ISUM Rectangular Plate Element with New Lateral Shape Functions (1st Report)

– Longitudinal and Transverse Thrust –

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Summary

New shape functions for the lateral deflection of rectangular plate elements for the Idealized Structural Unit Method (ISUM) are developed. First, the collapse behaviour of rectangular plates under longitudinal and transverse thrust is studied by elastoplastic large deflection FEM and the characteristics of the collapse modes are highlighted. Based on these characteristic deflection modes, new lateral shape functions for the large ISUM elements are assumed and implemented. Comparisons with FEM results show the applicability and good accuracy of these new ISUM elements. The importance of shape functions simulating the collapse mode accurately is demonstrated by comparing the new formulation with a previous one and by examining the influence of two yield functions.

1. Introduction

Ultimate strength assessment for ship structures is an important task to ensure the safety of ships. Much research has been dedicated to this topic, but nevertheless the numerical prediction of ultimate strength and collapse behaviour of large ship structures considering three-dimensional effects is still a great challenge. Despite the enormous development in computer technology, elastoplastic large deflection analyses with the conventional Finite Element Method (FEM) are too time consuming for large structures. An efficient computational tool is required for design and assessment of new structures. The Idealized Structural Unit Method (ISUM) proposed by Ueda and Rashed1) is one such method.

In ISUM, the elements are significantly larger than in FEM. The material and geometrical nonlinearities inside these ISUM elements are idealized and included in their formulation. Larger elements mean fewer nodes and fewer degrees of freedom to solve for. Therefore, calculation time with ISUM is much shorter than that with FEM.

The stiffened plate is the basic structural element of ships. To consider the buckling/collapse behaviour of stiffener and unstiffened panel including their interaction accurately, individual modelling of stiffeners by beam-column elements and panels by plate elements seems to be the more promising approach than modelling of stiffened plates as a whole.

For the unstiffened panel, a Simple Dynamical Model2,3) has been developed to simulate the buckling/plastic collapse behaviour of rectangular plates. This model basically consists of a combination of Elastic Large Deflection Analysis (ELDA) and Rigid Plastic Mechanism Analysis (RPMA).

Localization of plastic deformation is a very important issue for obtaining the correct behaviour of longitudinally compressed rectangular plates4,5). One part of the rectangular plate collapses whereas the rest of the plate unloads. Therefore, an unstiffened rectangular plate is divided into several elements to all of which the Simple Dynamical Model is applied6). This approach gives good results including localization and has been successfully extended to model stiffened plates in combination

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with beam-column elements\(^6\). However, the applicability of the Simple Dynamical Model is limited to longitudinal thrust.

The original ISUM rectangular plate element\(^7\) and an improved element\(^8\) are applicable to combined inplane loads; they are based on the concept of effective breadth. However, the original element cannot predict the decrease in load-carrying capacity beyond ultimate strength which takes place in reality. The improved element is able to predict this, but its formulation is rather complicated and makes it difficult to employ this element in large structures in combination with beam elements. Furthermore, localization of plastic deformation in the plate is not considered.

Another ISUM rectangular plate element in which the internal lateral deflection is treated as additional degree of freedom has been proposed\(^9\). This element can also be applied to combined inplane loads and its formulation is less complicated than the one of the previous element, but it has problems in accuracy for thin plates as will be shown in this paper. Localization of plastic deformation is not taken into account either.

A new ISUM rectangular plate element should incorporate:
- localization of plastic deformation
- good accuracy even in post-ultimate range
- possible combination with beams
- possible extension to combined loads

These requirements suggest a formulation which uses the internal deflection as degree of freedom, but localization has to be included and more accurate shape functions for this deflection have to be found.

In this paper, the characteristic collapse modes of rectangular plates under longitudinal and transverse thrust are studied by elastoplastic large deflection FEM. Based on these results, new shape functions for the lateral deflection of ISUM plate elements are developed. Localization of plastic deformation is taken into account. Two yield functions are examined: one is a fully plastic condition, and the other is a more accurate one which can cope with intermediate states between initial yielding and full yielding.

2. Collapse Behaviour of Rectangular Plates under Thrust

Elastoplastic large deflection FEM analyses have been carried out to clarify the collapse modes of rectangular plates under longitudinal and transverse thrust.

Figure 1 shows the definition of the coordinate system. All FEM calculations have been performed with the computer code ULSAS and material properties as follows:

- Young's Modulus \( E = 205.8 \) GPa
- Poisson's Ratio \( \nu = 0.3 \)
- Yield Strength \( \sigma_y = 313.6 \) MPa
- Hardening Rate \( H' = 0 \)

The simply supported plates are modelled with 4-node isoparametric bilinear degenerated shell elements with reduced integration. For calculations under longitudinal thrust, 10x10 elements per buckling half wave are used. In the case of transverse thrust, the number of elements is: 10 in \( y \)-direction and the next even number to \( a/b \cdot 10 \) \((a: \text{length}, b: \text{breadth of plate})\) in \( x \)-direction. Load is applied by forced displacements, and to simulate the continuity of the plating, all edges are kept straight. For square plates, initial deflection is given by one half wave. For rectangular plates, initial deflection in hungry-horse mode is applied.

In this chapter, the focus lies on finding typical collapse modes of rectangular plates, which can be used to design new shape functions for ISUM elements. More results of FEM analyses are presented in chapter 4 together with ISUM results.

2.1 Square Plate under Uniaxial Thrust

As a most fundamental case to understand collapse modes, a square plate under uniaxial thrust is considered. Figure 2 shows the deflected shapes and the spread of yielding for a rather thin plate \((1000x1000x15.5 \text{ mm})\) at ultimate strength and in the post-ultimate range. At ultimate strength, the deflected shape is still very similar to the sinusoidal buckling mode. However, the collapse takes place due to the spread of yielding along the unloaded edges. The inplane strain associated with this yielding straightens the sinusoidal shape in the direction of applied load. Beyond ultimate strength, deflection in a roof mode is clearly ob-
The deflected shape of a square plate can be expressed over the whole range of loading as

\[ w = \sum_{i} \sum_{j} A_{ij} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \]  

The coefficients of this Fourier series change with the state of loading. For the example considered here, the higher-order coefficients divided by \( A_{11} \) (the coefficient corresponding to the buckling mode) are shown in Fig. 3 depending on the average applied strain (nondimensionalized by yield strain). All other coefficients are either zero or very small. During the initial loading stages, all higher-order terms are negligible, the plate has a sinusoidal shape. Beyond ultimate strength, \( A_{31} \) causes the straightening in \( x \)-direction, leading to the roof mode. The increase in \( A_{13} \) tends to produce a plateau in \( y \)-direction, but in general the deflected shape stays sinusoidal in \( y \)-direction. \( A_{33} \) can be seen as coupling term of these effects.

The change from a sinusoidal shape to a roof mode beyond ultimate strength is a characteristic phenomenon for a wide range of thickness. Only very thick plates practically remain in a sinusoidal mode, and extremely thin plates, which are not considered here, collapse due to secondary buckling in a higher eigenmode. For all other plates, it can be said: the thinner the plate is, the more emphasized the roof mode becomes and the earlier it starts to develop.

### 2.2 Rectangular Plate under Longitudinal Thrust

A rectangular plate under longitudinal thrust buckles into several half waves. Figure 4 shows a rectangular plate (2400x835x15 mm) which has undergone elastic buckling in three half waves. The sinusoidal buckling mode is still present at ultimate strength, but due to the nonuniform initial deflection of hungry-horse mode, the deflection is not the same in different half waves. These differences in the deflection correspond to different extents of yielding. One half wave reaches its ultimate strength first. This half wave collapses whereas the rest of the plate unloads; the plastic deformation is localized beyond ultimate strength.

The collapsing half wave of the rectangular plate basically shows the same behaviour as an equivalent square plate. The deflection mode changes from a sinusoidal shape to a roof mode.

### 2.3 Rectangular Plate under Transverse Thrust

Under transverse thrust, a rectangular plate buckles in one big half wave, and the transverse buckling strength is lower than the longitudinal one. The same rectangular plate as under
longitudinal thrust is considered here. Unlike the case of longitudinal thrust, the sinusoidal form has already changed at ultimate strength, Fig. 5. The middle part of the plate has a cylindrical shape, sinusoidal in transverse direction, but longitudinally constant. The end parts collapse as half of a square plate each. Beyond ultimate strength, the shape is straightened in loading direction, also forming a kind of roof mode.

The most important feature of the collapse under transverse thrust is nevertheless that the cylindrical middle part is formed before reaching ultimate strength. Only plates which are so thick that they undergo plastic buckling even under transverse thrust start forming the cylindrical shape beyond ultimate strength.

3. Formulation of ISUM Element

3.1 General Formulation

The ISUM rectangular plate element has four corner nodes, each with the two inplane translations as degrees of freedom. The main difference of the formulation considered here from conventional plate elements is that the lateral deflection inside the element boundaries is treated explicitly as additional degree of freedom. Certain shape functions – as explained in sections 3.2 and 3.3 – are assumed for this internal deflection $w$ and its initial value $w_0$. In the present study, the performance of the ISUM element in simulating the local collapse of unstiffened panels is investigated. Therefore, a nodal degree of freedom in lateral direction is not included here.

The nonlinear contribution of the internal deflection $w$ to the inplane strain components is calculated by ELDA, i.e. the compatibility equation for large deflections

$$
\Delta \Delta \Gamma = E \left[ \left( \frac{\partial^2 w}{\partial x^2} \right)^2 - \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} \right] (2)
$$

has to be fulfilled. Having solved for Airy’s stress function $F$, the nonlinear strain is added to the linear one obtained by linear interpolation of nodal translations. The rest of the element formulation follows usual variational principles. Taking a total Lagrangian approach, Green’s strain increment for inplane strains and curvatures is calculated. The
principle of virtual work in an incremental form yields

$$[K][\Delta d] = \{\Delta F\} + \{\Delta F\} - \{R\}$$  \hspace{1cm} (3)

with tangential stiffness matrix $[K]$, incremental displacement vector $[\Delta d]$, applied force and its increment $\{F\}$, $\{\Delta F\}$, and internal force $\{R\}$. Stiffness matrix and internal force vector are integrated numerically by trapezoidal rule with equidistant integration points over the element area. To avoid through-thickness integration, stress resultants per cross section are used.

3.2 Shape Function for Longitudinal Thrust

The previous ISUM formulation\(^9\) employs one element for a rectangular plate under longitudinal thrust. The shape function for the internal deflection $w$ of that element covers all buckling half waves in the plate. In such an approach, localization cannot be considered because the deflection has the same amplitude in all half waves. As shown in Fig. 4, only one half wave of a rectangular plate finally collapses due to nonuniform initial deflection. Therefore, such a plate will be modelled by several elements, each covering one buckling half wave. As these half waves are nearly square, the results of section 2.1 for square plates will be used for finding an improved shape function.

The previous shape function\(^9\) assumes one sinusoidal mode of deflection

$$w = A \sin \frac{\pi x}{a_s} \sin \frac{\pi y}{b_s}$$  \hspace{1cm} (4)

as collapse mode of one buckling half wave of length $a_s$ and breadth $b_s$.

This shape function, Eq. (4), cannot cope with the change of the deflection to a roof mode. To simulate this change, at least one higher-order deflection component in loading direction, namely $A_{31}$ as shown in Fig. 3, has to be considered. According to some trial analyses, treating two components, $A_{11}$ and $A_{31}$, as independent degrees of freedom is not possible because their interaction in the framework of ELDA moves the deflection to a higher eigenmode, but not to a roof mode. Thus, a shape function with prescribed relation between the two components is proposed as

$$w = A_{11} \sin \frac{\pi x}{a_s} \sin \frac{\pi y}{b_s} + f \cdot A_{31} \sin 3\frac{\pi x}{a_s} \sin \frac{\pi y}{b_s}$$  \hspace{1cm} (5)

in which $f = A_{31}/A_{11}$. Following the curve for $A_{31}/A_{11}$ of Fig. 3, the factor $f$ is assumed depending on the nondimensionalized average strain $\epsilon_x/\epsilon_Y$ as

$$f = \frac{A_{31}}{A_{11}} = \begin{cases} 0, & \frac{\epsilon_x}{\epsilon_Y} < 1.0 - p \\ -m \cdot \ln(\frac{\epsilon_x}{\epsilon_Y} + p), & \frac{\epsilon_x}{\epsilon_Y} \geq 1.0 - p \end{cases}$$  \hspace{1cm} (6)

In the nonlinear solution process, the average strain $\epsilon_x$ in Eq. (6) is treated as constant during one iteration. The coefficients $m$ and $p$ of Eq. (6) depending on the slenderness of the plate are presented in section 4.1. Since further higher-order terms are not included, the factor $f$ in this formulation differs from the real ratio $A_{31}/A_{11}$ obtained by FEM and Fourier analysis.

3.3 Shape Function for Transverse Thrust

The previous shape function based on the elastic buckling mode under transverse thrust assumes Eq. (4)\(^9\) where $a_s = a$ and $b_s = b$ for the transverse thrust. As shown in Fig. 5, however, most rectangular plates under transverse thrust develop a collapse mode with cylindrical middle part even before ultimate strength. Therefore, as idealization of such a collapse mode, the following shape function is proposed.

$$w = \begin{cases} A \sin \frac{\pi x}{b_s}, & 0 \leq x \leq \frac{b}{2} \\ A \sin \frac{\pi x}{a_s}, & \frac{b}{2} \leq x \leq a - \frac{b}{2} \\ A \sin \frac{\pi \left(x-(a-b)\right)}{b_s} \sin \frac{\pi y}{b_s}, & a - \frac{b}{2} \leq x \leq a \end{cases}$$  \hspace{1cm} (7)

The first and third lines of Eq. (7) correspond to half the buckling mode of a square plate. The second line describes the cylindrical middle part.

In reality, the cylindrical collapse mode is developed during loading. Thus, blending of buckling and cylindrical collapse mode is proposed as third possibility. Denoting the shape function of the buckling mode, Eq. (4) where $a_s = a$ and $b_s = b$, with $w_{buckl}$ and the one of the collapse mode, Eq. (7), with $w_{coll}$, blending is accomplished by

$$w = g \cdot w_{coll} + (1 - g) \cdot w_{buckl}$$  \hspace{1cm} (8)

with $g$ as blending factor. Strain contributions of $w_{coll}$ and $w_{buckl}$ are calculated independently by ELDA without interaction terms. To ensure that after buckling the shape changes quickly enough from buckling to cylindrical mode, the blending factor is calculated as

$$g = \begin{cases} 0, & \epsilon_Y - \sigma_{Ycr}/E < c \cdot \epsilon_Y - \sigma_{Ycr}/E \\ 1 - \exp \left(-c \cdot \epsilon_Y - \sigma_{Ycr}/E\right), & \epsilon_Y < \sigma_{Ycr}/E \leq \epsilon_Y \geq \sigma_{Ycr}/E \end{cases}$$  \hspace{1cm} (9)

with average strain $\epsilon_Y$, yield strain $\epsilon_Y$, transverse buckling strength $\sigma_{Ycr}$, Young’s Modulus $E$, and an adjustment factor $c$ which will be discussed in section 4.3. If the elastic buckling strength exceeds $\sigma_Y$, $\sigma_{Ycr} = \sigma_Y$ is set.

The evaluation of the blending factor $g$ with Eq. (9) realizes a pure buckling shape until the average strain $\epsilon_Y$ reaches a linearly calculated buckling
strain $\sigma_{yy}/E$. Then $g$ asymptotically approaches unity with increasing strain, shifting the deflection from buckling to cylindrical collapse mode.

### 3.4 Yield Function

Yielding of the ISUM element is checked by a yield function in terms of stress resultants applied to each integration point. To assess the influence of the accuracy of such a yield function on the overall behaviour of the ISUM element, two yield criteria based on the von Mises hypothesis have been examined:

1. An integration point is yielded if the full cross section at this point is plastic, referred to as 'yield criterion 1'.
2. To allow for earlier fibre yielding, a yield function which considers the intermediate states between initial yielding and full yielding is employed\(^{10,11}\), referred to as 'yield criterion 2'.

Having yielded, the material matrix for the integration point is calculated according to the plastic flow law. A full description of both yield criteria is given in Reference 12).

### 4. Results and Discussions

#### 4.1 Square Plate under Uniaxial Thrust

The new proposed shape function, Eq. (5), has been applied to square plates of various slenderness. A square plate is modelled by only one ISUM element with 7x7 integration points. Length $a$ and breadth $b$ have been set to 1000mm, while varying the thickness $t$ to obtain different slenderness ratios $\beta = \frac{b}{t} \sqrt{\frac{\sigma_y}{E}}$. The calculations have been performed with an amplitude of initial deflection of 0.1 $t$. Through many comparisons with FEM results, the coefficients $m$ and $p$ for the factor $f$ of Eq. (6) have been obtained, as shown in Fig. 6.

For $\beta \leq 1$, the new shape function becomes equal to the previous one, Eq. (4), as with $m = 0$ the factor $f$ is identically zero. This corresponds to the fact that very thick plates do not develop a significant roof mode. With increasing $\beta$, also $m$ increases, reflecting the more emphasized roof mode of thinner plates. Additionally, the change from sinusoidal mode to roof mode starts earlier with higher slenderness. Thus, the offset $p$ is negative for small values of $\beta$ and positive for large ones.

Figure 7 compares the average stress-average strain curves obtained with two shape functions, Eqs. (4) and (5), and two yield criteria with FEM; three slenderness ratios, $\beta$, are shown. The choice of the yield function influences the accuracy of the solution to a certain extent, but the overall behaviour of the ISUM element is dominated by the shape function.
The previous shape function, Eq. (4), considering buckling mode only overpredicts the strength in the post-ultimate range; for very thin plates it does not give an ultimate strength value. The new shape function, Eq. (5), predicts the loss of load-carrying capacity beyond ultimate strength very well — regardless of the yield criterion. As the yield criterion 1 gives estimates of ultimate strength on the unsafe side of FEM, only the yield criterion 2 will be employed further.

Figure 8 demonstrates that the coefficients \( m \) and \( p \) for the fixed ratio \( f \), Eq. (6), which have been obtained by calculations with an initial deflection of 0.1 \( t \), are valid for all practically interesting values of initial deflection.

Although the results are omitted due to page limitation, calculations for plates which are not exactly square have shown that the new shape function is superior to the previous one in most cases. Only for thick plates which have an aspect ratio \( a/b \) below 0.8 and undergo plastic buckling, the previous shape function gives slightly better results.

4.2 Rectangular Plate under Longitudinal Thrust

Six rectangular plates under longitudinal thrust are presented here to show the applicability of the new shape function. The characteristics of the plates are given in Table 1. The initial deflection of hungry-horse mode expressed by

\[
 w_0 = \sum_{i} A_{0i} \sin \frac{i\pi x}{a} \sin \frac{i\pi y}{b} \tag{10}
\]

is considered with coefficients \( A_{0i} \) of Table 2. Initial deflections of plates No. 1-5 are according to the measurements of ship platings\(^{13}\). As no measured data exist for plate No. 6, the same coefficients of initial deflection are assumed as for plate No. 4.

The number of buckling half waves under longitudinal thrust, \( n \), is calculated considering the buckling behaviour. If the theoretical longitudinal buckling strength exceeds the yield strength, plastic buckling occurs and the aspect ratio of each buckling half wave tends to 0.7. Otherwise, elastic buckling with aspect ratios about 1.0 per half wave takes place.

<table>
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<th>( a )</th>
<th>( b )</th>
<th>( t )</th>
<th>( \beta )</th>
<th>( n )</th>
<th>( \sigma_{x\text{cr}} )</th>
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\( a, b, t \): length, breadth, thickness [mm]

\( \beta \): slenderness ratio \( (\beta = \frac{b}{t} \sqrt{\frac{E}{f}}) \)

\( n \): number of buckling half waves under longitudinal thrust

\( \sigma_{x\text{cr}} \): longitudinal elastic buckling strength

\( \sigma_{y\text{cr}} \): transverse elastic buckling strength

\( \sigma_y \): yield strength

<table>
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</tbody>
</table>
In FEM analyses, the initial deflection $u_0$ in hungry horse mode, Eq. (10), is directly applied to the model, causing the localization of plastic deformation. In ISUM analyses, the following model is used to simulate nonuniform deflection and localization:

- The rectangular plate is equally divided by as many ISUM elements as the number of buckling half waves, $n^5$, and $a/n$ is used as half-wave length as in the shape functions of Eqs. (4) and (5). Figure 9(a) shows a typical mesh.

- Only the component of the initial deflection which corresponds to the buckling mode, $A_{0n}$, is considered. The full value of $A_{0n}$ is given to one ISUM element, and half this value to the other ones$^5$.

- To the element with the full value of $A_{0n}$, the new shape function, Eq. (5), is applied. Only for plate No. 2 having an aspect ratio below 0.8 per element, the previous shape function, Eq. (4), is used, and plate No. 1 is so thick that new and previous shape function coincide.

- Only the element with the full value of $A_{0n}$ will collapse, and the other elements will unload. Thus, all other elements can still use the sinusoidal formulation of Eq. (4); this has proven to give stablest solutions.

Figure 10 compares the average stress-average strain relations obtained by FEM and ISUM. Three curves are shown regarding the new ISUM approach. The short dashed line indicates the average stress-average strain path of the collapsing element in which localization of deformation takes place, and the dotted line the path of one of the unloading elements. The long dashed line is the result for the overall plate. For comparison purpose, the average stress-average strain relationship obtained by the previous ISUM formulation$^9$ is plotted by the chain line.

All ISUM calculations have been performed with 7x7 integration points per buckling half wave and the yield criterion 2. As some stress-strain paths in ISUM show vertical or even backward directed segments at the point of ultimate strength, arc-length control is used here. This severer behaviour compared with FEM can be explained by the more sudden appearance of unloading in the ISUM model due to much fewer elements.

Fig. 10: Average stress-average strain relations of rectangular plates under longitudinal thrust
The previous ISUM formulation without localization cannot follow the FEM results. As it uses Eq. (4) even for thin plates, the load-carrying capacity beyond ultimate strength is almost constant in those cases. In the new approach, all but one element unload. Thus the overall stress-strain path, which can be seen as the average over one collapsing and all unloading elements, turns sharply downward where localization happens, basically following the FEM solution. With increasing strain the overall path departs from the FEM curve because - even with the new shape function - the collapsing element cannot reach values below a certain stress level. To improve the accuracy further, more higher-order components, as shown in Fig. 3, have to be included in the shape function. Nevertheless, the new approach considering localization is much more accurate than the previous one.

4.3 Rectangular Plate under Transverse Thrust

The same plates as under longitudinal thrust have been studied for the case of transverse thrust. In FEM analyses, the same initial deflection, Eq. (10), has been used for consistency. In ISUM, $A_{01}$ of Table 2 is given as amplitude of initial deflection. Figure 11 compares ISUM results for all three shape functions, Eqs. (4), (7) and (8), with FEM.

For the shape function following the transverse buckling mode, Eq. (4), only one ISUM element is used. To ensure the best possible solution, the same number of integration points is used for the single transverse buckling half wave as for all half waves under longitudinal thrust in total.

On the other hand, for the shape function directly assuming the cylindrical collapse mode, Eq. (7), a mesh of three elements is designed, Fig. 9(b). At each end of the plate, one element covers the part collapsing as half of a square plate with 7x7 integration points each. In the middle, one element is used for the cylindrical part, also with 7x7 integration points. Seven integration points in longitudinal direction are even enough for long cylindrical parts because there the deflection is constant in this direction. The lateral deflections of all three elements, corresponding to the amplitude $A$ in Eq. (7), are assigned to the same degree of freedom in the solution process, and thus the continuity of the deflection is guaranteed.

![Fig. 11: Average stress-average strain relations of rectangular plates under transverse thrust](image)
This approach gives good results for almost all plates. Only for plate No. 1, the solution with this shape function has failed in convergence resulting from zero stiffness of the cylindrical part due to yielding almost exclusively under inplane forces.

Blending of buckling and cylindrical mode, Eqs. (8) and (9), gives accurate results for all plates. The same mesh and integration point density as for the pure cylindrical collapse mode can be used because the shape becomes quickly cylindrical with increasing load. This blending is basically an interpolation between two modes, but as the plastic behaviour depends on the strain history, prediction of ultimate strength and post-ultimate behaviour is generally improved compared to both single-mode approaches. Unfortunately, no relation for the adjustment factor \( c \) in Eq. (9) can be given, the values of \( c \) used for the plates considered here are shown in Fig. 11.

Because of the difficulty in choosing the adjustment factor \( c \) and the very good results obtained by directly assuming the cylindrical collapse mode, this mode is recommended for rather thin plates. Assuming the buckling mode as shape function is strictly limited to very stocky plates.

5. Conclusions

The characteristics of the collapse behaviour of rectangular plates under longitudinal and transverse thrust have been pointed out with elastoplastic large deflection FEM analyses. New shape functions for ISUM rectangular plate elements have been developed based on the collapse modes obtained by FEM. The main findings can be summarized as

1. Square plates under uniaxial thrust change from their sinusoidal buckling mode to a roof mode beyond ultimate strength. An ISUM element incorporating this change in its shape function can predict the collapse behaviour very well.

2. Rectangular plates under longitudinal thrust show localized plastic deformation when collapsing. An ISUM approach using one element per buckling half wave can cope with this phenomenon. The collapsing half wave basically behaves like a square plate beyond ultimate strength.

3. Under transverse thrust, the collapse mode of most rectangular plates is significantly different from their buckling mode. Even before ultimate strength, a cylindrical shape in the middle of the plate develops. Taking this deflection into account in the shape function, the performance of the ISUM element is largely improved.

The use of shape functions taking account of the real collapse mode of plates is the key point for accurate results in ISUM. For rectangular plates under longitudinal thrust, good results can only be obtained when localization is included in the ISUM approach. Different yield functions influence the results only slightly.

The present report has clarified the principal behaviour under uniaxial thrust. Application of the new shape functions to biaxially compressed plates and combination with beam elements are currently under development.

References


