A QUASI-FIXED POINT OF INDIVIDUAL DISTANCE-VELOCITY CURVE OF HEIGHT DURING THE CHILDHOOD

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SUMMARY

The logistic model (1) is applicable to the individual height growth curve during the childhood, provided that his (or her) curve is well linearized by our method. Then by a slight modification of our approach (see Box) we can find the explicit relation between two parameters K and H .

The almost-one parameter hypothesis is slightly modified and shown in the Box on the next page.

Let $h(t)$ be the mean height at age t during the childhood. Then according to the logistic model, we have

$$(1) \quad h(t) = H/[1 + \exp\{-K(t - t_0)\}],$$

where H , K and t_0 are three positive constants. Then d being a given constant, we obtain a difference equation

$$(2) \quad Y_t = AX_t + B,$$

where we set $Y_t = 1/h(t + d)$, $X_t = 1/h(t)$, $A = \exp(-Kd)$, and $B = (1 - A)/H$.

The correlation coefficient (C.C.) r between X_t and Y_t is higher, if the size of sample n is larger, i.e. $r = 0.9$ for $n = 10^1$, $r = 0.99$ for $n = 10^2$ and $r = 0.999$ for $n = 10^3$ respectively [3].

When we applied this model to individual data, we found that, if the corresponding linearized growth curve contained one or more largely deviated points, at least one of the following inequalities were violated:

$$(3) \quad A > 0, \quad B > 0, \quad \text{and} \quad 1 < H(m) < 4.$$

There were only four exceptions among ca. 200 cases, i.e. the linearity holds but H was greater than $4(m)$. But their inclusion and exclusion do not influence very much the over-all estimates, so that these four cases are included in the following estimation.

According to our hypothesis, H and K are not independent, as shown in the references [3,4,5].

To find out their explicit relation, let us set

$$(4) \quad X_i(t) = d \ln h_i(t)/dt, \quad \text{and} \quad Y(t) = h_i(t)$$

for the i -th individual, to have a linear relation

$$(5) \quad X_i(t) = a_i + b_i Y(t)$$

with $a_i = K_i$ and $b_i = -K_i/H_i$.

Then C.C. between a_i and b_i equals almost one (v.i.), so that we obtain the following required relation [6,7]:

$$(6) \quad K_i = C + (K_i/H_i)D, \quad (C \text{ and } D \text{ are constants.})$$

Key words: Almost-one Parameter Hypothesis; Growth Curve; Logistic Model;
Linearized Growth Curve.

THE ALMOST-ONE PARAMETER HYPOTHESIS

THE BIOLOGICAL PROCESS $x(t)$ IS PRIMITIVE, OR FUNDAMENTAL FOR THE LIFE.

THE INDIVIDUAL VARIABILITY OF $x_i(t)$ FOR $i = 1, 2, \dots, N$ IS SUFFICIENTLY SMALL, WHERE THE VARIABILITY IS REPRESENTED BY A DIMENSIONLESS MEASURE. N NEED NOT BE SUFFICIENTLY LARGE.

THE i -TH INDIVIDUAL PROCESS IS A LINEAR FUNCTION OF THE REFERENCE PROCESS $Y(t)$, I.E.

$$x_i(t) = a_i + b_i Y(t) \dots (I)$$

IF WE TAKE $x_i(t)$ AS $Y(t)$, WE HAVE

$$a_i = 0, \text{ AND } b_i = 1.$$

THE DOT STANDS FOR THE MEAN WITH RESPECT TO i .

THE STANDARD DEVIATION OF $x_i(t)$ WITH RESPECT TO i , SAY $\sigma_x(t)$, IS A LINEAR FUNCTION OF $Y(t)$, NAMELY

$$\sigma_x(t) = \pm [\sigma_a + \sigma_b Y(t)] \dots (II),$$

WHERE σ_a AND σ_b DENOTE THE STANDARD DEVIATIONS OF a_i AND b_i RESPECTIVELY.

IN REALITY, THESE FUNDAMENTAL RELATIONS (I), (II), AND (III) HOLD APPROXIMATELY, SO THAT WE SAY 'ALMOST-ONE PARAMETER' INSTEAD OF 'ONE PARAMETER', AND 'QUASI-FIXED' INSTEAD OF 'FIXED'.

EXAMPLE. THE HEIGHT $h(t)$ AT AGE t DURING THE ADOLESCENT PERIOD, $CV_{est} = 3 \sim 4\%$, $\rho_{est} = -0.997 \sim -0.99999$.

(IF NECESSARY, $x(t)$ IS TRANSFORMED IN ADVANCE INTO A NEW VARIATE, WHICH IS DENOTED AGAIN BY $x(t)$.)

ONE (GENETIC) FACTOR IS PREDOMINANT IN THE PROCESS.

PARAMETERS CONTAINED IN THE EXPLICIT FORM OF $x_i(t)$ ARE MUTUALLY DEPENDENT.

THE CORRELATION COEFFICIENT ρ BETWEEN a_i AND b_i EQUALS -1 .

THERE EXISTS TWO POSITIVE CONSTANTS C AND D SUCH THAT

$$a_i = C - Db_i \dots (III)$$

THE TRAJECTORY OF $x_i(t)$ PASSES A FIXED POINT (D, C) ON THE $Y-x_i$ PLANE, WHEN t IS A CONTINUOUS PARAMETER.

DUALLY SPEAKING, POINTS (b_i, a_i) ARE ON A FIXED STRAIGHT LINE

$$v = C - Du$$

ON THE $u-v$ PLANE.

or

$$(7) \quad (K_i - C)(H_i - D) = CD.$$

Clearly the relation (III) does not hold in this case.

The C.C. between K_i and K_i/H_i is equal to 0.99 for $n = 59$ boys and again 0.99 for $n = 54$ girls.

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