Methodological Unification of the Real-time Algorithm for Wave Generation in a Test Basin and Consideration of its Calculation Error

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Summary

This paper describes the real-time algorithm of ocean waves generated by the snake type wave maker in a test basin. The developed real-time algorithm forms a unified computation process against various kinds of waves. Moreover, a lattice filter like structure of this algorithm is shown.

The algorithm takes an iterative formulation on its calculation process. An iterative calculation generally has a progressive error when the number of iteration becomes large. However, the algorithm causes only a little error after the one million times iteration. Further, the spectrum profiles generated by this method and the conventional one are discussed.

1. Introduction

The important of model testing using a wave maker is on the increase. Simulation of real ocean condition in the laboratory is indispensable to overcome the design limitations.

The author has developed the real-time algorithm which generates the multi-directional waves¹⁾²⁾. However, the former studies have been lacking in consideration of the calculation error depending on the iterative process. The error caused by the iterative process has not been cleared. The algorithm structure also has been in short consideration.

The following section summarizes several waves generated by the snake type wave maker and its mathematical formulae. Section 3 deals with the real-time algorithm of the wave synthesis corresponding to section 2. Section 4 discusses the error caused by the real-time calculation as well as the structure of the real-time algorithm. The final section describes the concluding remarks.

2. Ocean Waves and Generation Formulae in the Snake Type Wave Maker

A fundamental wave of ocean waves is formulated as a sinusoidal function, which is based on the potential wave theory³⁾. Therefore various kinds of ocean waves can be written as a linear superposition of a large number of sinusoidal waves. Above all, so-called short

crested waves are multi-directional irregular waves all traveling independently of one another in different directions with different frequencies.

The model test using multi-directional irregular waves is in the process of being a main current. However, other several waves must be prepared to obtain the basic data.

2.1 Regular waves

The regular wave is the artificial but fundamental wave for obtaining the basic data acting on the test body. The regular wave with its propagation direction of θ is generated by the following wave paddle motion. The wave paddle motion $\eta_i(t)$ of the i -th segmented wave paddle at time t can be formulated as

$$\eta_i(t) = \frac{a}{2F} \sin(\sigma t - ikb \cos \theta) \quad (1)$$

where a , σ , b and t denote amplitude, angular frequency, wave paddle width and time, respectively, and F indicates transfer function for wave generation. F is represented by

$$F = \frac{4 \sinh(kh)}{kh} \frac{1 - \cosh(kh) + (kh) \sinh(kh)}{2kh + \sinh(2kh)} \quad (2)$$

for flap type wave paddle⁴⁾, where h means the uniform water depth in the basin. Moreover, k is wave number and a solution of the dispersion relation as follows.

$$\sigma^2 = kg \tanh(kh) \quad (3)$$

Here g indicates the acceleration due to gravity.

2.2 Long-crested irregular waves

The long-crested irregular wave has its original spectrum such as ISSC⁵⁾, JONSWAP⁶⁾, and Bretschneider-Mitsuyasu⁷⁾ etc., therefore can be considered as a linear superposition of a large number of sinusoidal waves with different frequencies. However this wave also progresses toward a direction same as the regular wave.

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In the same manner as the regular wave, the flap motion $\eta_i(t)$ can be formulated as

$$\eta_i(t) = \sum_{n=1}^N \frac{a_n}{F_n} \sin(\sigma_n t - i k_n b \cos \theta + \varepsilon_n) \quad (4)$$

where N , σ_n , k_n , ε_n and F_n are the number of component waves in frequency, the angular frequency of n -th component wave, the wave number of n -th component wave, the random phase lag defined from 0 to 2π of n -th component wave and the transfer function of n -th component wave, respectively. F_n is represented by the following equation for flap type wave paddle⁹.

$$F_n = \frac{4 \sinh(k_n h)}{k_n h} \frac{1 - \cosh(k_n h) + (k_n h) \sinh(k_n h)}{2 k_n h + \sinh(2 k_n h)} \quad (5)$$

Here k_n is obtained by

$$\sigma_n^2 = k_n g \tanh(k_n h). \quad (6)$$

Moreover, a_n is the amplitude of n -th component wave and represented by

$$a_n = \sqrt{2S(\sigma_n) \Delta \sigma_n}, \quad (7)$$

where S denotes the frequency spectrum and $\Delta \sigma_n$ represents the minute spectrum bandwidth.

2.3 Multi-directional irregular waves

This wave is the so-called short crested wave which has a number of directional components added to the frequency components of the irregular wave. To generate this wave, the wave paddle motion $\eta_i(t)$ can be formulated by the double summation model⁸ as

$$\eta_i(t) = \sum_{n=1}^N \sum_{m=1}^M \frac{a_{nm}}{F_n} \sin(\sigma_n t - i k_n b \cos \theta_m + \varepsilon_{nm}) \quad (8)$$

where M , a_{nm} , θ_m , and ε_{nm} are the number of component waves in direction, the amplitude of nm -th component wave, the m -th propagation direction, and the random phase lag defined from 0 to 2π of nm -th component wave, respectively. The amplitude of nm -th component wave, a_{nm} , is represented by

$$a_{nm} = \sqrt{2S_\eta(\sigma_n, \theta_m) \Delta \sigma_n \Delta \theta_m}, \quad (9)$$

where $\Delta \theta_m$ represents the directional resolution and S_η denotes directional spectrum which is given by

$$S_\eta(\sigma, \theta) = S(\sigma) G(\theta). \quad (10)$$

$G(\theta)$ denotes directional distribution function which is represented by⁹

$$G(\theta) = \frac{\Gamma(\gamma+1)}{\sqrt{\pi} \Gamma(\gamma + \frac{1}{2})} \cos^{2\gamma}(\theta - \theta_0), \quad 0 \leq \theta \leq \pi, \quad (11)$$

where Γ , γ and θ_0 mean Gamma function, the index number of the directional concentration and main wave stream line, respectively.

2.4 Complex waves

This wave is defined by the summation of long-crested irregular waves and short-crested irregular waves. This wave corresponds to the situation: the wind waves are piled up the swell. The wind waves are normally short-crested irregular waves with short wavelength and created near by the current position and the swell is created at far away and becomes only low frequency (long wavelength) components as pro-

gressing with vanishing high frequency component waves.

The wave paddle motion $\eta_i(t)$ for the complex wave can be formulated as the summation of some regular waves and irregular waves, therefore mathematically written by as follows from equation (1) and equation (4):

$$\eta_i(t) = \sum_{n=1}^{\tilde{N}} \frac{\tilde{a}_n}{\tilde{F}_n} \sin(\tilde{\sigma}_n t - i \tilde{k}_n b \cos \theta + \tilde{\varepsilon}_n) + \sum_{n=1}^{\tilde{N}} \sum_{m=1}^{\tilde{M}} \frac{\tilde{a}_{nm}}{\tilde{F}_n} \sin(\tilde{\sigma}_n t - i \tilde{k}_n b \cos \tilde{\theta}_m + \tilde{\varepsilon}_{nm}) \quad (12)$$

In this equation, the circumflex $\tilde{\cdot}$ and $\sim$ means parameters of the long-crested irregular wave and the short-crested irregular wave, respectively. Each suffix of n and m must be carefully read.

3. Real-time Algorithm¹⁰⁾

Calculation of generation signal of waves, especially multi-directional irregular waves and/or complex waves, is troublesome work with the evaluation of many trigonometric functions. To complete the computation within a time-step of the wave generation is too hard even by a currently modernized computer. Therefore wave generation signals for all wave maker units usually have to be calculated beforehand. That is, an operator of a wave maker has to wait a long stand-by time to compute generation signals defined by a next wave condition, because the preparation of calculation irregular wave generation needs a much longer computation time than the duration of actual wave generation¹¹⁾. To eliminate the problem described above, the author has formulated and employed the iterative computation method for the generation of various kinds of waves in real-time. Moreover, the unified algorithm for the generation of various kinds of waves is induced.

3.1 Regular waves

Let equation (1) be changed to the discrete time equation as follows:

$$\eta_{ij} = \frac{a}{2F} \sin(j\sigma\Delta t - i k b \cos \theta), \quad (13)$$

where j denotes the j -th time step.

Here

$$\xi_i = \frac{a}{2F} \sin(-i k b \cos \theta) \quad (14)$$

$$\zeta_i = \frac{a}{2F} \cos(-i k b \cos \theta), \quad (15)$$

then

$$\eta_{ij} = \xi_i \cos(j\sigma\Delta t) + \zeta_i \sin(j\sigma\Delta t). \quad (16)$$

Now let φ_{ij} and Φ_{ij} be

$$\begin{pmatrix} \varphi_{ij} \\ \Phi_{ij} \end{pmatrix} = \begin{pmatrix} \xi_i & \zeta_i \\ \zeta_i & -\xi_i \end{pmatrix} \begin{pmatrix} \cos(j\sigma\Delta t) \\ \sin(j\sigma\Delta t) \end{pmatrix}, \quad (17)$$

then the following iterative equation is obtained. That is,

$$\begin{pmatrix} \varphi_{i,j+1} \\ \Phi_{i,j+1} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix} \begin{pmatrix} \varphi_{ij} \\ \Phi_{ij} \end{pmatrix}, \quad (18)$$

where

$$\alpha = \cos(\sigma \Delta t) \quad (19)$$

$$\beta = \sin(\sigma \Delta t). \quad (20)$$

As the results of that, by calculating equations (14), (15), (19), and (20) in advance, $\varphi_{i,j+1}$, i. e., $\eta_{i,j+1}$ is obtained by equation (18) without sinusoidal calculation.

3.2 Irregular waves

In the case of the irregular wave, similarly, the following are shown.

Let equation (4) be changed to the discrete time equation as follows:

$$\eta_{ij} = \sum_{n=1}^N \frac{a_n}{F_n} \sin(j\sigma_n \Delta t - ik_n b \cos \theta + \varepsilon_n) \quad (21)$$

Here

$$\xi_{ni} = \frac{a_n}{F_n} \sin(-ik_n b \cos \theta + \varepsilon_n) \quad (22)$$

$$\zeta_{ni} = \frac{a_n}{F_n} \cos(-ik_n b \cos \theta + \varepsilon_n), \quad (23)$$

then

$$\eta_{ij} = \sum_{n=1}^N \{ \xi_{ni} \cos(j\sigma_n \Delta t) + \zeta_{ni} \sin(j\sigma_n \Delta t) \}. \quad (24)$$

Now let φ_{nij} and Φ_{nij} be

$$\begin{pmatrix} \varphi_{nij} \\ \Phi_{nij} \end{pmatrix} = \begin{pmatrix} \xi_{ni} & \zeta_{ni} \\ \zeta_{ni} & -\xi_{ni} \end{pmatrix} \begin{pmatrix} \cos(j\sigma_n \Delta t) \\ \sin(j\sigma_n \Delta t) \end{pmatrix}, \quad (25)$$

then the following iterative equation is obtained. That is,

$$\begin{pmatrix} \varphi_{ni,j+1} \\ \Phi_{ni,j+1} \end{pmatrix} = \begin{pmatrix} \alpha_n & \beta_n \\ -\beta_n & \alpha_n \end{pmatrix} \begin{pmatrix} \varphi_{nij} \\ \Phi_{nij} \end{pmatrix}, \quad (26)$$

where

$$\alpha_n = \cos(\sigma_n \Delta t) \quad (27)$$

$$\beta_n = \sin(\sigma_n \Delta t). \quad (28)$$

By calculating equations (22), (23), (27), and (28) in advance, $\varphi_{i,j+1}$ is obtained by equation (26) without sinusoidal calculation.

Finally, $\eta_{i,j+1}$ is obtained by equation (24) as follows:

$$\eta_{i,j+1} = \sum_{n=1}^N \varphi_{ni,j+1}. \quad (29)$$

3.3 Multi-directional irregular waves

The case of the multi-directional irregular wave is shown similarly.

Let equation (8) be changed to the discrete time equation as follows:

$$\eta_{ij} = \sum_{n=1}^N \sum_{m=1}^M \frac{a_{nm}}{F_n} \sin(j\sigma_n \Delta t - ik_n b \cos \theta_m + \varepsilon_{nm}) \quad (30)$$

Here

$$\xi_{ni} = \sum_{m=1}^M \frac{a_{nm}}{F_n} \sin(-ik_n b \cos \theta_m + \varepsilon_{nm}) \quad (31)$$

$$\zeta_{ni} = \sum_{m=1}^M \frac{a_{nm}}{F_n} \cos(-ik_n b \cos \theta_m + \varepsilon_{nm}), \quad (32)$$

then equation (24) is obtained.

Now let φ_{nij} and Φ_{nij} be equation (25). Then equation (26) and equation (29) are obtained.

3.4 Complex waves

The calculation procedure is similar, therefore only results are shown.

The discrete time equation:

$$\eta_{ij} = \sum_{n=1}^N \frac{\tilde{a}_n}{F_n} \sin(j\tilde{\sigma}_n \Delta t - i\tilde{k}_n b \cos \theta + \tilde{\varepsilon}_n)$$

$$+ \sum_{n=1}^N \sum_{m=1}^M \frac{\tilde{a}_{nm}}{F_n} \sin(j\tilde{\sigma}_n \Delta t - i\tilde{k}_n b \cos \tilde{\theta}_m + \tilde{\varepsilon}_{nm}) \quad (33)$$

Parameters:

$$\xi_{ni} = [\tilde{\xi}_{ni} \quad \tilde{\zeta}_{ni}] = \left[\frac{\tilde{a}_n}{F_n} \sin(-i\tilde{k}_n b \cos \theta + \tilde{\varepsilon}_n) \right. \\ \left. \sum_{m=1}^M \frac{\tilde{a}_{nm}}{F_n} \sin(-i\tilde{k}_n b \cos \tilde{\theta}_m + \tilde{\varepsilon}_{nm}) \right] \quad (34)$$

$$\zeta_{ni} = [\tilde{\zeta}_{ni} \quad \tilde{\xi}_{ni}] = \left[\frac{\tilde{a}_n}{F_n} \cos(-i\tilde{k}_n b \cos \theta + \tilde{\varepsilon}_n) \right. \\ \left. \sum_{m=1}^M \frac{\tilde{a}_{nm}}{F_n} \cos(-i\tilde{k}_n b \cos \tilde{\theta}_m + \tilde{\varepsilon}_{nm}) \right] \quad (35)$$

The equation which is rewritten by using equation (34) and equation (35) in equation (33):

$$\eta_{ij} = \sum_{n=1}^N (1 \quad 1) \varphi_{nij}, \quad \bar{N} = \max(\bar{N}, \bar{N}) \quad (36)$$

where φ_{nij} , Φ_{nij} :

$$\varphi_{nij} = [\tilde{\varphi}_{nij} \quad \tilde{\Phi}_{nij}]^T \quad (37)$$

$$\Phi_{nij} = [\tilde{\Phi}_{nij} \quad \tilde{\varphi}_{nij}]^T \quad (38)$$

$$\begin{pmatrix} \tilde{\varphi}_{nij} \\ \tilde{\Phi}_{nij} \end{pmatrix} = \begin{pmatrix} \xi_{ni} & \zeta_{ni} \\ \zeta_{ni} & -\xi_{ni} \end{pmatrix} \begin{pmatrix} \cos(j\tilde{\sigma}_n \Delta t) \\ \sin(j\tilde{\sigma}_n \Delta t) \end{pmatrix} \quad (39)$$

$$\begin{pmatrix} \tilde{\Phi}_{nij} \\ \tilde{\varphi}_{nij} \end{pmatrix} = \begin{pmatrix} \xi_{ni} & \zeta_{ni} \\ \zeta_{ni} & -\xi_{ni} \end{pmatrix} \begin{pmatrix} \cos(j\tilde{\sigma}_n \Delta t) \\ \sin(j\tilde{\sigma}_n \Delta t) \end{pmatrix} \quad (40)$$

The iterative equation:

$$\begin{pmatrix} \varphi_{ni,j+1} \\ \Phi_{ni,j+1} \end{pmatrix} = \begin{pmatrix} \alpha_n & \beta_n \\ -\beta_n & \alpha_n \end{pmatrix} \begin{pmatrix} \varphi_{nij} \\ \Phi_{nij} \end{pmatrix}, \quad (41)$$

where

$$\alpha_n = \text{diag}(\tilde{\alpha}_n \quad \tilde{\alpha}_n) \quad (42)$$

$$\beta_n = \text{diag}(\tilde{\beta}_n \quad \tilde{\beta}_n) \quad (43)$$

$$\tilde{\alpha}_n = \cos(\tilde{\sigma}_n \Delta t) \quad (44)$$

$$\tilde{\alpha}_n = \cos(\tilde{\sigma}_n \Delta t) \quad (45)$$

$$\tilde{\beta}_n = \sin(\tilde{\sigma}_n \Delta t) \quad (46)$$

$$\tilde{\beta}_n = \sin(\tilde{\sigma}_n \Delta t). \quad (47)$$

4. Consideration of the Calculation Error

4.1 Structure of the calculation

The structure of the iterative equations (18), (26) and (41) is considered here. The matrixes of coefficients of equations (18), (26) and (41) are the orthogonal matrixes which represent rotation. So, φ^* and Φ^* are guaranteed to be stable numerically depending on this calculation process.

Figure 1 shows the structure of equations (18), (26) and (41) by expressing in the digital filters. This structure is similar to a lattice filter in the field of signal processing¹²⁾.

Fig. 1 Structure of equations (18), (26) and (41): a lattice filter like structure.

4.2 Calculation error

First of all, let's consider the calculation error when the iteration process is executed. A simple example is as follows:

The direct calculation is b/a , where both a and b are integers. The iterative process to obtain b/a is defined as the function $f_n(a, b)$

$$f_n(a, b) = \frac{b}{c_1} \frac{c_1}{c_2} \frac{c_2}{c_3} \dots \frac{c_i}{c_{i+1}} \dots \frac{c_n}{a} \quad (48)$$

Here, c^* and n are uniform random integer values and integer, respectively.

$f_n(a, b)$ equals b/a theoretically. However, the error e :

$$e = \left| \frac{b}{a} - f_n(a, b) \right| \quad (49)$$

becomes progressively larger when n becomes large. That is, the value of the error e depends on the value of n . Figure 2 shows the state of the increasing error as the iteration.

The calculation error of the real-time algorithm described in section 3 is considered next. In the case of the multi-directional irregular wave, the calculation error e_w is defined by the following equation.

$$e_w = \sum_{n=1}^N \sum_{m=1}^M \frac{a_{nm}}{F_n} \sin(j\sigma_n \Delta t - ik_n b \cos \theta_m + \varepsilon_{nm}) - \sum_{n=1}^N (1 \ 0) \underbrace{\begin{pmatrix} a_n & \beta_n \\ -\beta_n & a_n \end{pmatrix} \begin{pmatrix} a_n & \beta_n \\ -\beta_n & a_n \end{pmatrix} \dots \begin{pmatrix} a_n & \beta_n \\ -\beta_n & a_n \end{pmatrix}}_{j \text{ times}} \times \left(\sum_{m=1}^M \frac{a_{nm}}{F_n} \sin(-ik_n b \cos \theta_m + \varepsilon_{nm}) \right) \quad (50)$$

Fig. 2 State of the increasing error.

Figure 3 shows the time histories from 990,620 to 1,003,000 in the sampling number generated by the direct method: equation (30) with the parameters: $M=70$, $N=200$, $b=0.4$ [m], $\Delta t=4$ [msec], $H_{1/3}=0.3$ [m], $T_{01}=0.75$ [sec] and the generated spectrum: ISSC. Figure 4 shows the same one, but generated by the iterative method. Figure 5 shows the error defined by equation (50). The arithmetic calculation is executed by the double precision floating point. Figure 5 indicates that this iterative method causes only the error of the order of 10^{-3} after the one million times iteration.

Fig. 3 Generated waves by equation (30) : the direct method.

Fig. 4 Generated waves by the iterative algorithm.

Fig. 5 Calculation error between the direct method and the iterative algorithm defined by equation (50).

(a) Spectrum profile obtained from figure 3.

(b) Spectrum profile obtained from figure 4.

Fig. 6 Power spectrum : 512-point-FFT, 90% overlapping, 25 times averaging.

Fig. 7 Power spectrum of the calculation error.

The spectrum profile is more important rather than the absolute accuracy in the model test using a wave maker. So, the next is the discussion concerning the spectrum profile comparison between the direct method and the iterative method. Figure 6 shows the power spectrum obtained from Figure 3 and Figure 4. These power spectra are calculated by FFT algorithm indicated in the figure's caption. These profiles are in good agreement. Moreover, Figure 7 shows the power spectrum of the error : Figure 5. The error spectrum is very low ($-100[\text{dB}]$), so the iterative error has no effect on the spectrum profile of the iterative method.

5. Concluding Remark

The author has reached a methodological unification of the real-time algorithm for wave generation in a test basin. The structure of this algorithm has the orth-

ogonal matrix which represents rotation and is similar to a lattice filter.

The algorithm causes only the error of the order of 10^{-3} after the one million times iteration which equals 66.7 minutes. Moreover, the spectrum profile of the algorithm gives good agreement with the one which is obtained by the direct method.

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