Progressive Collapse Analysis of a Ship's Hull under Longitudinal Bending (2nd Report)

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Summary

This paper describes the further improvement of the simplified method of progressive collapse analysis of a ship's hull under longitudinal bending proposed by the authors. Here, the method of analysis of flexural behaviour of stiffener elements was extended to deal with coupled flexural-torsional behaviour of angle bar stiffeners welded to continuous plating.

A series of elastoplastic large deflection analysis was performed and the rationality of the proposed method was examined.

With the improved computer code 'HULLST', two alternative analyses were performed on existing bulk carrier including and neglecting tripping of stiffener elements. It was found that the ultimate strength of stiffener elements decreases when stiffener tripping takes place, which results in reduction of ultimate bending moment of the cross-section. The load carrying capacity of stiffener elements in the post-ultimate strength range was more decreased and so that of the cross-section. The allowable bending moment according to the Lloyd's Register of Shipping was calculated and the reserve strength above it until the initial local collapse was 22.4% of the allowable bending moment. Interaction diagram for longitudinal hull strength under bi-axial bending is also presented.

Progressive collapse analysis was performed also on a large scale frigate model tested by Dow, and the applicability of the present method was demonstrated.

1. Introduction

The ability to predict the progressive collapse behaviour of a ship's hull subjected to longitudinal bending is a very important aspect of ship structural design. This has been resulted in extensive research works performed on buckling/plastic collapse of isolated plates, stiffened plate assemblages and overall progressive collapse behaviour of box girders as well as ship's hulls. An important direction in these works is the developments of simplified methods offering computational efficiency and a sufficient accuracy of the predicted structural responses. Such simple and efficient method is the Smith's method for progressive collapse analysis of box girders including ship's hulls subjected to pure longitudinal bending11. In his method, the progressive collapse behaviour of a longitudinally stiffened box girder is simulated by considering the collapse behaviour of the individual elements composing its cross-section under the condition that the plane cross-section of the girder remains plane. The nonlinear behaviour of the individual elements is idealized by their average stress-average strain relationships. Because the finite element procedure is applied to derive the average stress-average strain relationships, this method may become inefficient when a number of elements is large.

In the first report2, from this point of view, an analytical simplified method was proposed to derive the average stress-average strain relationships of structural elements composing a ship's hull cross-section. In this method, combining the results of elastic large deflection analysis and rigid plastic mechanism analysis, the average stress-average strain relationship of an isolated plate was derived. After this, a simple analytical method was proposed to simulate interface flexural buckling of a stiffener with attached plating. The fundamental equations were derived for double span model introducing elastic and plastic deflection components. These deflection components were evaluated from the equilibrium condition of moments produced at both ends of a stiffener element. Based on this result, the average axial strain was evaluated and the average stress-average strain relationship of a stiffener element was derived. Then, with the derived stress-strain relationships of the individual elements, the progressive collapse behaviour of a cross-section subjected to pure bi-axial bending is simulated following the Smith's method.

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The method proposed in the first report to derive average stress-average strain relationships of individual elements may be valid as far as interframe flexural buckling of stiffened panel takes place. However, this may occur only when flat bar or tee bar stiffeners are provided and tripping does not take place. In the actual ship's hulls, angle bar stiffeners are widely used. In this case, compressive collapse behaviour of a stiffener component is complicated because it involves coupled flexural-torsional buckling accompanied by geometrical and material nonlinearities, which can be observed in test results.

In the present paper, firstly, the former method is extended to deal with the coupled flexural-torsional behaviour of angle bar stiffener with attached plating. A series of elastoplastic large deflection analysis is then performed to examine the rationality of the derived average stress-average strain relationships, and the fundamental behaviour of an angle bar stiffener is discussed.

The procedure to derive improved relationships between average stress and average strain of stiffener elements is implemented in the computer code 'HULL-ST'. With this code, progressive collapse behaviour of a 1/3-scale model of Leander class frigate tested by Dow is analyzed, and the obtained results are compared with those by experiment and calculation according to the original Smith's method.

At the end, the same bulk carrier in the first report is analyzed and the influences of stiffener tripping on progressive collapse behaviour is demonstrated. Based on the calculated results, the reserve strength above the design load is discussed. The ultimate longitudinal strength under combined vertical and horizontal bending is also shown.

2. Average Stress-Average Strain Relationship of Angle Bar Stiffener Element

2.1 Basic assumptions

In the present method, a stiffened plate is treated as an assemblage of stiffeners with attached strips representing the plating. A typical cross-section of a stiffener element is shown in Fig. 1(a). When this element is subjected to axial compression, its span may deflect as illustrated in Fig. 1(b). The double span model, 12, allowing the interaction between adjacent spans is considered here.

Two coordinate systems are defined at the cross-section of a stiffener with attached plating as indicated in Fig. 1(a). G is the center of geometry of the whole cross-section, and G' is that of only the stiffener part, BDE.

The y and y' axes are taken parallel to the plating, and z and z' axes perpendicular to those, both passing through the respective center of geometry, G and G', of the cross-section.

It is assumed that the whole cross-section of an assemblage of a stiffener and attached effective strips of plating resists against flexural buckling in the vertical direction, whereas only the stiffener part is effective against flexural buckling in the horizontal direction and torsional buckling. The center of rotation is located at point D, which is a shear center of the stiffener part. Considering continuity of the plating, no horizontal displacement takes place at point B. This produces a horizontal reaction at this point.

2.2 Derivation of average stress-average strain relationship of stiffener element

2.2.1 Flexural-torsional buckling strength

Let v and w denote the translations of the whole cross-section in y and z directions, respectively, in xy-coordinate system and \( \phi \) the rotation with respect to point D, the deflections at an arbitrary point \( i(y, z) \) in BDE are expressed as:

\[
\begin{align*}
    v_i &= v + (z_0 - z) \phi \\
    w_i &= w - (y_0 - y) \phi
\end{align*}
\]

where \( (y_0, z_0) \) represents the location of the shear center in yz-coordinate system. The displacements at the same point are expressed in y'z'-coordinate system as:

\[
\begin{align*}
    v_i &= v + (z_0' - z') \phi \\
    w_i &= w - (y_0' - y') \phi
\end{align*}
\]

where \( (y_0', z_0') \) represents the location of the shear center in y'z'-coordinate system.

According to Timoshenko, the torsional moment with respect to the shear center axis produced by the lateral forces acting on the fiber located at \( i(y, z) \) is represented as:

\[
\begin{align*}
    -\sigma_t ds (z_0' - z') \frac{d^2}{dx^2} \{v + (z_0' - z')\phi\} dx \\
    +\sigma_t ds (y_0' - y') \frac{d^2}{dx^2} \{w - (y_0' - y')\phi\} dx
\end{align*}
\]

In the above expression, \( w \) represents the translation of
the whole cross-section, whereas  and the displacement terms due to rotation, , appear only in the stiffener. Summing up the forces and moments along the respective cross-sections, the lateral forces  and in  and  directions, respectively, and the torsional moment  per unit length are derived. Denoting the reaction force due to constraint of horizontal displacement at point  as , the equilibrium conditions of forces in  and  directions and torsional moment are expressed as follows.

\[ \frac{d^2 M_y}{dx^2} = q_y - q_s \]  

\[ \frac{d^2 M_z}{dx^2} = q_z \]  

\[ \frac{dT}{dx} = -m_t + q_s (\zeta_0 - \zeta_0) + k_s \phi \]  

In Eq. (6), the last two terms represent the torsional moment produced by the reaction force, , and the constraint moment from the plating with the spring constraint, both acting at point .  and are the bending moments in  and  planes, respectively, and  and  torsional moment, which are represented as follows.

\[ M_y = \int z Ez/\rho dA + \int \zeta Ez/\rho dA' \]  

\[ = EI_z \frac{d^2 w}{dx^2} + EI_{z'} \frac{d^2 v}{dx^2} \]  

\[ M_z = \int y Ey/\rho dA + \int \zeta Ey/\rho dA' \]  

\[ = EI_z \frac{d^2 v}{dx^2} + EI_{z'} \frac{d^2 w}{dx^2} \]  

\[ T = C_r \frac{d^2 \phi}{dx^2} - C_w \frac{d^2 \phi}{dx^2} \]  

where  and  are the torsional and warping rigidities of BDE of the cross-section, respectively.  is the moment of inertia of the whole cross-section with respect to  axis, and  and  are the moment and product of inertia of the stiffener part with respect to  axis.  and  axes, respectively.

The reaction force, , is obtained from Eq. (4). From the constraint condition at point  that , the translation of a stiffener in the horizontal direction is derived as:

\[ v = -(\zeta_0 - \zeta_0) \phi = -(\zeta_0 - \zeta_0) \phi \]  

Eliminating  and  from Eqs. (5) and (6) reduce to the following two simultaneous differential equations for coupled buckling by bending and torsion.

\[ EI_z \frac{d^2 w}{dx^2} + P \frac{d^2 w}{dx^2} - EI_{w}(\zeta_0 - \zeta_0) \frac{d^2 \phi}{dx^2} + P' y_0 \frac{d^2 w}{dx^2} = 0 \]  

\[ \left[ C_w - EI_z(\zeta_0 - \zeta_0) \right] \frac{d^2 \phi}{dx^2} \]  

\[- \left[ C_r - P' \left( I_0 A - \zeta_0 + z_0 \right) \right] \frac{d^2 \phi}{dx^2} \]  

\[ + k_s \phi - EI_z(\zeta_0 - \zeta_0) \frac{d^2 w}{dx^2} - P' y_0 \frac{d^2 w}{dx^2} = 0 \]  

Here, the buckling modes are assumed as:

\[ w = W \sin \frac{\pi x}{a} \]  

\[ \phi = \Phi \sin \frac{\pi x}{a} \]  

Substituting Eqs. (13) and (14) into Eqs. (11) and (12), the following equilibrium equations are derived.

\[ (P - a_3) W - (y_0 y_0 P - a_3) \Phi = 0 \]  

\[ (y_0 y_0 P - a_3) W - (y_0 a_3 P - a_3) \Phi = 0 \]  

where

\[ a_1 = (\pi a)^2 E I_z \]  

\[ a_2 = (\pi a)^2 E I_{w}(\zeta_0 - \zeta_0) \]  

\[ a_3 = I_0 (A - \zeta_0 + z_0) \]  

\[ a_4 = (\pi a)^2 \left[ C_w - EI_z(\zeta_0 - \zeta_0)^3 \right] + C' + (a/\pi)^2 k_s \]  

\[ \gamma = (\text{area of BDE})/(\text{area of ABC+BDE}) \]  

From the condition that nonzero  and  exist, the determinant of the coefficient matrix of Eqs. (15) and (16) has to be zero. Hence,

\[ b_1 P^2 + b_2 P + b_3 = 0 \]  

where

\[ b_1 = \gamma (y_0 y_0 - a_3) \]  

\[ b_2 = (a_1 a_3 - 2 y_0 a_3) + a_4 \]  

\[ b_3 = a_3^2 - a_1 a_3 \]  

Buckling load, , is then obtained as:

\[ P_r = (-b_2 + \sqrt{b_2^2 - 4b_1 b_3})/2b_1 \]  

During buckling, plating resists against stiffener rotations by reaction moments. The spring constant, , is determined as the resultant bending moment per unit length to produce a unit angle of rotation.

2.2.2 Stress distribution in cross-section

Substituting Eq. (18) into Eq. (15), the ratio of the deflection coefficients at buckling is obtained as:

\[ \xi = \Phi/W = (P_r - a_3)/(y_0 y_0 P_r - a_3) \]  

According to the assumption that a stiffener part is the only effective cross-section with respect to horizontal bending and torsion, the strain distribution in the stiffener part is expressed in the following form.

\[ \varepsilon = -z_1 \frac{d^2 w}{dx^2} + \zeta_1 (y_1 - y) \frac{d^2 \phi}{dx^2} \]  

\[ - y_1 (h + \zeta_0 + \zeta_0) \frac{d^2 \phi}{dx^2} - \varepsilon_a \]  

\[ = -z_1 \frac{d^2 w}{dx^2} + \zeta_1 (y_1 - y(h + \zeta_0)) \frac{d^2 \phi}{dx^2} - \varepsilon_a \]  

Here, it is assumed that the deflection mode after buckling is the same with buckling mode. Then, substituting the relationship, , into Eq. (20), the strain is expressed as:

\[ \varepsilon = -z_1 \zeta_1 z_1 y_1 z_1 + y_1 (h + \zeta_0) \frac{d^2 w}{dx^2} - \varepsilon_a \]  

From this equation, the strains at points , , and  are derived as follow.

\[ \varepsilon_B = -z_0 \frac{d^2 w}{dx^2} - \varepsilon_a \]  

\[ \varepsilon_D = -(z_0 + y_0 h_0 \zeta_1) \frac{d^2 w}{dx^2} - \varepsilon_a \]  

\[ \varepsilon_E = -(z_e + y_0 h_0 \zeta_1 + b(h + \zeta_0) \frac{d^2 w}{dx^2} - \varepsilon_a \]  

Distributions of the strain at Cross-sections 1 and 2 under flexural-torsional deformation are schematically shown in Fig. 2(a), where the shaded parts correspond to the additional strains due to torsional deformation.
The horizontal and vertical curvatures are expressed as:

\[ K_w = (\varepsilon_x - \varepsilon_y) / b = - (h + z_0) \frac{d^2 w}{dx^2} \]

\[ K_z = (\varepsilon_y - \varepsilon_x) / h = - (1 + y_0 \xi) \frac{d^2 w}{dx^2} \]  \hspace{1cm} (23)

and the ratio of the vertical curvature of the stiffener web, \( K_w \), to the horizontal curvature of a flange, \( K_z \), is derived as:

\[ K_w = \frac{(h + z_0) \xi}{(1 + y_0 \xi)} K_z \]  \hspace{1cm} (24)

The stress distributions in a top flange at Cross-sections 1 and 2 of the double span model take one of the patterns shown in Fig. 2(b) according to the magnitude of strains. Those in a web plate are shown in Ref. 2.

2.3 Validation of proposed method

A series of elastoplastic large deflection analysis is performed to examine the accuracy of the average stress-average strain relationship of a stiffener element derived by the present method. A part of a longitudinally stiffened panel simply supported along frames is analyzed. To represent the local buckling mode accurately, the panel part is divided into \( 30 \times 10 \) elements as indicated in Fig. 3(a). The angle bar stiffener is also divided by plate elements as shown in Fig. 3(b). The panel is assumed to be accompanied by initial deflection in two modes, which correspond to overall grillage buckling and local panel buckling. The stiffener is also accompanied by initial deflection of a torsional buckling mode. The welding residual stresses are not considered in the present analysis. The dimensions and material properties are indicated in the figure. Elastic and plastic panel buckling takes place when the plate thickness is 15 mm and 20 mm, respectively.

Four combinations of plate thicknesses and stiffener heights are considered. The evaluated average stress-average strain relationships are plotted by solid lines in Fig. 4. In Case 4, the plate is thin and elastic panel
buckling takes place. For this reason, the inplane rigidity is decreased before ultimate strength has been attained. Cases 1, 2 and 3 show almost the same ultimate strength, although the load carrying capacity in the post-ultimate strength range is different. In general, as the cross-section of a stiffener becomes slender, the tripping effect becomes to appear significantly and the load carrying capacity decreases. This is reflected in the calculated results in Fig. 4, and Case 2 with the most slender stiffener cross-section shows the lowest capacity.

The average stress-average strain relationships are derived also by the simplified method both including and neglecting the stiffener tripping. The results are shown by chain and dotted-chain lines in Fig. 4. It is known that the ultimate strength is not affected by tripping when a plate is thick. It is slightly reduced in Case 4, which undergoes elastic buckling. Tripping effect is largely appear after the ultimate strength, and the load carrying capacity is much reduced in this range.

In Cases 1 and 4, the results by FEM analysis and present analysis show very good agreements. However, in Case 4, the inplane rigidity and the ultimate strength are a little underestimated by the present method. This may be because the stiffener resists against rotation associated with elastic plate buckling in the FEM analysis. In Cases 2 and 3, the behaviour up to the ultimate strength is well predicted by the present method. In Case 2, the load carrying capacity in the post ultimate strength range is overestimated by the present method. This is because of a local deformation in a stiffener web as a plate which is accurately simulated in the FEM analysis. Contrary to this, in Case 3, the load carrying capacity after ultimate strength is underestimated by the present method. In the FEM analysis, the torsional deflection mode in a stiffener changed from one half wave to three half waves according to the panel buckling mode. This results in a higher load carrying capacity of the stiffener.

In all cases, at least, the ultimate strength is well predicted. In two cases, the load carrying capacity in the post-ultimate strength range is not so well predicted. However, considering the complexity of the flexural-torsional interacting phenomena accompanied by material and geometrical nonlinearities, it may be said that the present method provides the average stress-average strain relationship of an angle bar stiffener element with fairly good accuracy.

3. Progressive Collapse Analysis of Hull Cross-Section Subjected to Pure Bending

3.1 Method of analysis

The progressive collapse analysis of a ship’s hull cross-section subjected to pure bending is performed fundamentally following the Smith’s method\(^1\). The procedure of analysis was described in the first report\(^2\), which may be summarized as follows.

(1) The cross-section is subdivided into small elements composed of plate and a stiffener.

(2) The average stress-average strain relationships of individual elements are derived by the analytical methods proposed in Ref. 2) and in the present paper before performing progressive collapse analysis.

(3) The curvatures of the cross-section in two perpendicular directions are given incrementally, assuming that the plane cross-section
remains plane and that bendings take place with respect to the instantaneous neutral axes.

(4) At a certain incremental step, the inplane rigidities of elements are obtained at each specified strain as a slope of stress-strain curve.

(5) With these inplane rigidities, the location of instantaneous neutral axes and flexural rigidities of the cross-section are evaluated.

(6) Increments of bending moments are evaluated corresponding to the applied increments of curvatures, and so the increments of strains and stresses in individual elements.

(7) At the end of the step, increments of curvatures and bending moments are summed as well as those of strains and stresses to provide their cumulative values. Then, return to step (3).

The computer code 'HULLST' was developed to perform such analysis considering stiffener tripping.

3.2 Analysis on 1/3-scale model of Leander class frigate

In the first report, a series of example analysis was performed on five box girder specimens tested under pure bending. However, the size of the test models were not so large and the number of stiffeners was at most twenty-six. In the present paper, a more realistic 1/3-scale model of a Leander class frigate tested by Dow is analyzed, although angle bar stiffeners were not provided in this model.

This model was tested under four point bending in sagging condition, which resulted in overall collapse accompanied by buckling/plastic collapse of structural components. The total dimensions of the model are $L \times B \times D = 18 \text{ m} \times 4.1 \text{ m} \times 2.8 \text{ m}$. The midsection of the tested part of the scale model is shown in Fig. 5 together with the principal dimensions of the main structural components, the measured values of initial imperfections due to welding and the material characteristics.

In the present analysis, two stiffener elements at the deck-side corner and the top plates of deep girders representing the internal deck structures are considered as hard corner elements at which buckling does not take place. Dow also performed the progressive collapse analysis on this test model applying the Smith's method. The results of analyses are plotted in Figs. 6(a) and (b).

The solid lines in Fig. 6(a), represent the average stress-average strain relationships in compression range for three typical elements 1, 2 and 3 in Fig. 5, derived by the present method and the chain lines by the FEM analysis by Dow. The dashed line represents the assumed material stress-strain relationship. Significant difference is observed in the elastic rigidities of elements 2 and 3 evaluated by both methods. Concerning the results obtained by the present method, the following comments may be given.

As indicated in the table in Fig. 5, plating is ac-

![Fig. 5 Midship section of 1/3-scale frigate model](image)

![Fig. 6 Results of analysis for 1/3-scale frigate model](image)

(a) Element stress-strain relationships

(b) Moment-curvature relationships under sagging condition
Progressive Collapse Analysis of a Ship's Hull under Longitudinal Bending (2nd Report)

companied by high welding residual stresses and large initial deflections. The buckling stresses by elastic calculation considering the influences of these residual stresses are 536.1, 48.0 and -30.1 N/mm² for elements 1, 2 and 3, respectively. The physical meaning of negative buckling strength is that the plate has been buckled due to compressive residual stresses without any external load. When such buckled plate is subjected to inplane compression, its inplane rigidity is equal to that above buckling load from the beginning of loading. Such behaviour is automatically simulated by the present method. For the plating analyzed here, the inplane rigidity above buckling load is 0.5E. This is the main reason for the reduced inplane rigidity of element 3 from the beginning of loading.

At the same time, a large initial deflection also reduces the inplane rigidity of plating from the beginning of loading as in the case of element 2.

The calculated and measured moment-curvature relationships under sagging condition are compared in Fig. 6(b), where the solid lines represent the calculated results by the present method and the chain lines those by Dow. The measured moment-curvature relationships are indicated by ○. Concerning the ultimate strength, the experimental and the two numerical results are in a very good agreement with each other. In the experiment, fairly large overall vertical deflections were observed at the main strength deck due to overall grillage buckling, but this buckling did not show a significant influence on the hull girder moment-curvature response according to Dow. He also described that "Approaching the ultimate load, local stiffener and plating deformations developed on the deck, superimposed on the overall deck deformation, precipitating deck collapse. The side shell clearly showed interframe buckling of the longitudinal stiffeners with associated plate buckling". The predicted collapse mode is also interframe flexural buckling of the deck stiffener elements.

Some differences appear between the measured and calculated flexural rigidities, especially at the higher load level. According to Dow, a possible source of error was in the computation of the experimental curvature of the hull calculated with the vertical displacements, which, when significant nonlinearities occur, would underestimate the local curvature of the hull in the region of collapse. The assumption that the plane section remains plane during hull bending may also lead to the differences between measured and calculated moment-curvature relationship after the ultimate strength when large deformations are produced.

3.3 Analysis on Existing Bulk Carrier

3.3.1 Influence of stiffener tripping on progressive collapse behaviour

To examine the influence of tripping of angle bar stiffeners on progressive collapse behaviour, analysis is performed on a midship section of Bulk Carrier with DWT 60,000. This ship was analyzed also in the first report where the dimensions of the cross-section, the material properties and the assumed initial imperfections are given.

The cross-section is divided into 102 structural elements represented by 65 different types. At the corner parts, hard corner elements, following the material's stress-strain relationship, are introduced. Two alternative analyses are performed considering and without considering torsional deformation of the angle bar stiffener components.

For three typical elements in the deck and double bottom plating, the stress-strain relationships with and without considering stiffener tripping are plotted in Fig. 7(a). It is seen that the ultimate strength is slightly reduced in the deck element due to stiffener tripping, whereas no reduction is observed in the bottom and inner bottom elements. On the other hand, the load carrying capacity is much reduced due to tripping in the
post-ultimate strength range in all elements.

The moment-curvature relationships are plotted in Fig. 7(b). The dashed line represents the result when all elements are assumed to follow the material's stress-strain relationship. In this case, the influences of initial imperfections and buckling effects are excluded, and the same results are obtained under sagging and hogging conditions. The ultimate strength is equal to the fully plastic bending moment, $M_p$, of the cross-section. The solid and the chain lines represent the results under hogging and sagging conditions when stiffener tripping is considered. The dotted-chain lines correspond to the case neglecting stiffener tripping. It is known that, when torsional deformation are considered, the ultimate strength is reduced by about 5% and the load carrying capacity in the post-ultimate strength range is more reduced.

3.3.2 Progressive collapse behaviour

The slope of the moment-curvature curve represents the tangential flexural rigidity of the cross-section. It is seen in Fig. 7(b) that flexural rigidity is reduced from the beginning of loading when initial imperfections are introduced. This is partly because of initial deflections, but mainly due to tensile welding residual stresses. When a plate with welding residual stresses is subjected to tensile load, the portions where tensile residual stresses exist cannot carry tensile load, since these portions are already yielded in tension\(^\text{25}\). This results in a reduced inplane rigidity of the elements in tension range, which is almost a half cross-section, and the flexural rigidity of the cross-section is reduced.

In Fig. 7(b), the numerals and alphabets on the curves indicate the loading steps, at which progressive collapse states of structural components are illustrated in Fig. 8.

In hogging condition, at point a, initial local collapse takes place at the deck plate by yielding in tension as indicated in Fig. 8(a). With further increase in the applied curvature, yielding spreads all over the deck and at the upper parts of side shell and sloped bulkhead of topside tank. At point b, the bottom girders buckle. At point c, the ultimate strength is attained. At this state, the whole topside tank is yielded in tension, the inner bottom stiffeners are collapsed by buckling, and some of the elements considered as hard corner elements are collapsed by yielding in compression. After the ultimate strength, buckled part spreads over the sloped bulkhead of hopper side tank and the lower part of the side shell but unloading takes place in tension side, as is known comparing the yielded parts at points c and d in Fig. 8(a). This is because the effectiveness of the buckled double bottom and hopper side tank decreases and the position of the neutral axis moves upward.

In sagging condition, at point 1, the initial collapse takes place at the uppermost stiffener of the sloped bulkhead of topside tank by buckling as indicated in Fig. 8(b). With further increase in the applied curvature, buckled part spreads and, at point 2, the ultimate bending moment is attained. The reserve strength after the first collapse by buckling is very small comparing to the case of hogging condition. This is because the load carrying capacities of buckled parts in compression decrease rapidly, whereas that of the yielded parts in tension does not. In the post-ultimate strength range, buckled part spreads downward. Yielding does not take place at the bottom plate in tension side within the curvature applied here because the neutral axis moves downward due to the reduced load carrying capacity of the buckled upper part of the cross-section.

3.3.3 Reserve strength above design moment

According to the Rules by Classification Societies, the allowable stress is specified. Denoting this allowable stress as $\sigma_a$, the allowable bending moment can be evaluated as:

$$M_a = \frac{Z \sigma_a}{k}$$  \hspace{1cm} (25)

where

- $Z$ - elastic section modulus at the deck,
- $\sigma_a$ - allowable hull girder bending stresses for mild steel,
Here, the allowable stress is taken as 16.4 kgf/mm² according to the Rule by Lloyd's Register of Shipping. The calculated allowable moment, \( M_a \), is given in Table 1 together with the evaluated bending moments at initial local collapse and ultimate strength under sagging condition. It is known from this table that the bending moments at initial local collapse and ultimate strength are 22.4% and 28.1% higher than the allowable bending moment. The specified stress of 16.4 kgf/mm² is based on the wave load with probability of exceedance of \( 10^{-8} \), and this ship may be regarded as safe enough.

3.3.4 Ultimate longitudinal strength under bi-axial bending

In general, a ship's hull may undergo not only vertical bending but also combined vertical and horizontal bending in the service condition. So, it is also important to know the load carrying capacity of a cross-section under bi-axial bending.

In the analysis, the vertical and horizontal curvatures are given incrementally with a specified constant ratio of \( \Delta \Phi_v / \Delta \Phi_h \). The ultimate strength is assumed to be attained when the resultant moment:

\[ M = \sqrt{M_v^2 + M_h^2} \]

shows its maximum value.

The results are summarized in the form of interaction diagram of ultimate strength in Fig. 9. It is known that vertical ultimate strength of a hull girder decreases with the increase in the horizontal bending.

4. Conclusions

In this paper, a further development of the simplified method was presented for the progressive collapse analysis of a ship's hull under longitudinal bending. Considering angle bar stiffeners welded to continuous plating, tripping was simulated by coupled flexural-torsional buckling applying analytical method. Example calculations were performed by the proposed method and FEM, and the rationality of the proposed average stress-average strain relationships of stiffener elements was confirmed.

With the computer code ‘HULLST’, progressive collapse behaviour of a large scale frigate model tested under pure bending was analyzed. Good agreement was observed between calculated and experimental results.

Then, two alternative analyses were performed on existing bulk carrier including and neglecting tripping of angle bar stiffeners. It was found that considering tripping, the ultimate strength of the stiffener elements is reduced, resulting in the decrease of ultimate bending moment of the cross-section by about 5%. The load carrying capacity in the post-ultimate strength range is more reduced due to stiffener tripping.

The allowable bending moment according to the Lloyd's Register of Shipping was calculated, and the reserve strength above it until the initial local collapse was 22.4% of the allowable bending moment.

Ultimate strength interaction relationships were derived under combined vertical and horizontal bending.

The tripping effects should be considered also for other types of stiffeners such as a flat bar and a tee bar in a different and/or in the same way. Influences of lateral loads acting on stiffener elements in bottom and side plating should also be considered. These remain as future works.

At the same time, the present method deals with only one cross-section. If the assemblage of elements in one cross-section is regarded as a larger element, and connecting these elements in the longitudinal direction, it may be possible to perform progressive collapse analysis of a whole ship's hull. In this case, however, the present method has to be improved from many aspects such as the introductions of the influences of loads in the transverse direction, the horizontal and vertical shear forces and torsional moment. This also remains as a future work.

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Reference


