MINCSコードによる二流体数値ベンチマーク問題の計算^{*}

Application of MINCS code to Numerical Benchmark Problems for Two-Fluid Model

平野 雅司^{**} 渡辺 正^{**} 田辺 文也^{**} 大崎 浩^{***}
 HIRANO Masashi WATANABE Tadashi TANABE Fumiya OSAKI Hiroshi

井上 英明^{***} 加茂 英樹^{***} 秋元 正幸^{**}
 INOUE Hideaki KAMO Hideki AKIMOTO Masayuki

ABSTRACT The one-dimensional transient two-phase flow analyzer MINCS has been developed as a computational tool for the development and assessment of constitutive models. In order to assess the numerics of MINCS, it has been applied to the numerical benchmark problems for the two-fluid model proposed by the Workshop on Two-Phase Flow Fundamentals under the auspices of the US Department of Energy and the Electric Power Research Institute.

Although the adequacy of constitutive equations has primary importance compared with the accuracy of numerics in two-phase flow simulation, numerical stability is required for computer codes in order to assess the adequacy of constitutive equations. Even with such a stable numerical solution method, difficulties related to the numerics often appear in a complex manner coupled with those of constitutive relations in, for example, safety analyses of light water reactors. Therefore, it is worthwhile to investigate each numerical difficulty by isolating it from physical problems. In this report, the benchmark calculations by MINCS are discussed focusing on the numerical treatment of the missing phase in a single-phase flow and spatial and/or temporal transitions from a single phase to two-phase and vice versa. The selected problems for this purpose are the disappearance of liquid droplets due to evaporation, inception of boiling due to external heating, expulsion of steam by subcooled water and sedimentation. A systematic numerical treatment of missing phase mentioned above and the transition has been developed and assessed through the benchmark calculations.

Keywords : Phase Transition, Safety Analyses, Transient Two-Phase Flow Analyzer
 MINCS, Constitutive Equations, Numerics, Numerical Benchmark Problems

1. INTRODUCTION

A large number of transient two-phase flow calculations with two-fluid models have been performed related to safety evaluation of LWRs (Light Water Reactors) using large computer codes such as RELAP5/MOD2⁽¹⁾ and TRAC-PF1⁽²⁾. In these computer codes, six conservation equations of mass, momentum and energy for each phase are solved, taking into account the mass, momentum

* 1987.6.20 受理
 ** 日本原子力研究所
 *** 日本情報サービス(株)

and energy exchanges at the interface and the momentum and energy exchanges at the walls of structures. For the development of these safety analysis codes, the adequacy of constitutive equations has primary importance, compared with the accuracy of numerics, for simulating complex two-phase flow phenomena, because the uncertainty associated with constitutive equations dominates numerical errors in most cases.

From this point of view, the transient two-phase flow analyzer mimics⁽³⁾⁻⁽⁷⁾ has been developed at JAERI (Japan Atomic Energy Research Institute) as a computational tool for the development and assessment of various constitutive equations mainly through analyses of separate effect tests. As to the numerics of the code, a fully implicit first-order non-linear scheme with a first-order staggered-mesh donor-cell method is applied in MINCS, which is in general considered to be a stability assuring method for solutions. Even with such a stable numerical solution method, there still exist various numerical difficulties coupled with constitutive equations, which should be overcome to accomplish its role.

In order to investigate the differences in numerics within the framework of the two-fluid model, numerical benchmark problems have been proposed by the Workshop on Two-phase Flow Fundamentals⁽⁸⁾ under the auspices of US DOE (Department of Energy) and EPRI (Electric Power Research Institute). The above objective is to be accomplished by comparing the solutions supplied by different participants using different numerics but with the specified constitutive relations. Since MINCS is designed so as to easily change or modify the constitutive equations for its purpose as a computational tool, the present benchmark problems can be easily handled with MINCS. It has, therefore, been applied to the benchmark problems to contribute to the objective of the workshop as well as to assess the numerics of the code.

Besides comparing the solutions, the focal point of our interest is the numerical treatment of the missing phase in a single phase flow and phase transition, which means a spatial and/or temporal transition from a single phase to two-phase and vice versa. The phenomena involved in the phase transition are discontinuous in nature, and introduce difficulties in the numerical simulation. Different remedies are usually resorted depending on the numerics applied. Since MINCS uses fully implicit non-linear scheme solved by the Newton-Raphson iteration, these discontinuities often bring about serious problems. A systematic numerical treatment of the missing phase and the phase transition has been developed and assessed through the benchmark calculations.

2. NUMERICS IN MINCS

2. 1 Basic Field Equations

The basic field equations for the two-fluid nonequilibrium model consist of mass, momentum and energy equations for each phase⁽³⁾⁻⁽⁶⁾. The equations are presented here in the form of the one dimensional stream tube with slowly varying cross sectional area and an assumption is made that the distribution parameters are equal to unity, which appears in the time and volume averaging. The field equations for phase k (k = g for gas and k = l for liquid) are

$$A(\partial/\partial t)(\alpha_k \rho_k) + (\partial/\partial z)(\alpha_k \rho_k U_k A) = A\Gamma_{ik} \quad (1)$$

$$\begin{aligned} \alpha_k \rho_k (\partial U_k / \partial t) + \alpha_k \rho_k U_k (\partial U_k / \partial z) = & -\alpha_k (\partial P_k / \partial z) - (P_k - P_i) (\partial \alpha_k / \partial z) \\ & + \Gamma_{ik} (U_{ik} - U_k) + \tau_{ik} - M_{ik} - \tau_{wk} + \alpha_k \rho_k b_z, \end{aligned} \quad (2)$$

and

$$\begin{aligned} & A(\partial/\partial t)\{\alpha_k\rho_k(h_k+(U_k^2/2))\}+(\partial/\partial z)\{\alpha_k\rho_kU_k(h_k+(U_k^2/2))A\} \\ & = \alpha_kA(\partial P_k/\partial t)+A(P_k-P_i)(\partial\alpha_k/\partial t) \\ & +A\Gamma_{ik}(h_{ik}+(U_{ik}^2/2))+Aq_{ik}+Aq_{wk}+A\alpha_k\rho_kU_kb_z, \end{aligned} \quad (3)$$

where A , α_k , ρ_k , U_k , P_k and h_k denote respectively flow area, volume fraction, density, z-component of velocity, pressure and enthalpy for phase k and P_i and b_k are the interfacial pressure and the z-component of body force, respectively. The sum of the volume fractions $\alpha_g + \alpha_l$ is unity. The variables Γ , τ , M and q denote the rate of mass transfer, friction, added mass force, and heat transfer rate, respectively, per unit volume and subscripts i and w denote the interface and wall, respectively.

The jump conditions for mass, momentum and energy transfer across the interface are

$$\sum_k \Gamma_{ik}=0, \quad \sum_k (\tau_{ik} + \Gamma_{ik}U_{ik} - M_{ik})=0 \quad \text{and} \quad \sum_k \{\Gamma_{ik}(h_{ik} + U_{ik}^2/2) + q_{ik}\}=0. \quad (4)$$

Under the assumption that the interface is in the saturated condition and the kinetic energy at interface can be neglected, the following relations are derived from Eq. (4).

$$\Gamma_{ig} = -\Gamma_{il} = -(q_{ig} + q_{il})/(h_{ig} - h_{il}), \quad (5)$$

where

$$h_{ig} = h_{sg}(P_i) \quad \text{and} \quad h_{il} = h_{sl}(P_i), \quad (6)$$

and the subscript s denotes the saturated condition. The interfacial velocities are assumed as follows:

$$U_{ig} = U_{il} = U_l \quad \text{for} \quad \Gamma_{ig} \geq 0 \quad \text{and} \quad U_{il} = U_{ig} = U_g \quad \text{for} \quad \Gamma_{ig} < 0. \quad (7)$$

Also, it is assumed that the interfacial momentum transfers due to friction and added mass force independently satisfy the jump condition as

$$\tau_{ig} = -\tau_{il} \quad \text{and} \quad M_{ig} = -M_{il}. \quad (8)$$

2.2 Constitutive Relations

A total of six conservation equations are solved for the six basic variables α ($=\alpha_g$), P ($=P_l$), h_g , h_l , U_g and U_l , together with the thermodynamic relations and constitutive relations stated below and in the previous section. The thermodynamic equations of state are given by

$$\rho_k = \rho_k(P_k, h_k) \quad \text{and} \quad T_k = T_k(P_k, h_k), \quad (9)$$

where T_k is the k-phase temperature. The interfacial heat and momentum transfer terms are defined as follows:

$$q_{ik} = A_i \lambda_{ik} \{T_s(P_i) - T_k\} \quad \text{and} \quad \tau_{ig} = (1/2) A_i f_i \rho_c |U_g - U_l| (U_g - U_l), \quad (10)$$

where A_i is the area concentration, λ_{ik} the heat transfer coefficient and f_i the friction factor at the interface. These variables are given by appropriate correlations.

The wall transfer terms are defined as follows:

$$q_{wh} = A_{wh} \lambda_{wh} \{T_w - T_h\} \text{ and } \tau_{wh} = (1/2) A_{wh} f_{wh} \rho_h |U_h| U_h \quad (11)$$

As for the added mass force M_{ik} and the pressure force at the interface $P_k - P_i$, the models proposed by Drew and Lahey⁽⁹⁾, and Hancox et al.⁽¹⁰⁾ and Stuhmiller⁽¹¹⁾, respectively, are implemented as described in Ref. (3).

2. 3 Numerical Solution Method

The finite difference mesh scheme applied in MINCS is a fixed, and staggered, spatial mesh^{(3),(4),(6)} as illustrated in Fig. 1. The state variables and velocities are defined at the cell center and cell edge, respectively. A fully implicit scheme for the time integration is employed to enhance numerical stability for a wide range of flow conditions for all the terms except the coefficients calculated by empirical correlations. By applying backward differencing for the time, non-linear algebraic equations are obtained for the variables of the new time level. Thus the time advancement is accomplished by solving these algebraic equations by the Newton-Raphson method. In the Newton-Raphson method, the equations are rewritten in terms of the increments of variables in between successive two iterations. Then, these non-linear equations are linearized by expanding into the Taylor's series about the increments. The linearized equations are solved by Gaussian elimination or by the LU decomposition method.

Fig. 1 Mesh cell configuration

2. 4 Treatment of Single Phase and Phase Transition

There are two types of methods to handle the single-phase flow within the framework of the two-fluid model. One is to solve only three equations for a single-phase flow and the other is to solve six equations assuming a very small volume fraction for the missing phase. When an implicit time integration scheme is applied, the latter method is often used as in MINCS because it is difficult to change the number of equations during the iterative calculations. The basic requirements for the numerical method to handle the missing phase are: (i) the volume fraction of the missing phase should be calculated to be positive and less than a threshold value to define the single phase, (ii) the phasic enthalpy of the missing phase should stay nearly at the saturated value, (iii) the phasic velocity of the missing phase should be nearly equal to that of the massive phase. The requirements (ii) and (iii) are necessary when the phase transition to two-phase is calculated to occur.

In addition to these requirements, (iv) the phase transition has to be possible. In the case of MINCS, discontinuities related to the phase transition bring about serious problems because all the state variables are solved together by the Newton-Raphson method. The numerical treatment in MINCS to handle the missing phase and phase transition developed through the present benchmark calculations is shown below.

First of all, a threshold volume fraction α_{min} , which is taken to be 10^{-5} in the present calculations, is given to define the single-phase flow. When the volume fraction α_k of phase k is less than α_{min} , the single-phase flow is assumed and all the source terms in the r.h.s's of Eqs. (1), (2) and (3) for the missing phase are assumed zero. This assumption is to satisfy the requirement (i) and is numerically realized by describing the r.h.s's of these equations in terms of newly introduced $\hat{\alpha}_k$ instead of α_k as follows:

$$A(\partial/\partial t)(\alpha_k \rho_k) + (\partial/\partial z)(\alpha_k \rho_k U_k A) = A \Gamma_{ik}(\hat{\alpha}_k), \quad (12)$$

$$\alpha_k \rho_k (\partial U_k / \partial t) + \alpha_k \rho_k U_k (\partial U_k / \partial z) = -\hat{\alpha}_k (\partial P_k / \partial z) + \tau_{ik}(\hat{\alpha}_k) + \tau_{wk}(\hat{\alpha}_k) + \hat{\alpha}_k \rho_k b_z \quad (13)$$

and

$$\alpha_k \rho_k (\partial h_k / \partial t) + \alpha_k \rho_k U_k (\partial h_k / \partial z) = \Gamma_{ik}(h_{ik} - h_k) + q_{ik}(\hat{\alpha}_k) + q_{wk}(\hat{\alpha}_k) + \hat{\alpha}_k \rho_k U_k b_z, \quad (14)$$

where the energy equation is expressed by a non-conservative form and the dynamic pressure force, added mass force and kinetic energy terms are neglected for clarification. The quantity $\hat{\alpha}_k$ should be almost identical to α_k , but should satisfy the following relation:

$$\hat{\alpha}_k / \alpha_k \rightarrow 0 \text{ as } \alpha_k \rightarrow \alpha_{min}. \quad (15)$$

In the present calculations, it is defined as

$$\hat{\alpha}_k = \begin{cases} 0 & \text{for } \alpha_k \leq \alpha_{min} \\ 1 & \text{for } \alpha_k \geq (1 - \alpha_{min}) \\ (\alpha_k - \alpha_{min}) / (1 - 2\alpha_{min}) & \text{for } \alpha_{min} < \alpha_k < (1 - \alpha_{min}) \end{cases} \quad (16)$$

The expressions $\Gamma(\alpha_k)$, $q_{ik}(\alpha_k)$, $q_{wk}(\alpha_k)$, $\tau_{ik}(\alpha_k)$ and $\tau_{wk}(\alpha_k)$ mean that each of these quantities is expressed as a function of $\hat{\alpha}_k$ and that the quantity divided by α_k should approach zero as α_k decreases to α_{min} .

In view of requirements (ii) and (iii) and Eq. (10), the interfacial areas should not approach zero, but should approach a small value ϵ . Furthermore, related to the requirement (iv), the nuclei are indispensable to the inception of boiling or condensation not only for the physics but also for the numerics. As a result of the above considerations, the following relationships are derived:

$$A_i^f \rightarrow \epsilon_i^f, \quad A_{ig}^h \rightarrow \epsilon_{ig}^h, \quad \text{and} \quad A_{il}^h \rightarrow \begin{cases} 0 & \text{for } T_l < T_s \\ \epsilon_{il}^h & \text{for } T_l \geq T_s \end{cases} \text{ as } \hat{\alpha}_g \rightarrow 0, \text{ and} \\ A_i^f \rightarrow \epsilon_i^f, \quad A_{il}^h \rightarrow \epsilon_{il}^h, \quad \text{and} \quad A_{ig}^h \rightarrow \begin{cases} 0 & \text{for } T_g > T_s \\ \epsilon_{ig}^h & \text{for } T_g \leq T_s \end{cases} \text{ as } \hat{\alpha}_l \rightarrow 0, \quad (17)$$

where two kinds of interfacial heat transfer areas i.e. A_{ig}^h and A_{il}^h between interface and gas, and between interface and liquid, respectively, are introduced for numerical convenience. An example of $A_{il}^h(\hat{\alpha}_g)$ with consideration of Eq. (17) is schematically shown in Fig.2 as a function of α_g , when $A_{il}^h(\alpha_g)$ is defined to be proportional to $\alpha_g(1 - \alpha_g)$.

Another advantage of the expressions in Eqs. (12) to (14) is that the pressure gradient and body force terms in the momentum equation of the missing phase disappear when α_k is less than α_{min} . When using α_k instead of $\hat{\alpha}_k$, these two terms often influence the numerical results adversely because they act on the missing phase.

Fig. 2 Heat transfer area concentration between interface and liquid as function of gas volume fraction

3. NUMERICAL BENCHMARK PROBLEMS

3. 1 Overview of Problems

The numerical benchmark problems proposed are briefly summarized in Table 1⁽⁸⁾. The characteristic features of the problems are (i) the basic field equations for the two-fluid model to be used are specified, (ii) the geometry is simple and the space discretization and, in transient problems, the time step width to be used are specified, (iii) the constitutive equations including fluid properties are specified in simplified forms, and (iv) the boundary conditions as well as the initial conditions for the transient problems are specified. Therefore, these problems are considered suitable to investigate capabilities among different numerical solution methods. In addition, the problems shown in what follows are appropriate to investigate the numerical method for the phase transition of our interest.

Table 1 Description of Numerical benchmark problems

Category	No.	Title	Brief Description
1-D Steady	1.1	Nozzle Flow	Laval nozzle of two-phase
	1.2	Monopropellant Rocket	Disappearance of liquid droplet due to evaporation
	1.3	Boiling in Pipe	Inception of boiling due to external heating
1-D Unsteady	2.1	Faucet Flow	Acceleration of liquid by gravity
	2.2	Oscillating Manometer	U-tube oscillation
	2.3	Expulsion of Steam by Subcooled Water	Subcooled water injection into superheated steam
	2.4	Sedimentation	Sedimentation of liquid due to gravity
	2.5	Kelvin-Helmholtz Instability	Instability of small disturbance in stratified flow
	2.6	Shock Tube	Shock tube of two-phase
	2.7	Stratified Flow	Propagation of small disturbance of water level in stratified flow
2-D Steady	3.1	Idealized Tee Junction	Phase separation at tee junction
	3.2	Boiling in Channel	Inception of boiling due to external heating
2-D Unsteady	4.1	Sedimentation	Sedimentation of liquid due to gravity
	4.2	Expulsion of Water by Air	Injection of air into rectangular box filled with water

3. 2 Calculated Results and Discussions

3. 2. 1 Monopropellant Rocket

This one-dimensional steady state problem considers the injection of monopropellant droplets through small holes in one end of a rocket motor. The droplet diminishes its diameter due to evaporation and finally disappears somewhere in the rocket motor. The schematic drawing of the geometry and spatial discretization is shown in Fig. 3 together with the boundary conditions. The fluids are assumed incompressible with

$$\rho_g = 10.0 \text{ kg/m}^3 \text{ and } \rho_l = 10000.0 \text{ kg/m}^3. \quad (18)$$

Only mass and momentum equations of each phase are specified to be solved by neglecting wall

Fig. 3 Input model for Monopropellant Rocket

friction and with the following forms of the frictional force and mass transfer rate, both per unit volume.

$$F_{ig} = C\alpha(1-\alpha)\rho_l(u_g - u_l)/d_2^2 \text{ and } \Gamma_{ig} = C_m F_{ig}/(u_g - u_l) \quad (19)$$

where C and C_m are 10×10^5 and 1.0 , respectively. From this, the volume fraction α_g is written below by the void fraction α . The non-dimensional droplet diameter d_2 is expressed as

$$d_2 = \{(1-\alpha)/(1-\alpha_0)\}^{1/3} \quad (20)$$

where α and α_0 are the vapor volume fractions at the point in question and at the inlet, respectively. By substituting Eq. (20) into Eq. (19), it is shown that F_{ig} and Γ_{ig} are proportional to $(1-\alpha)^{1/3}$, whose derivative with respect to α becomes infinite as α approaches to unity. This leads to a difficulty during the Newton-Raphson iteration. In order to avoid the difficulty, it is assumed that d_2 has a minimum value corresponding to $\alpha=0.99995$.

In Fig. 4, the spatial distribution of the liquid velocity calculated by MINCS using the time

Fig. 4 Liquid velocity profiles calculated by MINCS and Runge-Kutta Method

marching method is compared with that obtained by the Runge-Kutta method. The discontinuity at the vanishing point of the liquid droplet is clearly shown by the Runge-Kutta method since the two-phase equations are solved up to this point as the initial value problem in the flow direction. This result is very close to the exact solution⁽¹⁵⁾. However, this point is rounded off in the MINCS calculation because the mass transfer rate linearly decreases to zero when the liquid volume fraction decreases to α_{min} .

3. 2. 2 Boiling in Pipe

This one-dimensional steady state problem treats the boiling of saturated water entering a uniform rectangular duct due to external heating. Figure 5 schematically shows the geometry, spatial discretization and boundary conditions. The densities and saturated enthalpies of both phases are assumed constant as follows:

Fig. 5 Input model for Boiling in Pipe

$$\rho_g = 10.0 \text{ kg/m}^3, \rho_l = 10000.0 \text{ kg/m}^3$$

$$h_{sg} = 2.5 \times 10^6 \text{ J/kg, and } h_{sl} = 4.0 \times 10^4 \text{ J/kg.} \quad (21)$$

The six equations of the two-fluid model are specified to be solved by neglecting wall friction and with the following forms of the interfacial frictional force and mass transfer rate, both per unit volume.

$$F_{ig} = C\alpha(1-\alpha)\rho_l(u_g - u_l) \text{ and}$$

$$\Gamma_{ig} = F_{ig}/(u_g - u_l) \{C_l(h_l - h_{sl}) + C_g(h_g - h_{sg})\} \quad (22)$$

where C , C_g and C_l are 50.0, 0.01 and 1.0 respectively.

The time marching method is used to calculate this steady state problem. Figure 6 shows the void fraction profiles calculated by MINCS in the cases of horizontal and vertical ducts. This problem is suitable for studying the magnitude of ϵ_{il}^h in Eq. (17). Figure 7 shows the void fraction and pressure transients during the time marching calculation using two different values of ϵ_{il}^h where the coefficient $\alpha(1-\alpha)$ in Eq. (22) is taken as A_i^f and A_{ik}^h . When ϵ_{il}^h is too small, the calculation fails because the boiling inception is seriously delayed and the pressure spike becomes larger.

Fig. 6 Void fraction in vertical and horizontal cases

Fig. 7 Transients of void fraction and pressure up to steady state

3. 2. 3 Expulsion of Steam by Subcooled Water

This one-dimensional transient problem is to investigate the so-called water packing. Figure 8 shows the geometry, space discretization and boundary conditions specified. The subcooled water is injected from the bottom of the duct, which is initially filled with superheated steam. A null in flow velocity is specified for the first 2.0 s of the transient to allow hydrostatic conditions to be established. Since no specific modeling recommendation is made in this problem, the six equations of MINCS are solved with its steam table and by neglecting wall friction terms, and the following

Fig. 8 Input model for Expulsion of Steam by Subcooled Water

forms of the interfacial frictional force and heat transfer rate, both per unit volume, and selected to be used.

$$F_{ig} = C\alpha(1-\alpha)\rho_l(u_g - u_l)|u_g - u_l| \quad \text{and} \quad q_{ik} = C_k\alpha(1-\alpha)\lambda_{ik}(T_s - T_k) \quad (23)$$

where C , C_g and C_l are 1.0 and, λ_{ig} and λ_{il} vary from case to case.

The water packing is a general problem common to finite difference mesh schemes⁽¹²⁾⁻⁽¹⁴⁾. A sharp pressure spike is calculated to occur due to the time derivative of the liquid velocity at the time when the cell is filled with liquid. Therefore, the water packing is a difficulty related to phase transition, but it is not improved by the numerical treatment shown in 2.4. Figure 9(a) to 9(d) show the calculated pressure and void fraction transients in the cases of horizontal and vertical ducts with and without condensation. It is shown that the water packing is calculated to occur even without condensation, where the liquid is forced to drop by gravitational force but not by condensation just before the cell is filled with liquid. In the cases with condensation, the pressure spikes are enhanced and gravitational force is no longer important.

In addition to the water packing, the numerical instabilities of the phasic energy equations are studied through the case of large condensation rate. The model equations are shown below:

$$\alpha_k \rho_k \partial h_k / \partial t = \Gamma_{ik}(h_{ks} - h_k) + q_{ik} \quad (24)$$

where the enthalpy flux terms are neglected for simplicity because the mass transfer rate is assumed to play the dominant role. The non-dimensional forms of Eq. (24) is obtained assuming that the pressure and void fraction are maintained constant as follows:

$$(\partial / \partial t_g^*) (\Delta h_g^*) = -\Delta h_g^* \{ \Delta h_g^* + 1 - A_l^* \lambda_l^* \Delta h_l^* \} \quad \text{and} \quad (25)$$

Fig. 9 Pressure and void fraction transients

$$(\partial/\partial t_i^*)(\Delta h_i^*) = -\Delta h_i^* \{ \Delta h_i^* + 1 - \Delta h_g^* / A_i^{h*} \lambda_i^* \}, \quad (26)$$

where the non-dimensional variables are defined as follows:

$$\begin{aligned} t_g^* &= (A_{lg}^h \lambda_{lg}) / (\alpha \rho_g C_{pg}) t, \quad t_l^* = (A_{ll}^h \lambda_{ll}) / ((1-\alpha) \rho_l C_{pl}) t, \\ \Delta h_g^* &= (h_g - h_{sg}) / h_{gl}, \quad \Delta h_l^* = (h_{sl} - h_l) / h_{gl}, \\ A_i^* &= A_{il}^h / A_{lg}^h \text{ and } \lambda_i^* = (C_{pg} \lambda_{il}) / (C_{pl} \lambda_{lg}). \end{aligned} \quad (27)$$

In the above equations, C_{pk} , λ_{ik} and h_{sk} are assumed to be constant.

Although Eqs. (25) and (26) are coupled with each other, they are considered separately for simplicity. For the case where the void fraction is small and the liquid is subcooled, Eq. (25) is considered, assuming that Δh_l^* is constant. For the case where the void fraction is large and the gas is superheated, Eq. (26) is taken into account, assuming that Δh_g^* is constant.

The conditions that the small disturbances of Δh_g^* and Δh_l^* from zero, namely from the saturated states, do not grow are obtained as follows:

$$A_i^* \lambda_i^* \leq 1 / \Delta h_i^* \text{ for given } \Delta h_i^*, \text{ and}$$

$$A_i^* \lambda_i^* \geq \Delta h_g^* \text{ for given } \Delta h_g^*. \quad (28)$$

Two calculations have been performed to show the effect of the condition shown in Eq. (28), with the same λ_{il} but different λ_{ig} . The calculated results are shown in Fig. 10. When $\lambda_{ig}/\lambda_{il}$ is

Fig. 10 Gas enthalpy and mass transfer rate

small enough not to satisfy Eq. (28), the gas enthalpy is calculated to diverge.

It should be noted that when using Eq. (17), Eq. (28) is satisfied irrelevant to λ_i^* for the cases of $\lambda_g \leq \alpha_{min}$ and $\alpha_l \leq \alpha_{min}$, since A_i^* is zero and infinite, respectively.

3. 2. 4 One-Dimensional Sedimentation

Initially, the denser phase is maintained above the lighter phase and at time zero the denser phase starts to drop. Finally all the denser phase rests below the lighter phase due to gravitational force. Figure 11 shows the geometry, space discretization and, boundary and initial conditions.

Fig. 11 Input model for One-Dimensional Sedimentation

The densities and gravitational force are specified as follows:

$$\rho_g=0.999, \rho_l=1.0 \text{ and } g=0.5(\rho_l+\rho_g)/(\rho_l-\rho_g) . \quad (29)$$

The mass and momentum equations of each phase are to be solved, neglecting wall friction and mass transfer. The interfacial frictional force per unit volume is specified as follows:

$$F_{ig}=C_f\{\alpha\rho_g+(1-\alpha)\rho_l\}\alpha(1-\alpha)(u_g-u_l) \quad (30)$$

where C_f is 1000, 2.0 or 0.0 for high, intermediate and low interfacial friction cases, respectively.

In this problem, the phase transition is brought about not by phase change but by phase separation due to gravity. Figure 12 shows the temporal change of void fraction profile in the case of intermediate interfacial friction. Figure 13 shows the gas velocity and void fraction transients in the case

Fig. 12 Transient of void fraction profile

Fig. 13 Gas velocity and void fraction transients

of zero interfacial friction at cell number 10, which is filled with water after the sedimentation is completed. When using α_k instead of $\hat{\alpha}_k$ in the momentum equations, the hydrostatic pressure gradient of the liquid acts on the missing phase and accelerates it. Finally an oscillation is calculated to occur, where the large pressure gradient is compensated by the time derivative of the velocity in the momentum equation of the missing phase. On the other hand, the gas velocity calculated by using the model described in 2.4 approaches zero.

4. CONCLUSION

The numerics in MINCS of a fully implicit first-order scheme with a first-order staggered-mesh donor-cell method has been applied to the numerical benchmark problems proposed by the Workshop on Two-Phase Flow Fundamentals under the auspices of the US DOE and EPRI. Besides the comparison among the results by participants with different numerical solution schemes, the focal points of our interest is the numerical treatment of the missing phase in a single-phase flow and phase transition. A systematic numerical treatment has been developed through the calculations of the benchmark problems. The most important points of the numerical treatment are summarized as follows. In order to obtain a stable solution for the missing phase and phase transition from two-phase to a single phase, when the volume fraction of less massive phase decreases to a threshold value to define the single phase, (i) the interfacial heat transfer area between the interface and the massive phase should smoothly approach zero, (ii) the interfacial heat transfer area between the interface and the less massive phase should remain a small value, (iii) the interfacial frictional area should remain a small value, and (iv) the pressure gradient and body force terms in the momentum equation of the less massive phase should smoothly approach zero. In addition, for the phase transition from a single phase to two-phase, (v) a small value of the interfacial heat transfer area to act as nucleus is indispensable.

ACKNOWLEDGEMENT

The authors would like to thank Mr. A. Kohsaka for continuous encouragement and critical reading of the manuscript.

Nomenclature

A	flow area or area concentration
b_z	body force
C	constant
C_p	specific heat
d_2	nondimensional droplet diameter
f	friction factor
F	frictional force per unit volume
g	gravity
h	specific enthalpy
h_{gl}	latent heat
M	added mass force
P	pressure

q	heat transfer rate per unit mass
t	time
T	temperature
U	velocity
z	coordinate

Greek

α	volume fraction or void fraction
ϵ	defined in Eq. (17)
Γ	mass transfer rate per unit volume
λ	heat transfer coefficient
ρ	density
τ	friction per unit volume

Subscript

c	continuous phase
f	friction
g	gas
i	interface
k	phase
l	liquid
min	minimum value
s	saturated value
w	wall
O	inlet value

Superscript

f	friction
h	heat transfer
*	nondimensional value
$\wedge$	defined in Eq. (16)

References

1. Ransom, V. H. et al., 1985: RELAP5/MOD2 Code Manual, NUREG/CR-4312, EGG-2396.
2. Safety Code Development Group, 1986: "TRAC-PF1/MOD1: An Advanced Best-Estimate Computer Program for Pressurized Water Reactor Thermal-Hydraulic Analysis", NUREG/CR-3858, LA-10157-MS.
3. Akimoto, M. et al., 1986: Development of Two-Phase Flow Analyzer : MINCS, 2nd Int. Topical Mtg. on Nucl. Power Plant Thermal Hydraulics and Operation, 72-79.
4. Akimoto, M. et al., 1984: MINCS: A Computer Code for Transient Thermo-Hydraulic Analysis in a Light Water Reactor System – MINCS-PIPE: A Computer Code for Transient Two-Phase Flow Analysis in One-Dimensional Ducts – (in Japanese), JAERI-M84-202.
5. Hirano, M. et al., 1986: Evaluation of Interfacial Shear Model for Bubbly Flow Regime With

MINCS Code, 2nd Int. Topical Mtg. on Nucl. Power Plant Thermal Hydraulics and Operation, 80-87.

6. Akimoto, M., Hirano, M. And Nariai, H., 1987: Present Status of Numerical Analysis on Transient Two-Phase Flow, J. Atomic Soc. Japan (in Japanese), Vol. 29, No.5, 385-391.
7. Watanabe T., Hirano, M. and Tanabe, F., (to be published): Vapor Generation Model for Flashing in Initial Blowdown Phase, Nucl. Engrg. Design.
8. DOE/EPRI, (to be published): "Proc. 2nd Int. Workshop on Two-Phase Flow Fundamentals".
9. Drew, D. A. and Lahey, R. T., 1979: Application of General Constitutive Principles to Derivation of Multi-Dimensional Two-Phase Flow Equations, Int. J. Multiphase Flow, 5, 243-264.
10. Hancox, W. T. et al., 1980: One-Dimensional Models for Transient Gas-Liquid Flows in Ducts, Int. J. Multiphase Flow, 6, 25-40.
11. Stuhmiller, J. H., 1977: The Influence of Interfacial Pressure Forces on the Character of Two-Phase Flow Model Equations, Int. J. Multiphase Flow, 3, 551-560.
12. Mahaffy, J. H. and Liles, D. R., 1983: Numerical Induced Pressure Excursion in Two-Phase-Flow Calculations, Proc. 2nd Int. Topical Mtg. on Nucl. Reactor Thermal-Hydraulics, 470-473.
13. Johnsen, G. W., 1984: Status of RELAP5/MOD2 Development and Assessment, Proc. 12th Water Reactor Safety Research Information Mtg., NUREG/CP-0058, 153-166.
14. Heck, C. L. and Andersen, J. G. M., 1985: BWR Full Integral Simulation Test (FIST) Program TRAC-BWR Model Development Volume 1 - Numerical Methods, NUREG/CR-4127-1, EPRI NP-1987-1, GEAP-30875-1.
15. Spalding, D. B., 1959: Combustion in Liquid-Fuelled Rocket Motors, Aero Quaterly, 10, 1-27.