y \) = the elastic buckling coefficient in edge shear
- \( D = E t^3/12(1 - \nu^2) \)
- \( R_q \) = factors accounting for lateral load effects to the elastic buckling stress
4. A New Plate Buckling Design Formula

As mentioned, length and breadth of plates are defined by a and b, respectively. The longer edge of the plate element should be taken as x direction such that the aspect ratio \( \frac{a}{b} \) is always equal to or greater than 1.0.

Using the proposed formula for the plasticity correction, the plate buckling design formula suggested in the previous paper\(^1\) becomes

\[
\Gamma_b = \left( \frac{\sigma_x + \sigma_{rez}}{\sigma_{xcr} \cdot R_{ex}} \right)^{a_x} + \left( \frac{\sigma_y + \sigma_{rez}}{\sigma_{ycr} \cdot R_{ey}} \right)^{a_y} - \eta_a \leq 0 \quad (3)
\]

where
- \( \eta_a \) = safety factor against the buckling, may be taken as 1.0
- \( \sigma_x, \sigma_y \) = the applied compressive stresses in the longitudinal and the transverse directions, respectively
- \( \sigma_{xcr}, \sigma_{ycr} \) = the critical buckling stresses for the longitudinal and transverse compressions, respectively, defined in Eq. (1.a)
- \( \sigma_{rez} \) = the effective compressive residual stress in the x direction
  \[
  = \sigma_x (1 - \frac{\sigma_y}{(\sigma_0 + \sigma_y)})
  \]
- \( \sigma_{rez} \) = the effective compressive residual stress in the y direction
  \[
  = \sigma_y (1 - \frac{\sigma_x}{(\sigma_0 + \sigma_x)})
  \]

\( R_{ex} = 1 - 0.056Q \quad \text{for } 1 \leq a/b < \sqrt{2} \)

\[ = (1 + \eta Q)^{a_x} \quad \text{for } \sqrt{2} \leq a/b \]

\( R_{ey} = 1 - 0.056Q \quad \text{for all aspect ratios} \)

\[ \gamma = 0.025(a/b)^2 - 0.155(a/b) + 0.360 \quad \text{for } \sqrt{2} < a/b < 4 \]

\( Q = \) lateral load parameter

\[ = \frac{q (b^4)}{E \cdot t} \]

\( q = \) lateral pressure
\[ \sigma_{xz}, \sigma_{y} = \text{the compressive residual stress in the } x \text{ and } y \text{ direction, respectively} \]
\[ R_{cr} = 1 - (\tau_{cr}^2)^{\frac{1}{n}} \]
\[ R_{cr} = 1 - (\tau_{cr}^2)^{\frac{1}{n}} \]
\[ \tau = \text{the applied edge shear stress} \]
\[ \tau_{cr} = \text{the critical buckling stress for edge shear, defined in Eq. (1.b)} \]
\[ a = a_0 = 1.0 \quad \text{for } a/b < \sqrt{2} \]
\[ a = a_0 = 0.0293(a/b)^2 - 0.3364(a/b)^3 + 1.5854(a/b) - 1.0596 \quad \text{for } a/b > \sqrt{2} \]
\[ a = a_0 = 0.0049(a/b)^2 - 0.1183(a/b)^3 + 0.6153(a/b) + 0.8522 \]
\[ \sigma_{cr} = \left( \sigma_{cr}^2 + \sigma_{pl}^2 - \sigma_{cr} \sigma_{pl} + 3 \tau_{cr}^2 \right)^{1/2} \]
\[ \sigma_{pl} = \text{the proportional limit state of the material, may be taken as 60% of the yield stress} \]

### 5. Numerical Results and Discussions

#### 5.1 Effect of the Plasticity Correction Factor

Fig. 7 shows the comparison of buckling interaction relationships for simply supported plates. Here, “NEW I” and “NEW II” indicate the present buckling design curves obtained using the Johnson-Ostenfeld parabola and the proposed plasticity correction formula, respectively. Other curves of classification society such as ABS, DnV and LR are employing the Johnson-Ostenfeld parabola for the plasticity correction.

It is expected that in the case of thin plates under axial compression the plasticity correction factor does not affect the buckling interaction curves because the elastic buckling occurs. However, for thick plates and even for thin plates under edge shear, about 15% of the critical buckling strength is additionally admitted at the severe case when using the proposed plasticity correction formula.

#### 5.2 Effect of the Edge Condition

Fig. 8 shows the influence of edge conditions on the present plate buckling strength interaction relationship. In this case, the proposed plasticity correction formula is employed.

It is observed that the plate buckling strength increases so much if the plate edges take clamped conditions. It is known that the ultimate hull girder strength of a ship is very much dependent on the buckling strength of individual plate elements. Recently much portions of ship structures are made of the higher strength steel plates with thin thickness, and this gives rise to the plate buckling problem. It is strongly recommended that the plate edges should be designed and fabricated such that much restraints with regard to the rotational movement are guaranteed.

### 6. Concluding Remarks and Further Research

The plate buckling design formula suggested in the previous paper is advanced to make a more accurate plasticity correction. Also the effect of ideal edge conditions such as all clamped and combined simply supported and clamped in addition to all simply supported is accounted for.

The following conclusion can be made:
1) By using the proposed plasticity correction formula, about 15% of the critical buckling strength is additionally admitted at the severe case.
2) Since the plate buckling strength is very much dependent on the edge condition as well as the plate thickness, it is recommended that the plate edges should be designed and fabricated such that much restraints with regard to rotational movement are guaranteed, in particular for the use of thin higher strength steel plates.

In the present study, all edges simply supported, all edges clamped and two edges clamped/two edges simply supported are treated as the ideal boundary condition of plate elements. However, the amount of rotational restraints is dependent on the rotational stiffness and welding condition of the stiffening members. Thus in the real plates, such ideal edge conditions never occur.

Further research about this problem is necessary.

### References

Appendix: Approximate Formulas of the Elastic Buckling Coefficients

1. \( k_x \) in Longitudinal Compression

(1) All Edges Simply Supported:
\[
k_x = \left( \frac{a}{b} \right)^{m_0} \geq 1, \quad 0.1 \leq a/b \leq 3.0
\]
where
\[
\begin{align*}
m_0 &= 1 \quad \text{for } 1 \leq a/b \leq \sqrt{2} \\
m_0 &= 2 \quad \text{for } \sqrt{2} < a/b \leq \sqrt{6} \\
m_0 &= 3 \quad \text{for } \sqrt{6} < a/b \leq 3.0 \\
&= 4.0 \quad \text{for } 3.0 < a/b
\end{align*}
\]

(2) Loaded Edges Simply Supported and Unloaded Edges Clamped:
\[
k_x = 7.39(a/b)^2 - 19.6(a/b) + 20 \quad \text{for } 1.0 \leq a/b \leq 1.33
\]
\[
= 6.98 \quad \text{for } 1.33 < a/b
\]

(3) Loaded Edges Clamped and Unloaded Edges Simply Supported:
\[
k_x = -0.95(a/b)^2 + 6.4(a/b)^2 - 14.86(a/b) + 16.34 \quad \text{for } 1.0 \leq a/b < 2.0
\]
\[
= 0.2(a/b)^2 - 1.4(a/b) + 6.64 \quad \text{for } 2.0 \leq a/b < 3.0
\]
\[
= -0.05(a/b) + 4.4 \quad \text{for } 3.0 \leq a/b < 8.0
\]
\[
= 4.0 \quad \text{for } 8.0 \leq a/b
\]

(4) All Edges Clamped
2. $k_s$ in Transverse Compression

(1) All Edges Simply Supported:

$$k_s = 1.0 + (b/a)^2$$

$0.0 \leq b/a \leq 1.0$

(2) Loaded Edges Simply Supported and Unloaded Edges Clamped:

$$k_s = 1.0 + (b/a)^2 + 0.12$$

$0.0 \leq b/a < 0.34$

3. $k_s$ in Edge Shear

(1) All Edges Simply Supported:

$$k_s = 4(b/a)^2 + 5.34$$

$0.0 \leq b/a \leq 1.0$
(2) Long Edges Simply Supported and Short Edges Clamped:

\[ k_s = 2.25(b/a)^2 + 1.95(b/a) + 5.35 \]

\[ = 22.92(b/a)^3 - 33(b/a)^2 + 20.43(b/a) + 2.13 \]

\[ \cdots \cdots \cdots 0.4 < b/a \leq 1.0 \]

(3) Long Edges Clamped and Short Edges Simply Supported:

\[ k_s = 2.4(b/a)^2 + 1.08(b/a) + 9.0 \]

\[ \cdots \cdots \cdots 0.0 \leq b/a \leq 0.4 \]

(4) All Edges Clamped:

\[ k_s = 5.4(b/a)^2 + 0.6(b/a) + 9.0 \]

\[ \cdots \cdots \cdots 0.0 \leq b/a \leq 1.0 \]