Simulation of the Relationship between the Current Pattern and Particle Distribution in Electrical Resistance Tomography

JeEun CHOI**† Masahiro TAKEI** DeogHee DOH***

Abstract In the present study, cross-sectional images of the assumed particle distribution of a particle-liquid two-phase flow are simulated using a newly proposed algorithm and electrical resistance tomography (ERT) with four current patterns. The first step of the algorithm is a forward problem, which involves calculating the electrical potential on electrodes from the assumed particle distribution by solving the Maxwell equation. The second step of the algorithm is an inverse problem, which involves calculating the particle distribution from the electrical voltage calculated by the forward problem. In the simulation conditions of the four current patterns, namely, opposite, adjacent, one-reference, and trigonometric methods are used to discuss the most accurate current patterns in three types of particle-liquid two-phase flow, namely, annular, stratified, and dispersed flows. The simulation results reveal that the current pattern depends on the flow conditions. In particular, among the four current patterns, the trigonometric method is found to be preferable.

Keywords: Multiphase flow, Trigonometric method, Electrical resistance tomography

1. Introduction

Electrical resistance tomography (ERT) can be applied to the visualization of the resistive particle concentration distribution on a vertical cross-section of the particle and conductive liquid two-phase flow. Electrical resistance tomography has obvious advantages in industrial applications involving the visualization of particle-liquid two-phase flow because ERT has been used successfully to predict the particle distribution and flow regimes in particle-liquid two-phase flows [1]. Williams et al. have shown that ERT application can clarify the particle concentration and structure in a particle-liquid two-phase flow [2]. Another ERT

application was constructed for the visualization of the dispersion states of particle-liquid two-phase flow in a chemical reaction [3].

The ERT method consists of two steps, namely, calculation of the transfer resistance (ratio of electrical potential difference to injected electrical current) on the electrodes and reconstruction of the particle distribution images. The first step is performed by injecting an electrical current between electrode pairs and by measuring the electrical potential difference between the electrodes to calculate the transfer resistance on the electrodes, which is referred to as a forward problem. The forward problem measures the electrical potentials

* 2010.1.12 受付

** Mechanical Engineering Department, Nihon University, 101-8308 1-8-14 Kanda Surugadai Chiyoda-ku Tokyo
TEL: (03) 3259-0749 FAX: (03) 3259-0749 E-mail: cje_0205@hotmail.com

*** Division of Mechanical and Information Engineering, Korea Maritime University, KOREA

for a given particle distribution and electrical current by solving the Maxwell equation. The second step is to reconstruct the particle distribution images, based on the transfer resistance data using an algorithm, which is referred to as an inverse problem. The inverse problem involves calculating the particle distribution from the transfer resistance on the electrodes, which is iteratively adjusted until the calculated and transfer resistance agree to within some predetermined value [4]. Recently, ERT methods for various injected electrical current patterns have been proposed. Webster proposed the opposite, adjacent, one-reference, and trigonometric methods in the electrical current pattern [5]. The reconstruction of the particle distribution is depending on the injected electrical current patterns in the particle-liquid two-phase flow types [6]. For instance, among the current patterns, Hua et al. used the opposite method of data collection in particle-liquid two-phase flow [7]. The electrical current was injected through diametrically opposed electrodes. Brown and Seagar suggested the adjacent (neighboring) method, whereby the current is applied through neighboring electrodes and the electrical voltage is measured successively through all other adjacent electrode pairs in particle-liquid two-phase flow [8]. This method does not yield good sensitivity at the center because the electrical current density at the center is low. Hu et al. used the one-reference method of data collection of the two-phase flow [9], and the trigonometric method was used for the simulation of the two-phase flow [10, 11]. However, the results reported in these papers cannot be compared because there are no common conditions between these papers. A common feature of these papers is that the authors have studied only one or two conditions with different models and have used different reconstruction algorithms. Therefore, there is no clear insight available on the relative results between these electrical current patterns and particle-liquid two-phase flow based upon which to discuss the effect of current pattern.

In the present study, the cross-sectional images of the assumed particle distribution of the particle-liquid two-phase flow are simulated using ERT with four

electrical current patterns. The four electrical current patterns are used to discuss the simulation results for three types of particle-liquid two-phase flow. The four electrical current patterns are the opposite, adjacent, one-reference, and trigonometric methods. In addition, three types of particle-liquid two-phase flow are considered, namely, annular, stratified and dispersed flows.

2. Theory of Simulation

2.1 Governing equation with simulation algorithm

The simulation algorithm used to convert the electrical potential at the electrical field to a particle distribution is described in this section. The simulation algorithm consists of a forward problem and an inverse problem. The simulation algorithm for the ERT forward problem and inverse problem is shown in Fig. 1. The forward problem calculates the electrical voltage by the assumed particle distribution with the given electrical current. The forward problem can be derived from the Maxwell equations:

$$\nabla \cdot \left(\frac{1}{\rho} \nabla u \right) = 0, x \in \Omega \quad (1)$$

where Ω is the electrical field, ρ is the resistivity of particle distribution, and u is the electrical potential in the electrical field. The inverse problem calculates the resistivity of particle distribution with reconstruction algorithm. The reconstruction algorithm is used to calculate the Jacobian matrix $\mathbf{J}$ and the electrical voltage U in the forward problem. The Jacobian matrix is a sensitivity matrix that linearly links the resistivity of particle distribution and the electrical voltage. The Jacobian matrix is denoted as follows:

$$\mathbf{J} = \begin{pmatrix} \frac{\partial U_1}{\partial \rho_1} & \dots & \frac{\partial U_1}{\partial \rho_N} \\ \frac{\partial U_2}{\partial \rho_1} & \dots & \frac{\partial U_2}{\partial \rho_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_K}{\partial \rho_1} & \dots & \frac{\partial U_K}{\partial \rho_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial U_L}{\partial \rho_1} & \dots & \frac{\partial U_L}{\partial \rho_N} \end{pmatrix} \quad (2)$$

where L is the electrode number, K is the current pattern number and N is the element number. The Jacobian elements are determined by calculating each

particle distribution element sequentially and then calculating the corresponding change in each electrode. This must be done for each electrical current pattern and for all elements. By dividing the change in the electrical voltages by the change in the resistivity of particle distribution, a difference approximation for $\partial U_L^K / \partial \rho_N$ is obtained. Jacobian matrix and the electrical voltage can be written in the following form:

$$\delta U \approx \mathbf{J} \delta \rho \quad (3)$$

where $\delta U = U(\rho) - U(\rho_0)$ and $\delta \rho = \rho - \rho_0$.

Fig.1 ERT forward and inverse problem

2.2 Forward problem and the electrode model

When electrical currents I are injected to the electrical field through the electrodes on the boundary electrical field, and the particle distribution over the electrical field is known. The electrical potential on the electrical field can be determined uniquely from a partial differential equation, which can be derived from the boundary conditions. The boundary condition is considered to be given by the complete electrode model (CEM), which takes into account the discrete electrodes and the effects of the contact resistance on the electrodes [12,13]. The

CEM consists of Maxwell equation and the boundary conditions are obtained as:

$$u + z_l \frac{1}{\rho} \frac{\partial u}{\partial \nu} = U_l, x \in e_l, l = 1, 2, \dots, L \quad (4)$$

$$\int_{e_l} \frac{1}{\rho} \frac{\partial u}{\partial \nu} dS = I_l, x \in e_l, l = 1, 2, 3, \dots, L \quad (5)$$

$$\frac{1}{\rho} \frac{\partial u}{\partial \nu} = 0, x \in \partial \Omega \setminus \bigcup_{l=0}^L e_l \quad (6)$$

where z_l is the contact resistance between the l th electrode, U_l is the electrical voltage on the l th electrode, e_l is the l th electrode, ν is the normal vector, and L is the electrode number. dS is the electrode boundary and I_l is the electrical current on the l th electrode. $\bigcup_{l=0}^L e_l$ is the notation in the union of the collection. In addition, the following two conditions for the given electrical current and the calculated electrical voltages are needed in order to ensure the uniqueness of the solution:

$$\sum_{i=1}^L I_i = 0 \quad \text{and} \quad \sum_{i=1}^L U_i = 0 \quad (7)$$

In general, the forward problem cannot be calculated analytically for an arbitrary particle distribution. Therefore, the forward problem is need to a numerical method. In the present study, the Finite Element Method (FEM) is used to obtain a numerical solution. In the FEM, the object area is discretized into sufficiently small elements and the electrical voltage within each element is calculated [14].

2.3 Inverse problem

Mathematically, the image reconstruction algorithm is an ill-posed inverse problem [15]. With the aid of boundary conditions and the FEM used in the forward problem, the equation can be written as

$$U_i = U_f(\rho, I) = R(\rho)I \quad (8)$$

where U_i are the given electrical voltages on the electrodes in the inverse problem, $U_f(\rho, I)$ is the calculated electrical voltage between the particle distribution and the given electrical current in the forward problem, and $R(\rho)$ is the part of the Jacobian matrix that was used in the FEM calculations. The ERT image reconstruction algorithm was developed in order to find the particle distribution inside an

electrical field when the calculated electrical voltage and the injected currents are known. Iterative methods that are used to solve the resistivity by the Levenberg-Marquardt algorithm (LMA). The LMA is based primarily on the Gauss-Newton algorithm (GNA). The LMA interpolates between the GNA and the method of gradient descent. The LMA is more robust than the GNA, which means that in many cases it finds a solution even if it starts very far off the final minimum [16,17]. The LMA is

$$\rho^{k+1} = \rho^k + \delta\rho^k \quad (9)$$

$$\delta\rho^k = \left[(\mathbf{J}^k)^T \mathbf{J}^k + \alpha \mathbf{I} \right]^{-1} \left[(\mathbf{J}^k)^T (U_i - U_f(\rho)) \right] \quad (10)$$

and k is the iteration number. In these cases, the resistivity of particle distribution is calculated by the iteration of Jacobian with the calculated electrical voltage in the forward problem. The iteration of the Jacobian is only computed implicitly. The reconstructed image has the high quality after several iteration steps; the maximum number of iterations is set to five.

3. Simulation Conditions of the Current Pattern and the Two-phase Flow

3.1. Current pattern condition

Most ERT systems are designed such that electrical current injection and given electrical voltages. The current is injected into the body through electrodes. Fig.2 shows a cross-section of the 2 dimensional mesh and electrode positions in the simulation model. This model has 16 electrodes, which are indicated in green. An electrical current source requires switches that choose electrodes are used for the electrical current injection. Fig.3 shows the element and the node in the 2 dimensional mesh of simulation model. The simulation model has 492 elements and 1,049 nodes. The electrical current patterns were introduced in the four types method.

(1) Opposite method

Fig.4(a) shows the opposite method. The electrical current is injected through diametrically opposed electrode pairs. The electrode adjacent to the electrical current injecting electrode is used as the electrical voltage reference. The electrical voltage is

calculated for all other electrodes except the current electrodes (electrodes 1 and 9), for a total of 13 electrical voltage calculations. The next set of 13 electrical voltage calculations is performed after selecting electrodes 2 and 10 as the electrical current electrodes. When 16 electrodes are used, the opposite method yields $8 \times 13 = 104$ data points.

(2) Adjacent method

Fig.4(b) shows the adjacent method. The electrical current is injected through two adjacent electrodes, and the electrical voltage differences between the electrodes are calculated for all other electrode pairs. The electrical current is first applied through electrodes 1 and 2. The electrical voltage is calculated for electrode pairs 3-4, 4-5, ..., 15-16. This is repeated for all of the electrodes. The adjacent method involves $16 \times 13 = 208$ electrical voltage calculations.

(3) One-reference method

Fig.4(c) shows the one-reference method. The electrical current is first applied through electrodes 1 and 2. The electrical voltage is calculated successively with electrode pairs 3-4, 4-5, ..., 15-16. The next set of 13 electrical voltage calculations is obtained by feeding the electrical current through electrodes 2 and 3. For a 16-electrode system, obtaining the one-reference method involves $16 \times 13 = 208$ electrical voltage calculations.

(4) Trigonometric method

Fig.4(d) shows the trigonometric method. The trigonometric method is

$$I_l^K = \begin{cases} \cos(K\zeta_l) & l=1, \dots, L \quad K=1, \dots, L/2 \\ \sin((K-L/2)\zeta_l) & l=1, \dots, L \quad K=L/2, \dots, L-1 \end{cases} \quad (11)$$

where $\zeta_l = 2\pi l/L$, K is spatial frequency, and L is the electrodes number. The electrical current injects the result of Eq. (11) for each electrode. The electrical voltage is calculated from all of the electrodes except for the electrical current injection electrode. The electrical voltage calculation involves fifteen electrode pairs, so that obtaining the trigonometric method involves $15 \times 16 = 240$ electrical voltage calculations.

Fig.2 Cross-section of mesh and electrode positions

Fig.3 Mesh of element and nodes

Fig.4 Current patterns

3.2. Particle dispersion patterns

In order to consider the electrical current patterns, three types of particle-liquid two-phase flow are simulated. First, the annular flow has three cases by different thickness r , the cases are $r=1$, $r=3$, and $r=5$, as shown in Fig.5(a). In the annular flow, the particles are marked from cross-section around. In the Fig.5, a green indicates the particle distribution, and a white indicates the liquid distribution. Second, the stratified flow has three cases by different height h of the particle distribution from the cross-section bottom, the cases are $h=3$, $h=8$, and $h=12$, as shown in Fig.5(b). Finally, the dispersed flow has three cases by different pixel number n of the arbitrary dispersed particle distribution, the cases are $n=40$, $n=120$ and $n=200$. In the case of the dispersed flow, the arbitrary

dispersed particle distributions grow bigger and bigger in the case $n=40$, as shown in Fig.5(c).

Fig.5 Simulation conditions for the particle-liquid two-phase flow

3.3. Simulation Conditions

The particle distribution is assumed in the particle-liquid two-phase flow in the section 3.2. The resistivity of the assumed particle distribution was $200\Omega\cdot m$, and the resistivity of the assumed liquid distribution was $400\Omega\cdot m$. The electrical current was injected from $-1A$ to $1A$ in each electrical current pattern. The reconstructed particle distribution is simulated for each electrical current pattern and each particle-liquid two-phase flow condition based on the forward problem and inverse problem. The correlation coefficients of the images are compared based on a correlation function, which is given as follows:

$$C = \frac{\sum_{i=1}^N [\rho_i^k - \bar{\rho}^k] (\rho_i^0 - \bar{\rho}^0)}{\sum_{i=1}^N \sqrt{(\rho_i^k - \bar{\rho}^k)^2} \sum_{i=1}^N \sqrt{(\rho_i^0 - \bar{\rho}^0)^2}} \quad (12)$$

where N is the element number, k is the iterative number, the overbar $\bar{\cdot}$ indicates the spatial average for the resistivity, ρ_i^k is the particle resistivity after k

iterative number and i th element, and ρ_i^0 is the resistivity of assumed particle distribution in i th element, as shown in Fig.5.

4. Simulation Results

4.1. Electrical voltage by the forward problem

The electrical voltage is calculated for the assumed particle distribution with electrical current. The electrical current is used by the trigonometric method among the electrical current patterns. Fig.6 shows the electrical voltage results for the particle-liquid two-phase flow in the case of trigonometric method as one example. The electrical current is injected through 16 electrodes. The electrical voltage is

calculated for 15 electrode pairs. The next set of 15 electrical voltage calculations is obtained by each electrode pairs. In the Fig.6, the x -axis shows the fifteen electrical current number, the y -axis shows the sixteen electrode number, the z -axis shows the electrical voltage. The small occupation ratio of particles in the cases of $r=1$, $h=3$ and $n=40$ shows high electrical voltage than the large occupation ratio of particle according to Ohm's law, because the small particle occupation ratio means large liquid occupation ratio. The inverse problem is used to calculate the particle distribution from the electrical voltage obtained from the forward problem with the reconstruction algorithm.

Fig.6 Electrical voltage results for a particle-liquid two-phase flow in the case of trigonometric method

4.2. Image reconstruction for a two-phase flow by the inverse problem

The annular, stratified, and dispersed flows were simulated using the four current patterns. Firstly, the simulation results of annular flow are shown in Fig.7. In the figure, red color indicates the high resistivity of the liquid distribution, and blue color indicates the low resistivity of particle distribution. The yellow and the green parts are distributed around the boundary between the particle and the liquid which is an inevitable error generated by the inverse problem. Fig.8 shows the correlation coefficient for the annular flow obtained by Eq. (12). The correlation coefficient is increased as the pixel thickness is increased in the case of opposite method. The other three current patterns indicate the highest correlation coefficient in $r=3$. The correlation coefficient is decreased as the particle pixel is very thick in the annular flow. In the case of $r=5$, all electrical current patterns have almost the same correlation coefficients. The trigonometric method has the highest correlation coefficient among the four electrical current patterns. This can be explained by the reason that the trigonometric method uses all electrodes for the electrical current injection and the electrical voltage calculation, while the electrical voltages are calculated only on noncurrent-carrying electrodes in the other electrical current injection methods. The trigonometric method is slightly less sensitive to errors than the other electrical current method.

Secondly, the simulation results for the stratified flow are shown in Fig.9. Fig.10 shows the correlation coefficient for the stratified flow. The stratified flow has a high correlation coefficient in each electrical current pattern. It is difficult to discriminate among the four electrical current patterns. It means ERT measurement technique takes an advantage for the stratified flow. The simulation results reveal that the electrical current pattern with these modeling errors can be partly compensated.

Finally, Fig.11 shows the simulation results for the dispersed flow. Fig.12 shows the correlation coefficients for the dispersed flow. The correlation coefficient is increased as the pixel number is increased in the dispersed flow. Among the electrical

current patterns, trigonometric method has the highest correlation coefficient in all electrical current patterns. In order to discuss the relationship between the pixel number and the correlation coefficient, the correlation coefficients in the case of the simple pixel number from $n=1$ to $n=6$ are calculated in the case of trigonometric method as an example. Fig.13 shows the simulation results in the representative case of $n=1$, $n=3$ and $n=6$. In the case of $n=1$, the particle distribution is blurred in the liquid distribution.

Fig.7 Simulation results for the annular flow

Fig.8 Correlation coefficient for the annular flow

Fig.9 Simulation results for the stratified flow

Fig.10 Correlation coefficient for the stratified flow

The particle distribution is more clarified as the simple pixel number is increased in the liquid distribution. Fig.14 shows the correlation coefficient for the simple pixel number in Fig.13. The correlation coefficient is increased as the simple pixel number is increased in the trigonometric method. Fig.15 shows the resistivity for in each element in the representative cases of $n=1$, $n=3$ and $n=6$. The element number 1 means the center element, the higher number is the near wall in a clockwise direction. The black circle indicates the resistivity of particle distribution including particle boundaries.

Fig.11 Simulation results for the dispersed flow

Fig.12 Correlation coefficient for the dispersed flow

The red circle indicates the resistivity of liquid distribution. The resistivity of particle distribution is decreased as the pixel number is increased. It means that the resistivity is closed to the assumed particle resistivity $200\Omega\cdot\text{m}$ as the pixel number is increased in the simulation conditions. The calculated resistivity is close to the assumed resistivity in the case the simple pixel number is high. Therefore, the correlation coefficient of dispersed flow is increased as the pixel number is increased.

Fig.13 Simulation results for the simple pixel number in trigonometric method

Fig.14 Correlation coefficient for the simple pixel number in trigonometric method

Fig.15 Resistivity in the element for the simple pixel number

5. Conclusions

This study simulates particle distributions of particle-liquid two-phase flows by using electrical resistance tomography with four electrical current patterns. Three types of particle-liquid two-phase flow are considered, namely, annular, stratified and dispersed flows. The four current patterns are opposite, adjacent, one-reference and trigonometric methods.

In the annular flow, the correlation coefficient is decreased in the very thick particle distribution. The stratified flow has high accuracy of the simulated

particle distribution. In the dispersed flow, the correlation coefficient is increased as the pixel number is increased. The calculated particle resistivity is closed to the assumed particle resistivity $200\Omega\cdot\text{m}$ as the pixel number is increased.

The simulation results revealed that the electrical current pattern depends on the flow conditions. The trigonometric method is preferable among the four current patterns.

Acknowledgements

The present study was supported in part by the Academic Frontier Promotion Program of the Japan Ministry of Education, Culture, Sports, Science and Technology and by a Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science and the Academic Frontier Promotion Program of MEXT Japan (#21360088). Additional support was provided by the National Research Lab of Korea Science & Engineering Foundation (R0A-2008-000-20069-0).

Nomenclature

C	: correlation coefficient	[-]
I	: electrical current	[A]
K	: current pattern	[-]
L	: electrode number	[-]
N	: element number	[-]
R	: resistivity	[$\Omega\cdot\text{m}$]
S	: boundary of the electrode	[-]
u	: electrical potential	[-]
U	: electrical voltage	[V]
ω	: angular frequency	[Hz]
z	: contact resistance	[Ω]

Greek letters

a	: coefficient	[-]
ρ	: resistivity column vector	[$\Omega\cdot\text{m}$]
ρ^0	: resistivity column vector in water	[$\Omega\cdot\text{m}$]
Ω	: electrical field	[F/m]
σ	: conductivity	[S/m]
ν	: normal vector	[-]

Subscripts

k	: iteration number	[-]
T	: transpose matrix	[-]
l	: l^{th} electrode	[-]

References

- [1] Dong, F., Jiang, Z.X., Qiao, X.T. and Xu, L.A., Application of Electrical Resistance Tomography to Two-Phase Pipe Flow Parameters Measurement, *Flow Measurement and Instrumentation*, Vol. 14(4), 183–192 (2003).
- [2] Williams, R.A. and Jia, X., Tomographic Imaging of Particulate Systems, *Advanced Powder Technology*, Vol. 14(1), 1–16 (2003).
- [3] Kaminoyama, M., Taguchi, S., Misumi, R. and Nishi, K., Monitoring Stability of Reaction and Dispersion States in a Suspension Polymerization Reactor Using Electrical Resistance Tomography Measurements, *Chemical Engineering Science*, Vol. 60(20), 5513–5518 (2005).
- [4] William, D. and Abelardo, R., Electrical Resistance Tomography, *Society of Exploration Geophysicists*, Vol. 23(5), 438–442 (2004).
- [5] Webster, J.G., *Electrical Impedance Tomography*, Adams Hilger, Bristol, 97–135 (1990).
- [6] Kaipio, J.P., Seppänen, A., Somersalo, E. and Harrio, H., Posterior Covariance Related Optimal Current Patterns in Electrical Impedance Tomography, *Inverse Problems*, Vol. 20(3), 919–936 (2004).
- [7] Hua, P., Webster, J.G. and Tompkins, W.J., Effect of the Measurement Method on Noise Handling and Image Quality of EIT Imaging, *Proceedings of the 9th International Conference IEEE Engineering in Medicine and Biology Society*, Vol. 2, 1429–1430 (1987).
- [8] Barber, D.C. and Seagar, A.D., Fast Reconstruction of Resistance Images, *Clinical Physics and Physiological Measurement*, Vol. 8(A), 47–54 (1987).
- [9] Hu, M.Q., Hua, P. and Webster, J.G., Measurement Error in An Electrical Impedance Tomography System, *Proceedings of the 9th International Conference IEEE Engineering in Medicine and Biology Society*, Vol. 9, 1425–1426 (1987).
- [10] Hua, P., Iterative Reconstruction Methods Using Regularization and Optimal Current Patterns in Electrical Impedance Tomography, *IEEE Transactions on Biomedical Engineering*, Vol. 10(4), 621 (1991).
- [11] Kim, K.Y., Kim, B.S., Kim, M.C., Lee, K.J., Ko, Y.J. and Kim, S., Electrical Impedance Imaging of Two-Phase Flows Undergoing Rapid Transient Part II. Effect of input current pattern, *Heat and Mass Transfer*, Vol. 32(5), 648–656 (2005).
- [12] Vauhkonen, M., Vadasz, D., Karjalainen, P.A., Somersalo, E. and Kaipio, J.P., Tikhonov Regularization and Prior Information in Electrical Impedance Tomography, *IEEE Transactions on Medical Imaging*, Vol. 17(2), 285–293 (1998).
- [13] Vauhkonen, P.J., Vauhkonen, M., Savolainen, T. and Kaipio, J.P., Three-dimensional Electrical Impedance Tomography Based on the Complete Electrode Model, *IEEE Transactions on Biomedical Engineering*, Vol. 46(9), 1150–1160 (1999).
- [14] Vauhkonen, M., Kaipio, J.P., Somersalo, E. and Karjalainen, P.A., Electrical Impedance Tomography with Basis Constraints, *Inverse Problems*, Vol. 13(2), 523–530 (1997).
- [15] Lionheart, W.R.B., EIT Reconstruction Algorithms: Pitfalls, Challenges and Recent Developments, *Physiological Measurement*, Vol. 25(1), 125–132 (2004).
- [16] Levenberg, K., A Method for the Solution of Certain Problems in Least Squares, *Quarterly of Applied Mathematics*, Vol. 2, 164–168 (1944).
- [17] Marquardt, D., An Algorithm for Least-squares Estimation of Nonlinear Parameters, *SIAM Journal on Applied Mathematics*, Vol. 11(2), 431–441 (1963).