A STUDY ON MOTIONS AND DRIFT FORCES OF A MULTIBODY FLOATING SYSTEM IN WAVES

by Yoshiyuki Inoue*, Member, M. Rafiquel Islam*
M. Murai*, Member

Summary

In this paper, the application of three dimensional source technique for computations of motion responses and drift forces for multi-body floating system with zero forward speed in waves are investigated. The mean wave drift forces are obtained by far field and near field (direct pressure integration) methods. Numerical results for the first order motion responses and the mean wave drift forces for a parallel arrangement of a FPSO (Floating Production Storage and Offloading Unit) and a LNG carrier are discussed. Some comparisons are made for a single floating body (semi-submersible) as well as for a simple multi-body system of a cylindrical body with a box shaped hull. Numerical results for motion responses are compared with experimental ones for the FPSO-LNG system and also results for mean wave drift forces obtained from both of the methods are compared with experimental ones. The trend of the results as well as correlation between both of the methods found to be comparatively well in agreement. The effects of roll damping to the second order wave drift forces for sway and yaw are also discussed.

1. Introduction

The hydrodynamic problems for multi-body systems in waves are nowadays important to the ocean engineers and naval architects; such as the cases for a crane barge in the vicinity of a offshore platform, assembly process of huge floating structures or a supply vessel connected to an offshore platform. Study of drift forces and moments for such kind of multi-body system in waves are important to maneuvering and sea keeping analyses, practically when mission requirements include station keeping.

There are many investigations of the second order drift forces in the past decade by Newman2), Faltinsen3), Pinkster5) and others, but they are mostly limited to single body, while the studies of the multi-body floating systems are rare; Ohkusu4), Oortmerssen6) and Loken7). Maruo5) has derived the equations for the drift forces in terms of Kochin function for a single body and, since then many developments have been done in this field. Newman5) derives the equation of the drift forces and moments for an arbitrary floating body by slender body assumption and presents some results for series 60 ships. Fang12) derives the equation of the drift forces and moments for twin hull body by phase transfer technique.

On the other hand Oortmerssen6) presents a concept of direct integration of second order pressure as given by Pinkster5). It is extended to multi-body floating system and, some computational and experimental results have been presented for a cylinder and a box. Loken7) extends the far field technique derived by Faltinsen5) to multi-body system and, he shows some numerical results for the same model used by Oortmerssen6).

This paper represents an application of 3-D source technique for the computations of motions and drift forces for multi-body floating system in waves. The second order drift forces are obtained by far field and near field approaches. The technical approach reported in this paper is the same as Inoue et al11) for the far field and, the basic approach for the near field method are similar to Kagemoto8), but the calculation method of velocity potentials are different. Results obtained from both of the methods are compared to experiments and, it shows that the trends as well as correlation are found to be satisfactory.

2. Drift forces and Moments in regular waves

There are two methods of calculating the second order drift forces on floating bodies: one is far field approach and the other is near field method. Here in this paper the equations for the far field approach are similar to those of Inoue et al 11) and, assuming that the fluid is irrotational, homogeneous and incompressible and, the simplified formulas using Kochin function \( H^m(\theta) \), \( H^m(x) \) for \( m \) th
body taking a right hand Cartesian co-ordinate system (Fig. 1.1) are as follows:

\[ F_m = \frac{1}{2} \rho g \xi_m \left( \frac{2k h + \sin 2k h}{1 + \cosh 2k h} \right) \mathcal{K}(\chi) \cos \theta \rho \left( \frac{2k h + \sin 2k h}{1 + \cosh 2k h} \right) \int_0^\pi \mathcal{K}(\chi) \cos \theta d\theta, \]

(1)

\[ F_m = \frac{1}{2} \rho g \xi_m \left( \frac{2k h + \sin 2k h}{1 + \cosh 2k h} \right) \mathcal{K}(\chi) \sin \theta \rho \left( \frac{2k h + \sin 2k h}{1 + \cosh 2k h} \right) \int_0^\pi \mathcal{K}(\chi) \sin \theta d\theta, \]

(2)

\[ F_m = \frac{1}{2} \rho g \xi_m \left( \frac{2k h + \sin 2k h}{1 + \cosh 2k h} \right) \Im \mathcal{K}(\chi) \rho \left( \frac{2k h + \sin 2k h}{1 + \cosh 2k h} \right) \int_0^\pi \Im \mathcal{K}(\chi) \rho d\theta, \]

(3)

where superscript 'm' means 'm'th floating body  
\[ \xi_m = \text{incidence wave amplitude} \]
\[ h = \text{depth of water} \]
\[ \chi = \text{wave heading angle from X axis} \]
\[ \rho = \text{mass density of water} \]
\[ \Omega = \text{circular frequency (rad/sec)} \]
\[ H''(\theta), H''(\chi) \] are the corresponding derivatives of the Kochin function \( H''(\theta), H''(\chi) \). \( H''(\theta) \) is the complex conjugate of \( H''(\theta) \).

In regular wave, the Kochin function can be written as:

\[ H''(\theta) = -\frac{k^2}{k^2 + k - k'k'} \int_0^\pi \mathcal{K}(\chi + h) \cos h dh, \]

(4)

where \((\xi, \eta, \zeta)\) is the point on the body surface.

And as dispersion relation,

\[ K = k \tan h, \]

where \[ K = \frac{\omega^2}{\rho} \quad \text{and} \quad k = 2\pi / \lambda. \]

\[ \lambda = \text{wave length} \]

\[ O'' = O''^m + \sum_{j=1}^n \left( \sum_{m=1}^n \left( \sum_{j=1}^n \sigma''_{m} X''_{m} + \sum_{n=1}^m \sigma''_{j} X''_{j} \right) \right), \]

which is the total source density at body 'm' due to diffraction and radiation potentials and the effect of interacting bodies \((m \neq n)\) and \(X''_{j}\) is the motion amplitude of the body 'm'in three translational oscillations and three rotational oscillations about the co-ordinate axes.

The motion responses of the floating bodies are obtained by using 3-D source technique, which is described in our previous paper\(^{10, 11}\).

For the near field approach, the derivation of the second order drift forces and moments are based on direct integration of the second order pressures. To derive it, let us consider the Bernoulli equation including velocity square term:

\[ P = -\rho g z - \rho \frac{\partial \phi}{\partial t} - \rho \frac{(\nabla \phi)^2}{2}, \]

(5)

where \( \phi \) is the total velocity potential in the field.

And the corresponding force on body 'm'

\[ F_k = -\int_{s_m} P n_x dS, \]

(6)

where subscript \( k = 1, 2, 6 \) (: surge, sway and yaw directions).

\[ n_1 = \cos(\phi, X_m), \]
\[ n_2 = \cos(\phi, Y_m), \]
\[ n_3 = (X_m - X_m) n_2 - (Y_m - Y_m) n_1, \]

where \( X_G^m \) and \( Y_G^m \) are the co-ordinate of the horizontal center of gravity of the body 'm' and, \( X_m \) and \( Y_m \) are the co-ordinates of the investigating points on its wetted surface and, \( n \) is the outward unit normal vector on the surface of the body 'm'.

The integration of equation (6) is difficult to calculate because the integral surface varies as time interval. Therefore assuming the motion to be small parameter \( \epsilon (\epsilon<<1) \) and the fluid motion may be described by means of expansion series of velocity potential \( \phi \) of order \( \epsilon \) :

\[ \phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + O(\epsilon^3). \]

(7)

The quantities \( 0(1), 0(\epsilon), 0(\epsilon^2) \) are expressed within superscript \( 0) \), \( 1) \) and \( 2) \) respectively. Therefore, the forces on the body can be expressed according to Kagemoto\(^8\) as:

\[ F_k = F_k^{(0)} + \epsilon F_k^{(1)} + \epsilon^2 F_k^{(2)} + O(\epsilon^3), \]

(8)

where

\[ F_k^{(0)} = \int_{S_m} \rho g z n_x dS, \]

(9a)

\[ F_k^{(1)} = \int_{S_m} \rho \frac{\partial \phi}{\partial t} n_x dS, \]

(9b)

\[ F_k^{(2)} = \int_{S_m} \left( \rho \frac{\partial^2 \phi}{\partial t^2} + \rho \frac{\nabla \phi \nabla \phi}{2} \right) n_x dS + \int_{S_m} \left( \rho g z + \rho \frac{\partial \phi}{\partial t} \right) n_x dS. \]

(9c)

Equation (9a), (9b) and (9c) give the total hydrostatic, first order and second order fluid forces acting on the body 'm'. \( S_m \) is the wetted surface of the body 'm' under the mean water line (MWL) and, \( \Delta S_m \) is that varies with time due to wave elevation.

Therefore second order force can be written as:

\[ F_k^{(2)} = \int_{S_m} \left( \rho \frac{\partial \phi}{\partial t} + \rho \frac{\nabla \phi \nabla \phi}{2} \right) n_x dS + \int_{S_m} \left( \rho g z + \rho \frac{\partial \phi}{\partial t} \right) n_x dS. \]

(10)

Assuming the body is wall sided \(( \int_{\Delta S_m} dS = \int_{\Delta S_m} dz f dl \text{ MWL} )\) and, disregarding the second order hydrostatic reaction forces due to second order displacements under the influence of the mean second order wave exciting force, then the above equation can be written as:

\[ F_k^{(2)} = \int_{S_m} \left( \rho \frac{\partial \phi}{\partial t} + \rho \frac{\nabla \phi \nabla \phi}{2} \right) n_x dS + \int_{f_{\text{MWL}}} \frac{1}{g} \left( \frac{\partial \phi}{\partial t} \right) n_x dS. \]

(11)

As \[ \zeta = -\frac{1}{g} \frac{\partial \phi}{\partial t} \text{ from linearized Bernoulli's equation} \]
Now if the contribution from the first and the second order potential can be separated then

\[
F_{k}^{(2)} = F_{k1}^{(2)} + F_{k2}^{(2)},
\]

where

\[
F_{k1}^{(2)} = \int_{S_k} \rho \left( \frac{\nabla \phi_0}{2} \right)^2 n_k ds + \int_{\text{muw}} \frac{1}{2} \rho g \left( \frac{\partial \phi_0}{\partial t} \right)^2 n_k dl
\]

and,

\[
F_{k2}^{(2)} = \int_{S_k} \rho \frac{\partial \phi_2}{\partial t} n_k ds.
\]

In this paper, the first order motion and the mean second order forces are concerned, so the contribution of the second order potential disappears or can be ignored. The mean value of \( F_{k2}^{(2)} = 0 \) (for more detail see for instance ref Salvessen13) and \( F_{k1}^{(2)} \) can be calculated with \( \phi_1 \) (linear potential) and can be written as:

\[
F_k^{\infty} = \text{Mean value of } F_{k1}^{(2)} = \int_{S_k} \frac{\nabla \phi_1 \nabla \phi_1}{4} n_k ds \int_{\text{muw}} \frac{1}{2} \rho g \phi_1 n_k dl.
\]

The first order potential \( \phi_1 \) consists of the sum of three potentials associated with the undisturbed incoming waves, the diffraction waves and the radiation waves due to the first order body motions respectively:

\[
\phi_1 = -i \omega \left[ (\phi_0 + \phi_7) \xi_a + \sum_{m=1}^{N} \sum_{j=1}^{6} (X_j^m \phi_j^m) \right],
\]

where

- \( \phi_0 \) = incident wave potential and for long crested harmonic progressive waves,
- \( \phi_7 \) = diffraction wave potential
- \( \phi_j^m \) = potential due to motion of the body \( \cdot m \) in \( j \) th mode i.e. radiation wave potentials
- \( \xi_a \) = incident wave amplitude
- \( X_j^m \) = motion amplitude of the body \( \cdot m \) in \( j \) th mode.

The incident wave potential is as follows:

\[
\phi_0 = \frac{g}{\omega^2} \cosh[\sqrt{kh}] e^{-i \omega t} \cosh kh
\]

The individual potentials must satisfy the Laplace equation in the fluid domain and moreover satisfy the linearized boundary condition on the free surface, on the sea bottom and the boundary conditions on the wetted surface of the floating bodies and, the details of which have been given in our previous paper Inoue et. al.10).

The most difficult portion of the equation (13) is to evaluate fluid velocity, which is equal to the spatial derivative of the velocity potentials, and it is calculated by the following procedure.

\[
\begin{bmatrix}
\phi_{x_1} \\
\phi_{y_1} \\
\phi_{z_1}
\end{bmatrix} = \frac{1}{4 \pi \sigma} \int_{S_k} \int_{\text{muw}} G(x, y, z; \xi, \eta, \zeta) ds,
\]

where \( \sigma_j \) is the source strength distributed over the mean wetted surface area of the body and \( G(x, y, z; \xi, \eta, \zeta) \) is the Green function describing the influence at a point \((x, y, z)\) due to a source strength at a point \((\xi, \eta, \zeta)\) and, the suffix \( X, Y, Z \) denote the corresponding component of fluid velocity in their respective directions.

### 3. Comparative study

A computer program in order to calculate the motions and the second order drift forces for multi-body system has been developed. In order to check the validity of the program, the numerical calculations are carried out for a cylindrical body in vicinity of a box shaped hull as shown in fig. 1.2, the same model as used by Oortmerssen9 and for a single semi-submersible body examined by Takezawa14) as shown in fig.1.3. The numerical results for both of the cases are compared with experimental results as shown in fig.2.1 through 2.5. In these figures the force components have been non-dimensionalised by dividing \( \rho g \zeta^2 L \).

Fig. 1.1 An example sketch of the multi-body floating system

Fig. 1.2 Mesh arrangement of a Box and cylinder

In this case for the wave-heading angle of 180°, the
cylinder is on the lee side. As mesh size i.e. number of panels is an important parameter for the 3-D source technique, so in order to examine the effect of mesh sizes, four types of meshes are examined for the cylinder. The number of panels varies from 84 to 136. From figs. 2, 3 and 2.5, it is shown that the higher number of elements does not always give better agreement.

Fig. 1.3 : Meshing of semi-submersible

From these figures, it can be observed that the numerical results are well agreement with experiments for 112 number of panels. Although the experimental results are scatter in high frequency region, the numerical results are comparatively well agreement with the experimental ones. The very high peak value of drift force does not appear in the experiment near the resonance frequency of heave motion as shown in fig. 2.5.

In the present calculations frequency interval is taken as 0.025 rad/sec. However in both of the cases, it reveals that the calculated results and experimental ones are varied slightly. The reason for inconsistency is that the exact prediction of drift forces is difficult to calculate numerically for very short waves.

Fig. 2.1 Sway Drift Force on Box for Wave heading angle 270° (Separation distance 50 m)

Fig. 2.2: Comparison of Sway drift force of Semi-submersible in beam sea condition

Fig. 2.3: Surge drift force on Cylinder (wave heading 180°)

Fig. 2.4: Surge motion of Cylinder (wave heading 180°)

Fig. 2.5: Heave motion of Cylinder (wave heading 180°)
4. Numerical results and Discussions

Computations have been carried out in a wide range of wave frequencies for a parallely connected FPSO and an LNG carrier (separation distance 8.68 m). The mesh arrangements are shown in fig. 4.1. The wetted surface of the FPSO and the LNG are subdivided in to 300 and 280 elements respectively, which satisfy the mesh sizes for calculation of the motions and the second order drift forces. The principal dimensions are given in Table-1. The wave conditions are considered correspond to 260.8 m in depth of water for the wave-heading angle of 180° and 270° respectively. In this paper the motions as well as second order drift forces have been shown only for the LNG carrier.

The numerical results for motion responses of the LNG carrier are compared with the experimental ones as shown in figs. 4.2 through 4.13. For the wave heading angle of 180°, from figs 4.2, 4.4, 4.6, it is shown that the motion responses for surge, heave and pitch are good in agreement between the computations and experiments. From figs. 4.3, 4.5, 4.7 in sway, roll and yaw motions the numerical results deviate from the experimental ones because of neglecting viscous roll damping. So, to confirm this matter, 10% of critical roll damping is added to potential roll damping. These numerical results are also presented in the figures. The effects of roll damping are significant in the case of sway and yaw motions due to coupled response of roll motion and due to hydrodynamic interaction between the bodies. For the wave-heading angle of 270°, from figs. 4.8–4.13, it is shown that the agreement between the computations and experiments are fairly good. The interaction effects are clear in the motions of the LNG carrier, which is on the lee side of the FPSO. As in beam sea, due to the interaction of radiation and diffraction waves, the wave amplitude may sometimes double of the incident wave amplitude if their phase comes closer to each other and the sway motion amplitude shows almost double of incident wave amplitude. The discrepancy between the computations and experiments for the roll and yaw may be due to neglecting of viscous roll damping. In order to check its effect, 10% of critical roll damping is added in this case. It is clear from dotted curves in figs. 4.11 and fig. 4.13 the numerical results of the roll and sway motions reduce significantly due to adding 10% of critical roll damping and are comparatively well agreement with experiments.

The results of the numerical and experimental wave drift forces are shown in figs. 4.14 through 4.19 for the wave heading angle of 180° and 270° respectively where, the computations have been carried out by the far field and the near field (direct pressure integration) methods. In general, it can be stated that the agreement between the numerical and experimental results are quite well except sway drift forces in 270° wave heading angle, but the trends are the same between both of the methods. As in 270° wave direction viscous roll damping is important which is realized in motion response curves (figs. 4.9, 4.11 & 4.13), so drift forces are also computed including this effect and are plotted in figs. 4.17 through 4.19. From these figures, it is clear that due to adding 10% of critical roll damping to potential damping, surge drift force and yaw drift moment significantly reduce in high frequency zone and are quite well with the experimental results. For wave heading angle 180°, the results for the surge and sway drift forces (figs. 4.14, 4.15) show very good correlation between the far field approach and experimental results. The peak value of surge drift forces appears between the range of 0.46–0.52 of $\frac{\lambda}{L}$ and this corresponds to heave motion resonance as shown in fig. 4.4. The worst discrepancies are found in the higher frequency ranges for the near field approach (see figs. 4.15, 4.17). It is observed that the numerical results deviate considerably from the experimental results for high frequency range but this deviation is reduced for the far field approach. As shown in fig. 4.14 and fig. 4.18 the drift forces are negative which means the vessel drift against the incoming waves. Ohkusu also observes the similar trends for certain frequencies.

Table 1

<table>
<thead>
<tr>
<th>Specification</th>
<th>FPSO (m)</th>
<th>LNG (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (L)</td>
<td>295</td>
<td>260.3</td>
</tr>
<tr>
<td>Breadth (B)</td>
<td>60.0</td>
<td>46.10</td>
</tr>
<tr>
<td>Depth (D)</td>
<td>25.0</td>
<td>(46.5)</td>
</tr>
<tr>
<td>Draft (T)</td>
<td>8.5</td>
<td>11.6</td>
</tr>
<tr>
<td>Displacement (W)</td>
<td>143,845</td>
<td>97,960</td>
</tr>
<tr>
<td>Center of Gravity from</td>
<td>0.75</td>
<td>-1.94</td>
</tr>
<tr>
<td>Center of Gravity above</td>
<td>18.16</td>
<td>17.78</td>
</tr>
<tr>
<td>Metacentric height (GM)</td>
<td>23.24</td>
<td>3.21</td>
</tr>
<tr>
<td>Transverse radius of</td>
<td>19.80</td>
<td>17.09</td>
</tr>
<tr>
<td>gyration ($K_{mg}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal radius of</td>
<td>82.60</td>
<td>64.79</td>
</tr>
<tr>
<td>gyration ($K_{mg}$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4.1: Meshing and arrangement of parallely connected FPSO and LNG carrier.
Fig. 4.2: Surge motion of LNG (wave heading 180°)

Fig. 4.3: Sway motion of LNG (wave heading 180°)

Fig. 4.4: Heave motion of LNG (wave heading 180°)

Fig. 4.5: Roll motion of LNG carrier (wave heading 180°)

Fig. 4.6: Pitch motion of LNG carrier (wave heading 180°)

Fig. 4.7: Yaw motion of LNG carrier (wave heading 180°)

Fig. 4.8: Surge motion of LNG carrier (wave heading 270°)

Fig. 4.9: Sway motion of LNG carrier (wave heading 270°)
A STUDY ON MOTIONS AND DRIFT FORCES OF A MULTIBODY FLOATING SYSTEM IN WAVES

Fig. 4.10: Heave motion of LNG carrier (wave heading 270°)

Fig. 4.11: Roll motion of LNG carrier (wave heading 270°)

Fig. 4.12: Pitch motion of LNG carrier (wave heading 270°)

Fig. 4.13: Yaw motion of LNG carrier (wave heading 270°)

Fig. 4.14: Surge drift forces on LNG carrier (wave heading 180°)

Fig. 4.15: Sway drift forces on LNG carrier (wave heading 180°)

Fig. 4.16: Yaw drift moments on LNG carrier (wave heading 180°)

Fig. 4.17: Surge drift force on LNG (wave heading 270°)
5. Conclusion

3-D source technique is applied for computations of the first order motion and the second order drift forces in regular waves. The validation of the mean wave drift forces are studied by comparing the results obtained from near field and far field methods with those of experimental ones. It is shown that the hydrodynamic interaction effects are significant for the wave heading angle of 270°.

It can be stated that the first order motion and the second order drift forces can be predicted with fair accuracy by means of 3-D source technique and, the effect of viscous roll damping is significant for the case of motion responses as well as for the second order drift forces.

In general, the results obtained from the near field method, the far field method and the experiments are comparatively well in agreement. However, the results obtained from both of the numerical methods show that the far-field method predicts better agreement with the experimental ones. Although the direct pressure integration method has the advantage of easy resolution in to components of the drift forces, it is very sensitive to deal with the numerical derivative.

Acknowledgement

This research was supported by the grant-in-aid for Scientific Research of the Ministry of Education, Science, Sports and Culture (Monbusho), Japan.

References


Fig. 4.18 Sway drift force on LNG (wave heading 270°)

Fig. 4.19 Yaw drift moments on LNG (wave heading 270°)