Estimation of the Motions of Semisubmersibles in Combined Heel and Trim Condition by the Application of Boundary Element Method

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Summary

Motion of offshore structure in damaged condition is important from the view point of safety. For the response estimation of damaged condition, some research works are reported, but most works treat only heeled or trimmed condition. The combined heeled and trimmed condition seems not to be treated. One reason is that there are some considerations that even in damaged condition, the frequency response functions will not be changed largely and another reason will be that this condition is very complicated in space and not only calculation but also experiments are very difficult. But by experiments in damaged condition, it is found out that some responses are changed largely within the range of ocean wave frequency.

Here, we challenged this problem and a computer program was developed in order to estimate linear (infinitesimal) motion responses of floating structures especially in combined largely heeled and trimmed condition in waves and compared with experiments. Calculated results showed relatively good coincidents with experiments.

1. Introduction

Estimation of motion of offshore structure in damaged condition is important from the view point of safety. For the damaged condition, some research works are reported but most works treat only heeled or trimmed condition and concentrated on slow oscillations from the consideration of that linear responses are not changed largely even in inclined condition. So the combined heel and trim condition is not treated. Another reason will be that this condition is very complicated in the transformation between coordinate systems and comparison of calculation with experiments are very difficult. Furthermore, non-linear phenomena occur easily.

For combined largely heeled and trimmed condition, some motion responses change largely, so numerical estimation is worthy to carry out even for linear case.

Here, we challenged this problem and a computer program was developed in order to calculate motion responses of floating offshore structures especially in combined heeled and trimmed condition in waves.

For numerical estimation of hydrodynamic forces and moments, Boundary element method (panel method) was used by distributing the sink and sources on the immersed volume of the structure.

At first, for confirmation of computation program, calculations were done in intact condition for a floating structure composed of twelve Columns with Underwater Footing (CUF) which are connected to each other by rectangular pipe beams from above the structure. Calculated wave exciting forces and motion responses are presented and compared with the experiments. It was found out that the results are congruent.

The findings of this research is significant considering the dearth of information on the problem of large permanent list angle in trimmed condition. For this purpose one column of the conventional semisubmersible was damaged and motion responses were calculated and compared with the experiments.

For the intact condition, the results obtained from the theoretical calculations were congruent to the results obtained from the experiment. With the damaged condition however, results from theoretical calculations slightly differ from the results obtained in the experiments. This may be accounted to the nonlinear effect of restoring coefficients on the motion. Motions for different wave propagation directions were also calculated.

In order to calculate motion responses of an arbitrary shaped floating structure, Boundary Element Method which is also called panel method because of the result-
The method is based on five expressions for the Green's Function utilized for the free-surface part of Green Function. In recent developments, Noblesse (1982) and Newman (1974) have been extended. This is to provide robust algorithms, for all possible positions of the field point relative to the panel, with a uniform tolerance of six decimals accuracy. Separate expressions exist for the free-surface part of Green Function for infinite and finite (constant) depth of the fluid, but their forms are similar and infinite-depth limit can be recovered as a special case of the finite-depth integral representation.

The numerical evaluation of these integral representations shows that they are extremely time-consuming. Therefore other algorithms which are more efficient were developed. First research was published by Hess & Wilcox (1969) by automatically reducing the original three-dimensional flow problem in an infinite fluid domain to a two-dimensional computational problem in a finite domain, namely the mean immersed body surface. These two non-dimensional coordinates are the radial and vertical separation between the field point and the image source point above the free surface, normalized by the wave number.

The result is a double infinite series with positive powers of radial distance and the negative powers of vertical distance. Noblesse (1982) and Newman (1985) were also developed efficient algorithms in the same way, where Noblesse's expressions are not defined when the ratio of vertical distance to horizontal distance goes to 1. After few years, Noblesse published a paper with Telste where they evaluated the calculation for the full domain. In this report, linear wave theory and Green's second identity were utilized in order to obtain necessary formulations. Because there are a lot of publications about the boundary element formulations, it is not repeated here. Detailed explanation can be found Taylor (1982) and Garrison (1979) where same procedure is applied here. For completeness we present some basic formulations which mainly correspond to the formulas of aforementioned publications.

The velocity potential $\phi$ due to the whole (continuous) distribution of sources over the body surface is given as

$$\phi(\vec{x}) = \frac{1}{4\pi} \int_{S_b} f(\vec{\xi}) G(\vec{x}; \vec{\xi}) dS \quad (1)$$

where $\vec{x}$ represents the field point $(x, y, z)$, $G(\vec{x}; \vec{\xi})$ is the Green's function of a point wave source of unit strength located at the point $\vec{\xi} = (\xi, \eta, \zeta)$; $f(\vec{\xi})$ is the source strength distribution function and $dS$ is a differential area on the immersed body surface $S$.

The source densities $f(\vec{\xi})$ are found by satisfying the boundary condition, the Neumann condition on the submerged body surface $S_b$ follows from the requirement that there be no flow through the surface. From the normal gradient of $\phi(x)$ on just off $S_b$ we obtained the following two-dimensional Fredholm integral equation of the second kind for $f$:

$$2\pi f(\vec{x}) + \int_{S_b} f(\vec{\xi}) \frac{\partial G(\vec{x}; \vec{\xi})}{\partial n} dS = 4\pi \frac{\partial \phi(\vec{x})}{\partial n} \quad (2)$$

where

$$\frac{\partial \phi(\vec{x})}{\partial n} = n_j, \quad j = 1, 6$$

for radiation potential

$$\frac{\partial \phi(\vec{x})}{\partial n} = - \frac{\partial \phi(\vec{x})}{\partial n}$$

for diffraction potential

and $n_j$ in eqn. (2) is the normal at $\vec{x}$ directed out of the fluid domain.

3. Discretization of The Integral Equations

The integral equation (2) may be solved numerically beginning with the subdivision of $S_b$ into $N$ quadrilateral facets or panels of area $dS, (j = 1, 2, \ldots, N)$ and identifying as node points the centroid of each panel. Thus, in the discretization process eqn. (2) is replaced by the $N$ equations,
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Furthermore, the surface integral in equ. (3) may be written as the sum of the integrals over the N panels of area \(dS_i\), and as an approximation, the source strength function \(f(\xi, \eta, \zeta)\) may be taken as constant over each panel so that equ. (3) becomes,

\[
\delta_{ij} \phi_i + a_{ij} \phi_j = 2v_n, \quad i, j = 1, 2, \ldots, N
\]

where the repeated index denotes summation and where

\[
a_{ij} = \frac{1}{2\pi} \iint_{S} \nabla G(x_i, y_i, z_i; \xi, \eta, \zeta) dS
\]

In equ. (4) the function \(v_n\) is considered specified being the mode of body motion for radiation potential and negative normal derivative of the incident wave potential for diffraction potential, where \(i\) denotes the number of the panel element.

The elements of the \(a\)-matrix are defined through the Green's function according to eqn. (5). The unknown source strength vector \(f_j\) is, therefore, obtained from the matrix equation using a Gauss Elimination method with LU decomposition and backsubstitution.

Using the same numerical scheme as outlined above, equ. (1) may be expressed as the sum

\[
\phi_i = a_{ij} \phi_j, \quad i, j = 1, 2, \ldots, N
\]

in which

\[
\beta_{ij} = \frac{1}{4\pi} \iint_{S} G(x_i, y_i, z_i; \xi, \eta, \zeta) dS
\]

Equation (6) gives the value of at the \(i^{th}\) node point on \(S_i\).

The definitions of \(a_{ij}\) and \(\beta_{ij}\) indicate that \(\partial G/\partial n\) and \(G\) are to be integrated over the \(j^{th}\) panel.


The evaluation of the \(a\) and \(\beta\) matrices represents an important part of the numerical procedure because a great deal of CPU time is consumed in the process.

4.1 Panelling of Body Surface

The body surface is defined by a set of points, which are presumably exactly on the surface. These are associated in groups of four to form the quadrilateral surface elements as given in Fig. 1.

The procedure given by Hess & Smith (1964) is used in forming the plane quadrilateral element from four given points.

4.2 Evaluation of Rankine Singularity and Its Derivatives

Integral of the Rankine singularity \(1/R\) is done by following a procedure similar to Faltinsen & Michelsen (1974). The integrals are directly evaluated through Romberg numerical integration in most cases.

Hess & Smith (1964) have carried out the derivatives of \(1/R\) analytically in terms of the local coordinates. Their results are used here.

Approximate formulas are also used if the point

where Rankine singularity and velocity components are being evaluated is sufficiently far from the elements inducing the velocity. The quadrilateral is replaced by a point source alone.

4.3 Evaluation Of The Harmonic Part of Green Function

The integrands of the harmonic part of Green function are regular throughout the fluid domain and oscillate approximately with wave length \(L\). In practice \(L\) is generally large, (i.e.), at least comparable to the dimensions of the immersed surface, so harmonic part vary slowly over \(S\) and, therefore, are nearly constant over a panel. Thus, the integrals of the harmonic part were evaluated at the centroid (null point) of the panel and simply multiplied by \(dS\). This was done by following a method for numerically evaluating the Green Function, and its gradient, of the theory of water-wave radiation and diffraction, given by Noblesse & Telste (1986). For completeness, their method is given briefly in the following.

The method is based on five expressions for the Green Function that are useful in complementary regions of the quadrant in which the Green Function is defined. These expressions consist of asymptotic expansions, ascending series, two complementary Taylor series, and a numerical approximation based on a modified form of the Haskind integral representation.

These series express the Green Function and its gradient as sums of power series and terms involving functions of only one variable. The power series is evaluated quickly by using recurrence relations; and the functions of one variable, by using rational approximations. The method permits the Green Function and its gradient to be evaluated with an absolute error smaller than \(10^{-8}\) very efficiently. So this is consistent with the Romberg numerical integration that one can control the absolute error \((10^{-8})\) which is used for the calculation of \(1/R\).
The Green Function $G = G(x, y; f_i)$ can be expressed in the form given below (its derivatives are neglected to show here).

$$4\pi G = -\frac{1}{R} - \frac{1}{R} + 2f_i[Rd(h, v) + i\pi \phi(h)e^\beta]$$  \hspace{1cm} (8)

$f_i$ is the frequency parameter (wave number) defined in terms of the acceleration of gravity $g$ by

$$f_i = \frac{\rho^2}{g}$$  \hspace{1cm} (9)

the notation

$$R = \sqrt{(x - \xi)^2 + (y - \eta)^2}, R' = \sqrt{\rho^2 + (x - \zeta)^2}$$

$$v = f_i(x + \xi), d = (h^2 + v^2)^{1/2} = f_iR'$$

$\phi(h)$ is the usual Bessel functions of the first kinds, and $Rd(h, v)$ is the real function.

The Bessel function $\phi(h)$ in the imaginary part is evaluated numerically in an efficient and accurate manner by using the polynomial approximations given in Abramowitz & Stegun (1970) and Newman (1984) or by using rational approximations within various segments of the positive $h$-axis.

The real functions $Rd(h, v)$ is defined by complementary integral and series representations. Detailed formulations of these functions can be found in Noblesse & Telste (1986).

Following them, the five complementary expressions for the functions $Rd$ is adopted.

5. Numerical Applications and Comparative Analysis

The expressions for Rankine singularities and its derivatives and for the harmonic part of Green Function have been used in a Fortran program which calculates the motions of arbitrary floating structures. Calculated motion responses are presented in this section for different type of semisubmersibles both in intact and damaged conditions, which are compared with the experimental results.

5.1 Co-ordinate system and equation of motion

The coordinate system and the translatory and angular displacements conventions are shown in Fig. 2.

Equation of motion in frequency domain can be written as follows and coupling terms are important for inclined condition.

Calculations are done by space fixed co-ordinate system and converted to the body fixed axis when comparing with experiments.

$$\sum_{j=1}^{6}(a_{ij} + a_{ij})\dot{\eta}_j + b_{ij}\dot{\eta}_j + c_{ij}\eta_j = F_j e^{-i\omega t},$$

$$i = 1, 2, 3, \ldots, 6$$  \hspace{1cm} (11)

5.2 Intact Condition (Column & Footing Unit Type of Semisubmersible)

For basic validation of computer program, calculations were done for the aforementioned type of structure which is composed of 12 columns with underwater footings because experimental data are available including exciting forces and moments. Discretization of its immersed surface is shown in Fig. 3. Geometrical data in model scale are given in Table 1. Calculated results for wave exciting forces and moments and motions in...
head waves are compared with the experimental results where the model was moored by catenary mooring system and presented in Figs. 4 to 6.

As shown in Fig. 4, calculated wave exciting forces and moment are congruent with the experiments in Transient Water Waves.

In Fig. 5, the peaks at heave and surge motions correspond to the measured natural frequencies, whereas in pitch motion, there are two peaks. The first peak is around the surge natural frequency and the second peak is at the pitch motion natural frequency. This shows that pitch motion has coupling with surge motion.

Fig. 6 shows the wave exciting forces. Sway force is not measured in experiments. Calculated motion characteristics were not changed significantly between head sea (180 degrees) and quartering sea (x = 135 degrees), so figures are omitted here.

As can be seen from the figures, results are satisfactory and congruent with the experiments in both head sea and quartering sea conditions. Measured restoring mooring coefficients are included and viscous damping coefficients are not considered in the motion calculations.

5.3 Damaged Condition (Conventional Semisubmersible)

For damaged condition, the model used is shown in Fig. 7 (under water) and principal dimensions are shown in Table 2 as intact condition.

5.3.1 Stability Curves

In order to investigate the response in damaged condition, the initial heel and trim condition were selected as follows. When the one column is flooded, equilibrium angles of heel and trim are obtained by the point of minimum potential energy namely dynamical stability as shown in Fig. 8. Upper figure corresponds to intact condition and the lower to damaged condition. Ordinate is direction of inclining axis and abscissa is heel or list angle around inclining axis. Contour curves of dynamical stability are shown by the percentage of maximum value. From this figure the equilibrium direction of inclination is -44 degrees and inclination angle is 26 degrees, and from this results, motion calculation

![Fig. 4 Exciting Forces and Moment on 12 CUFs Type of Structure in Head Sea (Intact)](image)

![Fig. 5 Motions of 12 CUFs Type of Structure in Head Sea (Intact)](image)
Fig. 6 Exciting Forces on 12 CUFs Type of Structure in Quartering Sea (Intact)

Fig. 7 Discretization of the Wetted Surface of the Conventional Semisubmersible in intact condition by 1132 Panels

Table 2 Geometrical Data for Conventional Semisubmersible (Intact Condition)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1.797 m.</td>
</tr>
<tr>
<td>Breach</td>
<td>1.172 m.</td>
</tr>
<tr>
<td>Draft</td>
<td>0.313 m.</td>
</tr>
<tr>
<td>C.G. (x, y, z)</td>
<td>(0 m, 0 m, 0.273 m.)</td>
</tr>
<tr>
<td>Weight</td>
<td>132.0 kg</td>
</tr>
<tr>
<td>k_x</td>
<td>0.516 m.</td>
</tr>
<tr>
<td>k_y</td>
<td>0.557 m.</td>
</tr>
<tr>
<td>k_z</td>
<td>0.666 m.</td>
</tr>
<tr>
<td>G_Mx</td>
<td>0.045 m.</td>
</tr>
<tr>
<td>G_My</td>
<td>0.037 m.</td>
</tr>
</tbody>
</table>

Table 2 continues

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Natural Frequencies (rad/sec)</td>
<td></td>
</tr>
<tr>
<td>Surge</td>
<td>0.115</td>
</tr>
<tr>
<td>Sway</td>
<td>0.184</td>
</tr>
<tr>
<td>Heave</td>
<td>2.108</td>
</tr>
<tr>
<td>Roll</td>
<td>0.966</td>
</tr>
<tr>
<td>Pitch</td>
<td>0.866</td>
</tr>
<tr>
<td>Yaw</td>
<td>0.180</td>
</tr>
</tbody>
</table>

and experiment were carried out around this condition. Coupling terms in restoring forces and moments are very important for damaged condition.

5.3.2 Responses

Calculations were done for the aforementioned conventional semisubmersible where its immersed surface is shown in Fig. 7 panelled by 1132 facets. Its geometrical data for damaged condition is given in Table 3.

Fig. 9 shows the wetted surface in damaged condition as considered just before, and discretized by 1340 panels. In order to automatically discretize the immersed body surface in damaged condition, another computer program was also developed. This program made discretization fast which would have been otherwise tedious and difficult if done manually.

At first, for intact condition, calculated and non-dimensionalized sway, heave and roll motions of the structure are given in Fig. 10 where each were compar-
Table 3 Geometrical Data for Conventional Semi-
submersible (Damaged Condition)

| Listing Angle | 12.5 deg. |
| Trim Angle    | 17.5 deg. |
| C.G. (x, y, z) | -0.0074 m, -0.0526 m, -0.1046 m |
| Weight        | 141.866 kg |
| k_x           | 0.5143 m |
| k_y           | 0.5594 m |
| k_z           | 0.6317 m |
| k_yz          | 0.1611 m |
| k_zx          | 0.1019 m |
| k_zy          | 0.2446 m |
| GM_x          | 0.356 m |
| GM_y          | 0.357 m |
| Measured Natural Frequencies (rad/sec) |
| Surge         | 0.108 |
| Sway          | 0.188 |
| Heave         | 0.095 |
| Roll          | 1.256 |
| Pitch         | 1.174 |
| Yaw           | 0.298 |

Fig. 9 Discretization of the Wetted Surface of the Conventional Semi-submersible in Damaged Condition by 1340 Panels

ed with the experimental results in Transient Water Waves (mark of ○). Here, calculations were done both with (-----) without (○) mooring restoring coefficients and viscous damping coefficients were included which were obtained from free oscillation experiments. The trend shows that there is no difference between the presence and the absence of mooring restoring coefficients and this is the same as common recognition about mooring line effects on body motion. The difference occurs only in the natural frequencies of sway and roll motions. Especially, sway motion peak is evident when mooring coefficients were used.

Further, roll motion shows high responses near the natural frequency of sway motion because of the coupling effect of sway motion.

Next, in order to verify the accuracy of the calculated results in damaged condition, calculated motions are compared with the experiment. Here, the model was listed and trimmed by placing 9.866 kg weight at the left-back column of the model as already mentioned. This roughly corresponds to the condition that one column was flooded.

In Fig. 11, an example of measured time-history in regular waves are shown. Coupling effects or non-linear responses are seen.
This case, the model was damaged so badly that some parts of the lower hull (pontoon) went out to the water surface while some parts of the deck were immersed. This kind of damage gives rise to non-linear effects particularly in restoring coefficients. This effect is strong especially in angular motions. Further, waves sometimes cover the same part of the deck and when the water turns back from the deck, strong and circular radiated waves are created. There were also difficulties in the experiment in damaged condition while measuring and analyzing the motions. Comparison are done in Fig. 12 with the experiment results in damaged condition. Sway and heave motions fit the experiment results which are obtained both in regular and transient water waves. There are some differences for roll and pitch motions. One of the possible reason may be the effect of the nonlinear restoring coefficients. In the experiment, the water-plane area changes non-linearly such that the coupling restoring coefficients $C_{34}=C_{43}$, $C_{55}=C_{53}$ and $C_{45}=C_{54}$ became negative, zero or positive depending on $x$. 

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**Fig. 11** Example of measured time-history for damaged condition. In body fixed axis.

**Fig. 12** Motions of Conventional Semisubmersible in Damaged Condition (Beam Sea)

**Fig. 13** Motions of Conventional Semisubmersible Comparison of Intact and Damaged condition. (All Calculations)
the shape of the water-plane area. On the average the value is mostly zero because the center of the floating is close to zero even if the water-plane area is asymmetric. In calculating the motion, the value zero gives the closest curve trend with the experimental result. It must be pointed out however that when the negative or positive values were used in the calculation, the results are divergent from the experimental results.

Calculated and non-dimensionalized motions in the damaged and the intact conditions are shown side by side for comparative illustration in Fig. 13. The 90 degrees correspond to beam sea, that means x axis fixed to the body is perpendicular to incident wave direction. Motions are generally bigger in damaged condition.

Finally, calculations were done for the different wave propagation directions in damaged condition in order to understand the dangerous direction.

For this reason, two wave propagation angles were chosen. One was same as the intact condition (beam sea) and the other was the angle which is close to the damaged direction.

Calculated and non-dimensionalized motions are given in Fig. 14. For conventional semisubmersibles, there is difference between these two directions, particularly for translatory motions. Understandably, motions are bigger for surge, heave and roll if the waves come from the direction which is close to the damaged side.

6. Conclusions

For evaluating the linear response in damaged (combined largely trimmed and heeled) condition of semisubmersible offshore platforms, we applied so called panel method for numerical linear calculation and compared with experiments which were undertaken at...
the Towing Tank of the Yokohama National University. Obtained results are as follows in the condition for infinite water depth with Froude number zero.

(1) By the Panel Method, predictions of the motions in damaged conditions, especially in combined large listing and trim angles for conventional semisubmersible is satisfactory for the translatory motions which are compared with the experimental results.

(2) For angular motions, there are some differences between calculations and experiments, and coupling restoring coefficients seems to have large effects for this differences.

(3) Conclusive evidence shows that semisubmersibles will have larger motions in damaged condition than in intact condition. It is therefore important to predict the motions of the semisubmersibles in case of damage. The worst direction angle of coming waves was also investigated in damaged condition.

Acknowledgment

The authors would like to acknowledge the support given by Prof. S. Ueno and the cooperation received from Mr. K. Miyakawa and Mr. T. Takayama in preparation of experimental set-ups and execution of experiments. The authors also appreciate the help received from graduate and undergraduate students of Seakeeping Laboratory of Yokohama National University in carrying out experiments.

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