The Crushing Mechanics of Bow Structure in Head-on Collision

(2nd Report) A Simple Calculation Formula

by Ge Wang, *Member*, Katsuyuki Suzuki, *Member*
Hideomi Ohtsubo, *Member*

Summary

The purpose of this paper is to derive an approximation method, which is both simple in mathematics and reasonable in mechanics, for predicting the crushing strength of ship bow structures.

The first emphasis of this paper is on the analysis of the crushing strength of a parallel structure whose axis direction is the same as the direction of the compressing load. It is found that the dimensionless crushing strength is not sensitive to the slenderness ratio of its plate flanges. The energy absorption factor α_E is then introduced to represent the energy absorption ability of a parallel element.

The second part is an analysis of energy absorption reduction which is caused by inclination of element with respect to external compressing load. This reduction effect is represented by an inclination correction factor α_I . It is found that the energy absorption ability of an element depends strongly on the angle of its initial axial inclination.

The third part is the proposition of a simple formula for approximating the crushing strength of ship bow structures, using the two factors α_E , α_I introduced above. During collision, the crushing strength of a ship bow can be derived by multiplying the flow stress of the material, the cross-sectional area, the energy absorption factor α_E and the inclination correction factor α_I . The effectiveness of this one-term formula is validated by the fact that calculation results show good agreement with model test data. Because of its simplicity, this formula is sure to be able to find its usages in practical designs.

1. Introduction

Cargo spillage from a damaged ship may cause severe environmental pollution of sea and coast. Many efforts have been done to reduce the risk of cargo spillage, but at present there are still no design rules about how to evaluate structural damages during ship impacts. The purpose of this paper is to derive an approximation method, which is both simple in mathematics and reasonable in mechanics, for predicting the crushing strength of ship bow structures.

During collision, the axial crumpling behaviours of intersections of plate flanges play a very important role in energy absorption. The crushing strength of a particular ship bow can be derived by analyzing the energy absorption characteristics of structural elements.

Wierzbicki and Abramowicz^{1)~4)} have done some fundamental researches on the crushing behaviours of plate intersections. Amdahl⁵⁾, Yang and Caldwell⁶⁾ extended this theory to bow structures, assuming that a section between two frames is crushed frame by frame from the front end of the bow. Kierkegaard⁷⁾ has used an orthotropic theory to take into consideration the effect of

stiffeners. Ohtsubo and Suzuki⁸⁾ have extended Yang and Caldwell's method to the structural element whose plate flanges have different thickness. Pedersen⁹⁾ has used these prediction methods to estimate the collision forces between conventional merchant vessels and large volume offshore structures.

Noticing that all the methods by now are far from practical usage because of their complexities in calculation procedures, the present authors will in this paper propose a simple one-term formula for predicting the crushing strength of ship bow structures, through introducing two factors which represent energy absorption ability of structures and energy absorption reduction effect which is caused by inclination of plate intersection with respect to collision load.

Section 2 of this paper outlines a calculation method for predicting the crushing strength of ship bow. Based on the analysis of section 3, the coefficient of energy absorption factor is introduced. Crushing strength of structure whose axis is parallel to the external load is then expressed as the product of its squash load and the energy absorption factor. Section 4 reconsiders the energy absorption ability of inclined element. The reduction effect which is caused by inclination of element's axis is clarified through the introduced coefficient of inclination correction factor. In section 5, a simple formula is proposed for predicting the crushing strength of ship bow structures, using the two factors introduced in

* Department of Naval Architecture and Ocean Engineering, University of Tokyo

section 3 and 4. This one-term formula is intended to eliminate the difficulties in calculating the crushing strength of bow structures, through representing the geometrical attributes of ship bow with just two factors. The calculation results of this simplified method are checked against model test data in section 6. And finally, in section 7, the analysis and conclusions are summarized.

2. Crushing Strength of Bow Structures

During initial stage of collision, buckling of plate flanges is the primary failure. After buckling occurs, structural elements absorb the impact energy through plastic deformation (Figure 1). The elastic absorption energy contributes so little that usually this part of energy absorption can be neglected. Therefore the behaviour of structures is often idealized as rigid-plastic.

When analyzing the crushing strength of structural element, we do not mean to follow the fluctuation of the resistance force. In fact, it is the amount of energy the element can absorb that is the most important to the impacted structure as a whole, because during the process of collision or impact, structural elements behave just as energy absorbers. On the other hand, the complexities of ship bow structures make it very difficult to calculate the detailed resistance force-indentation curve. So we in this paper, for convenience, adopt the mean crushing load as the representation of the energy absorption ability of a structure.

In the previous paper (1st report⁸⁾), the present authors extended Yang and Caldwell's calculation equation of mean collapse force to where there exists different plate thickness. Here we propose a calculation method for predicting the crushing strength of ship bow structures, which is summarized in the following paragraphs:

- (1) It is assumed that the ship under consideration

Fig. 1 Resistance-Indentation Curve of Structural Element

collides perpendicularly towards an infinitely large and infinitely stiff plane.

- (2) Conceptual cuts are introduced so that the bow structure can be treated as an assembly of L , T and X -shaped structural elements. Classification of structural element is based on its cross-sectional shape. For examples, intersection of upper deck and side plate makes L -shaped element, joint of stiffener and side plate makes T -shaped element, cross point of longitudinal bulkhead and inner deck makes X -shaped element.
- (3) Failure, or formation of folding waves progresses frame by frame from the head of the bow.
- (4) Folding lengths are the same for all the structures in a same frame space. This assumption is out of the requirement of geometrical compatibility between adjacent elements which share plate flanges.
- (5) The mean crushing strength of a frame space is defined by dividing the energy dissipated in all the structural elements with the effective crushing distance:

$$P_m = \frac{E}{\delta_{\text{effective}} 2H} = \frac{\sum E_i}{\delta_{\text{effective}} 2H} \quad (1)$$

where

P_m : the mean crushing strength during indentation;

E_i : energy absorbed by a particular structural element, taking into account the effect of axial inclination;

$\delta_{\text{effective}} 2H$: the effective crushing indentation.

- (6) The length of folding wave $2H$ of a particular frame space is determined by the minimization of the total mean crushing strength:

$$\frac{\partial}{\partial H} P_m = 0 \quad (2)$$

If the wave length calculated by minimization is bigger than the actual frame spacing, then only one fold occurs in one frame space, with the wave length being the frame spacing.

3. Crushing Strength of Parallel Element

According to whether its initial axis is parallel to the direction of the compressing load or not, elements can be catalogued into either "parallel" ones or "inclined" ones. The emphasis of this section is on simplifying the formulas for calculating the crushing strength of a parallel element. In section four, the energy absorption ability of an inclined element is discussed.

For simplicity, we consider here only the element which is made by same plate flanges. The crushing strength of L , T or X -shaped element is the function of the yield flow stress of the material σ_0 , plate width b , plate thickness t and the folding length of the element $2H$.

$$P_m = F(\sigma_0, b, t, 2H) \quad (3)$$

For a particular structural element, it is reasonable to

postulate that it fails into a collapse mechanism which has the least possible amount of mean crushing force. Indeed, minimization of equations (3) with respect to H results in, P_m^* , the minimum crushing strength of L , T or X -shaped element. Non-dimensionalization of P_m^* with respect to P_s , the squash load of the section ($P_s = \sigma_0 A$), shows that P_m^*/P_s , the dimensionless crushing strength is the function of only one variant: β , the slenderness ratio of the plate flange ($\beta = b/t$):

$$\frac{P_m^*}{P_s} = f(\beta) \quad (4)$$

Non-dimensionalization of $2H^*$, the folding length when P_m^* is obtained, with respect to plate flange width b leads to a function which has also only one variant β :

$$\frac{2H^*}{b} = g(\beta) \quad (5)$$

Figures 2, 3 are the $f(\beta)$ curves and $g(\beta)$ curves. From these curves it can be concluded that when β is not too small, the dimensionless crushing strength P_m^*/P_s and the dimensionless folding length $2H^*/b$ are basically

Fig. 2 Dimensionless Crushing Strength versus Slenderness Ratio for Parallel Elements

Fig. 3 Dimensionless Folding Length versus Slenderness Ratio for Parallel Elements

not sensitive to the slenderness ratio of plate flange β . When the slenderness ratio β of plate flange is big enough, the minimum mean crushing strength P_m^* of a structural element is approximately proportional to its squash load P_s , or the area of the transverse section of the element.

It is reasonable to define an energy absorption factor α_E ($\alpha_E = P_m^*/P_s$) to represent the energy absorption ability of a parallel element. Then P_m^* can be expressed by the product of the element's squash load and the energy absorption factor:

$$P_m^* = \alpha_E \sigma_0 A \quad (6)$$

Of course, α_E is determined by the shape of the element and the slenderness ratio of its plate flange. For many types of ships, L and X -shaped elements in bow structures are relatively few in numbers so that we can replace their energy absorption factors with that of T -shaped element, because this replacement gives only small difference in the amount of total energy which can be absorbed by the bow structure as a whole.

4. Crushing Strength of Inclined Element

In this section, we shall discuss the reduction effect of energy absorption which is caused by the inclination of the element with respect to the compressing load.

Most of the researches by now dealt only with the crushing behaviors of parallel elements. Within our knowledge, there are only two exceptions^{7),8)} in which the effect of inclination was briefly discussed. Basically these two papers accepted a similar assumption which states that the element's resistance is the projected component of the mean crushing strength of the structural element when it is a parallel one.

There are two reasons for which the energy absorption ability of an inclined element declines. First, with the development of compression, the inclination of the element becomes bigger and bigger, hence its resistance to the external load becomes smaller and smaller. In method of either Kierkegaard⁷⁾, or Ohtsubo and Suzuki⁸⁾, the initial inclination is used throughout the compression process to calculate the resistance force of the inclined element. Obviously both of their calculations inevitably overestimate the crushing strength of an inclined element. Second, when there is an initial inclination, the final length between the element's two ends will be longer, resulting in a shortened effective length of energy absorption.

Figure 4 shows the problem under consideration: structural element AB which has an initial inclination θ with respect to the compressing load is compressed into its final shape of $A'B$. Through careful observation of the crumpling phenomenon, the following assumptions are introduced:

- (1) Both ends of the inclined element, point A and B , are fixed on very stiff planes. Their y -coordinates (Figure 4) are not variant during compression.

Fig. 4 Crushing of Inclined Element

- (2) Failure mode of an element is not affected by its angle of inclination. Or the element plastically deforms into the same shape as that when it is a parallel one.
- (3) The resistance of a parallel element is assumed to be P_m , its mean crushing strength.

The energy absorbed by an element when its front end moves from point A to point A' can be derived by integrating its axial force along its axial direction, or by integrating the resistance force along x-axis, the direction of the compressing load:

$$E = \int_L^{L'} P dl = \int_{l_x}^{l'_x} F dx \quad (7)$$

where

- E : energy absorbed by the element during compression;
- P : axial resistance force;
- L : initial axial length of the element;
- L' : final length between the element's two ends;
- F : resistance force along x-axis, the compressing direction;
- l_x : initial length of the element in the direction of x-axis;
- l'_x : final length of the element in the x-axis.

Using geometrical relationships, the mean resistance force along the direction of the external load F_m is derived by the following equation:

$$F_m = \alpha_l P_m, \quad \alpha_l = \frac{1 - \sqrt{\sin^2 \theta + \delta^2 \cos^2 \theta}}{(1 - \delta) \cos \theta} \quad (8)$$

where

- α_l : inclination correction factor, $\alpha_l = F_m / P_m$;
- δ : effective crushing length factor, $\delta = l'_x / l_x$;
- P_m : mean crushing strength when the element is a parallel one.

Figure 5 shows the relationship between the inclination correction factor α_l and the parameters δ and θ . From these curves, it is clear that:

- (1) The energy absorption capability of an element

Fig. 5 Inclination Correction Factor versus Inclination Angle

depends strongly on the angle of its axial inclination. In ship bow structures, the inclination of an element with respect to the ship's longitudinal direction may be bigger than 45 degree, which means that less than half of the energy absorption ability of this element is utilized during the process of head-on collision.

- (2) The inclination correction factor α_l is insensitive to parameter δ .

5. One-Term Formula for Approximating Crushing Strength of Bow Structures

Even though the method for predicting crushing strength of bow structures summarized in section 2 is relatively satisfactory, it is still too complicated to be used by ship designers and ship classification societies. In this section, basing on the previous analysis of this paper, we will propose a simplified formula which is both simple in mathematics and reasonable in mechanics for approximating the crushing strength of bow structure.

Ship bow is a very complicated assembly of structural elements. The structural elements are L , T or X -shaped, parallel or non-parallel to the longitudinal direction of the ship. Different ships may have quite different layouts of bow structures. Hence basically the crushing strength of ship bow structures is sensitive to structural design.

From the analysis in the previous sections, we find that the mean crushing strength of ship bow can be approximated by the following one-term formula:

$$P_m = \alpha_l \alpha_E \sigma_0 A \quad (9)$$

Here,

- P_m : mean crushing strength of ship bow;
- σ_0 : yield flow stress of material;
- A : total cross-sectional area of bow structures;
- α_E : energy absorption factor, representing the energy absorption ability when all the structural elements are parallel ones;

α_l : inclination correction factor, representing the energy absorption reduction caused by inclination of structural elements.

The inclination correction factor α_l can be determined with the help of Figure 5, after the inclination angle of ship bow structures is known. α_E the energy absorption factor, on the other hand, will be determined either by a previously obtained constant, or according to Figure 2, the calculation curves of energy absorption ability of parallel elements.

Although different designs of ship bows may have different α_E , in view of the characteristics of ship bow structures, energy absorption factor can be determined in a very simple way, which is detailed as follows:

From the $2H^*/b \sim \beta$ curves in Figure 3, it is clear that for many bow designs, despite the fact that frame spacing is usually smaller than the width of plate flange, folding length $2H$ is still smaller than frame spacing, which means that folding of bow structures in one frame space is basically multi-waved. The conclusion is verified by calculations carried out by the authors.

Usually, plate flanges used in ship construction range widely from very stocky ones, which have small β , to relatively very slender ones, such as β equals some hundreds. However, to a great proportion of the practical designs, the slenderness ratios β of main bow structures are not sparsely distributed. Therefore it is possible for us to use an "average" slenderness ratio $\bar{\beta}$ to represent all the structures in a particular frame space of ship bow.

In view of the total energy which can be absorbed by whole bow structures, the calculation curve of T -shaped element is used to determine the energy absorption factor for a ship bow. This is because L and X -shaped elements are fewer than T -shaped elements.

As a conclusion, α_E , the energy absorption factor of a ship bow is determined by the calculation curve of T -shaped element in Figure 2, with the help of an "average" slenderness ratio $\bar{\beta}$ to represent the geometrical attributes of all the plate flanges of the bow's cross-section.

Of course, for such a complicated structure as a ship bow, it is very difficult to represent all its structural details and layouts using only two coefficients α_E and α_l . There are many uncertainties involved in modeling and determining the energy absorption factor α_E and the inclination correction factor α_l . So this approximation method is still under discussion. Further efforts should be done to clarify these uncertainties before this one-term formula (equation 9) can be put into practical usage.

Because the introductions of α_E and α_l make the approximation of the crushing strength of bow structure easy to do with the help of a common calculator, the authors believe that this consideration is sure to be able to find its position in practical designs in the near future.

6. Calculation Examples

The proposed calculation method and the one-term formula are tested against the crushing experiments of ship bow models, given in the literatures by Hagiwara et al¹⁰, Nagasawa¹¹ and Ohnishi¹². All these experiments were carried out quasi-statically. All the scaled models were built up by unstiffened plates, with many structural details omitted in construction.

Here we take the test of the bulbous ship bow model from Hagiwara et al¹⁰ as the first example to check the proposed approximation method. This bow model is shown in Figure 6.

In Figure 7, with solid lines, the results based on the calculation method proposed in section 2 are plotted against the test data, which show that $P_m/\sigma_0 A = 0.068 \sim 0.088$.

It is obvious from Figure 7 that this modified calculation method gives good prediction of the energy dissipated in ship bow structure. According to the calculation results, foldings of the structures between two frames are multi-waved. By carefully observing the photographs of the experiments, we found the evidences of this multi-waved phenomenon. Also the calculated numbers of folding waves are the same as those counted from the photos.

The slenderness ratios of plate flanges in this bulbous bow model are mainly in the range from 100 to 200, therefore the "average" slenderness ratio $\bar{\beta}$ is approximately 150. According to Figure 2, energy absorption factor α_E of this model bow is about 0.11. On the other

Fig. 6 Bulbous Bow Model from Hagiwara et al

Fig. 7 Calculation Results Compared with Test Data for Bulbous Bow Model From Hagiwara et al

hand, the inclination angle is about 30, 15, or 0 degree, which means that α_i , the averaged inclination correction factor is approximately 0.76. Therefore the product of α_E and α_i is approximately 0.083. The mean crushing strength of this bulbous bow can be formulated by the following one-term expression :

$$P_m = 0.083 \sigma_0 A \quad (10)$$

It is clear that approximation by the one-term formula proposed in section 5 correlates well with test data. This approximation results are also plotted in Figure 7 as dotted lines.

Another example is shown in Figure 8 : normal bow model from Hagiwara et al¹⁰⁾. In Figure 9, some discrepancies are found between calculation results and test data in the first two sections of the model bow. After having checked the experiment photos, we noticed that while all the other structural elements fail into multi-waved deformation shapes, stem buckles into only one wave between two frames. Therefore this part of structure can absorb more energy than when it deforms multi-waved. Because fashion plate occupies relatively big portions of the total cross-sectional areas of the first two sections, its one-wave deformation may give an increase in the total energy which can be absorbed by the total structure.

From the above discussion, it seems that big inclination may lead to a different failure shape. So the assumption 2 of the analysis on the crushing strength of inclined element may be not valid for big inclination angle.

7. Conclusions

In this paper, a one-term formula (equation 9) is proposed for predicting the crushing strength of ship bow structures, which is both simple in mathematics and reasonable in mechanics. Two coefficients are introduced in this formula, they are : α_E , the energy absorp-

Fig. 8 Normal Bow Model from Hagiwara et al

Fig. 9 Calculation Results Compared with Test Data for Normal Bow Model From Hagiwara et al

tion factor and α_i , the inclination correction factor. Through comparison of the calculation results with test data of ship bow models, on the simple one-term formula has been validated.

Because this formula (equation 9) makes the prediction of crushing strength very easy to do with the help of a calculator, it is sure to be able to find its practical usage in wide ranges, such as prediction of damage extend of striking vessel, struck ship or offshore structures during collision, design of new-generation tanker which gives least or no pollution to environment, etc.

The mean crushing strength of a parallel structural

element P_m^* can be expressed as the product of its squash load and the energy absorption factor α_E . It is found that if the slenderness β of plate flange is not too small, P_m^* is basically not sensitive to β .

The energy absorption ability of a structural element is found to depend strongly on the inclination of its axis with respect to the compressing load. The mean crushing strength of an inclined element can be expressed as the product of the inclination correction factor α_i and the crushing strength when it is a parallel one.

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