5. HIGH ENERGY NUCLEUS–NUCLEUS INTERACTIONS

National Institute of Radiological Sciences (NIRS)
L. Sihver

ABSTRACT

I first give a short historical background of heavy ion research. Nucleus–Nucleus reactions at intermediate, relativistic and ultrarelativistic energies are then discussed and target and projectile fragmentation processes are described. A short summary of existing semiempirical total and partial cross section formulas is given. These formulas are compared with those developed by Sihver et al. The agreement between the calculated cross sections by Sihver et al. and the experimental data is much better than all earlier published results.

I. INTRODUCTION

What is matter and where does it come from? These questions have probably always interested mankind. However, it is not until this century we got "tools" to "break up" the matter itself, so we could start to see and try to understand the basic components of our world. Nuclear disintegrations with artificially accelerated protons were achieved for the first time in 1930 by Cockcroft and Walton at the Cavendish Laboratory. The interaction and propagation of high-energy heavy ions in matter is now a subject of much current interest and activity. The first recorded heavy–ion–induced nuclear interactions were those of 96 MeV $^{12}$C$^{6+}$ beams with emulsions. These experiments were performed in the early forties by Cornelius Tobias at the 60° cyclotron at Berkeley. In 1950 Ghiorso et al. succeeded to produce $^{244}$Cf and $^{246}$Cf by bombarding natural uranium targets with carbon from the internal carbon beam of the same cyclotron. By 1955 the Berkeley group and their collaborators had synthesized all the elements from $Z=93$ through $Z=101$. Currently elements with atomic number up to $Z=109$ have been produced by heavy ion bombardment of suitable heavy element targets. The use of heavy ions nucleus as projectiles was also stimulated in the mid–fifties by the use of Coulomb excitation, in which the strength of the interaction is proportional to the product of the charges of the two colliding nuclei. These interactions are naturally much stronger when two heavy nuclei collide than in proton induced reactions. There is no well–defined borderline between light and heavy ions. The interaction of $\alpha$–particle with nuclei exhibit some features characteristic of much heavier complex projectiles, and therefore the $\alpha$–particle has sometimes been referred to as the "lightest heavy ion". From a historical
and technical standpoint most physicists think of ions heavier than $\alpha$-particles as heavy ions. Around 1960 heavy ion beams at energies above the Coulomb barrier became available. In the seventies there was a new breakthrough in heavy ion physics when the deep inelastic collision was discovered (it had been seen by Kaufman and Wolfgang in 1959, but was then ascribed to a "grazing contact mechanism"). At the same time, the Bevalac accelerator for relativistic ion ($500$ MeV/nucleon $\leq E_{\text{projectile}} \leq 2$ GeV/nucleon), was constructed at Berkeley. After the reconstruction of the CERN Synchro–Cyclotron (SC) accelerator in 1980, followed by the construction of other facilities, such as the Grand Accélérateur National d’Ions Lourds (GANIL) accelerator in Caen, it became possible to study reactions in the intermediate energy regime ($10$ MeV/nucleon $\leq E_{\text{projectile}} \leq 100$ MeV/nucleon). In 1986, heavy ion beams of 60 and 200 GeV per nucleon were delivered for the first time at the CERN Super Proton Synchrotron (SPS). In the same year, the Tandem/AGS facility, at Brookhaven National Laboratory (BNL), started to deliver heavy ion beams of 13.6 GeV per nucleon, making it possible to study ultrarelativistic energy ($E_{\text{projectile}} > 10$ GeV/nucleon) nucleus–nucleus collisions.

II. NUCLEAR REACTIONS

![Reaction Phase Diagram](image)

Figure 1. Reaction phase diagram for a heavy ion collision, from ref 4.
In Figure 1, I show a reaction phase diagram over the various reaction types which can take place in a nucleus–nucleus collision, depending on the impact parameter and the bombarding energy. The coordinates are the bombarding energy per nucleon and the impact parameter of the reaction (target is at rest in the lab. system). The full drawn curve separates the interaction zone (below the curve) from the zone where the ions miss each other. The Coulomb excitation occurs for reactions just over this curve. Just below finds the nuclear inelastic and transfer processes. The lower energy regime in the interaction zone is dominated by complete fusion (CF) and deep inelastic collisions (DI). When the energy rises towards the binding limit a process in between, the incomplete fusion (ICF), takes over. Above the fragmentation threshold the violent processes take place. For the more peripheral collisions the colliding system separates into hot participants and two colder coherent spectators (PS). For the central collisions the total system undergoes a total explosion (TE) with preferable high multiplicity of small fragments in the most central events (gas), while the fragments become bigger (droplets) in the slightly less central events.

A. Intermediate energy reactions

Studies of intermediate energy nuclear–nuclear collisions (10 MeV/nucleon \( \leq E_{\text{projectile}} \leq 100 \) MeV/nucleon) are interesting because of the "transitional" character of this regime. Nuclear reactions at energies only a few MeV above the interaction barrier are dominated by the mean nuclear field. At these energies the de Broglie wave-length for the relative motion of the one nucleon of the target is much larger than the intranuclear distance. At low energies near the Coulomb barrier (\( \approx 5 \) MeV per nucleon for heavy targets), collisions with relatively light projectiles at small impact parameters lead to the formation of equilibrated compound nuclei which subsequently decay by particle emission, fission or both. The fusion process between two heavy ions has been extensively studied at low energies. It is characterized by a complete transfer of linear and angular momentum, with the total available energy being transferred into excitation energy. Peripheral collisions in this energy regime consist in part of quasi–elastic reactions. In reactions induced by projectiles with \( A \leq 24 \), the quasi–elastic processes are observed to contribute up to \( \approx 30\% \) of the total reaction cross sections at energies close to the Coulomb barrier \(^7,^8\). In quasi–elastic processes the target and the projectile leave the interaction region with relatively small excitation energies and mass transfers. Some years ago many research groups started to pay much attention to the study the quasi–elastic reactions induced by ions with \( A \geq 40 \). It was found that \(^9,^{10}\) that at energies in the vicinity of the Coulomb barrier quasi–elastic reactions are the dominant reaction channels. The heavy products of quasi–elastic scattering have characteristic sideward–peaked angular distributions. As the projectile energy increases into the intermediate energy regime one begins to observe evidence for precompound processes, i.e. the system decays before reaching full statistical equilibrium. These nonequilibrium ("incomplete fusion") events first
become evident at incident energies of \( E_{\text{projectile}/\text{nucleon}} = 10 \text{ MeV} \), beyond which they grow in importance until, for small impact parameters, they dominate the collision process in the vicinity of the Fermi energy in nuclear matter (\( E_{\text{projectile}/\text{nucleon}} = 35 \text{ MeV} \)).

Intermediate energy reactions at larger impact parameters, intermediate between those for purely Coulomb interactions and those leading to compound-nucleus formation, are often assumed to proceed through deep inelastic reactions for heavier projectiles. There is, however, still a discussion whether this assumption is valid or not. The discovery of deep inelastic processes changed the scope of heavy ion physics. It was observed that for energetic collisions there could be a big loss of the relative kinetic energy of the two ions, all the way down to the Coulomb barrier, without neither the projectile nor the target really losing their identities. Deep inelastic reactions are usually assumed to take place for initial values of the orbital angular momentum \( \ell \), lying between \( \ell_{\text{crit}} \), the critical angular momentum for fusion, and values close to \( \ell_{\text{max}} \), the maximum angular momentum leading to nuclear reactions. For \( \ell \)-values smaller than the \( \ell_{\text{crit}} \) a compound nucleus is usually assumed to be formed. There exists, however, some cases where \( \ell_{\text{crit}} \) is larger than \( \ell_{\text{Br}} \), the value of the angular momentum for which the fission barrier of the compound nucleus vanishes. It has been suggested that the \( \ell \)-values located between \( \ell_{\text{Br}} \) and \( \ell_{\text{crit}} \) (when \( \ell_{\text{crit}} > \ell_{\text{Br}} \)), could lead to a long-lived deep inelastic component which is called "fast fission" \(^{11} \), see Figure 2, because its properties are very close to those of compound-nucleus fission. However, since a compound-nucleus is not formed, "fast fission" would occur on a shorter time scale. It has been estimated that "fast" means \( 2-3 \times 10^{-23} \text{ sec} \), which is the time estimated\(^{12} \) for establishing statistical equilibrium in an excited nuclear system. The characteristics of "fast fission" would be a large momentum transfer (due to \( \ell < \ell_{\text{crit}} \)) and a large mass asymmetry due to the fact that the fusing nuclei re-separate on a fast time scale. The mass distributions following such a mechanism would be expected to be broader than those for fission after compound-nucleus formations because mass equilibrium has not yet been established\(^{13} \).

Reactions in the intermediate energy regime offer an opportunity to study the transition from the mean field dominated low energy regime to a high energy regime where two body collisions are important. Collisions in this energy regime have successfully been investigated using the so-called Boltzman–
Uehling–Uhlenbeck (BUU)\textsuperscript{14,15,16,17} and Vlasov–Uehling–Uhlenbeck (VUU)\textsuperscript{18,19,20,21} models. These theories include the nuclear mean field dynamics and the effect of nucleon–nucleon collisions which obey the Pauli principle.

**B. Relativistic and ultrarelativistic energy reactions**

For nucleus–nucleus collisions at relativistic energies, mean field aspects become less important, as the de Broglie wavelength for the relative motion of the one nucleon of the projectile to one nucleon of the target is shorter than the intra–nuclear distance. The reactions are dominated by effects of individual nucleon–nucleon collisions leading to rapid thermalization. At these energies, the geometrical definition of "participant" and spectator" regions becomes a useful concept for larger impact parameters, and the reactions are often discussed in terms of local statistical equilibrium. This is illustrated in Figure 3.

![Figure 3. Nuclear collision at high energies, from ref 22.](image)

Central collisions are expected to lead to multifragmentation\textsuperscript{4}. The transition from low energy mean field dominated nuclear reactions to high energy nucleon–nucleon collision dominated reactions is expected to take place at intermediated energies, and has been investigated by examining the systematic development of linear momentum transfer with increasing energy etc. This region is therefore very interesting with many competing reaction processes, depending on the projectile–target combinations, the projectile energy and the impact parameter.

At very high energies, there are two limiting hypotheses developed to describe elementary particle interactions. The first hypotheses is Limiting fragmentation, which predicts that spectra and yields of fragments will become independent of bombarding energy at sufficiently high energies. The onset of limiting fragmentation appears to be at \(\approx 10\) GeV for proton induced reactions and \(\approx 2.1\) GeV/nucleon for the heavy ion reactions\textsuperscript{22}. The limiting fragmentation hypotheses for proton induced reactions has been shown to be valid for incident energies up to 800 GeV\textsuperscript{23}. The second hypotheses is factorization, which asserts that spectra and yields of projectile (or target) fragments will depend on the target (or projectile) only via
a total cross section term. The distinction between target and projectile fragmentation depends only on the reference frame chosen.

The availability of ultrarelativistic heavy ions raised expectation of finding new phenomena in nuclear–nuclear collisions. Central collisions at these energies provide a means to create nuclear matter in conditions of extreme temperature or high baryon density. In these regimes of either high temperature or high baryon density a phase transition is expected, from a state of nuclear matter consisting of deconfined quarks and gluons. This new state of matter is called quark–gluon plasma (QGP). Most of the studies of ultrarelativistic nucleus–nucleus collisions have focused on the search for this quark–gluon plasma. In Figure 4, I show an expected phase diagram for nuclear matter in the plane of density ($\rho$) and temperature ($T$). The diagram is taken from ref. 24.

![Phase Diagram](image)

Figure 4, I show an expected phase diagram for nuclear matter in the plane of $(\rho,T)$. The diagram is taken from ref. 24.

III. TOTAL REACTION CROSS SECTION

One fundamental observable of a nuclear collision is the total reaction cross section, which is defined as the total minus the elastic cross sections for nucleons incident on a nucleus

$$\sigma_R = \sigma_T - \sigma_{el}.$$ (1)
To understand the nuclear strong interactions, the basic properties of the reaction cross sections are needed. To know the reaction cross sections is also very important in many other research areas, including shielding against heavy ions originating from either space radiations or accelerators, cosmic ray propagation, radiobiological effects resulting from work or clinical exposures. The total reaction cross section has therefore been extensively studied both theoretically and experimentally for more than 45 years. A detailed list of references is found in ref. 26, by Kox et al.

From geometrical arguments, one expects the total reaction cross section, \( \sigma_R \), for proton–nucleus reactions to be proportional to the area seen by the projectile, i.e. to \( \pi R^2 \), \( R \) being the nuclear radius. Since \( R \propto A^{1/3} \), one expects a dependence of the form \( \sigma_R \propto A^{2/3} \). For nucleus–nucleus reactions one expects a dependence of the form \( \sigma_R \propto (A_p^{1/3} + A_t^{1/3})^2 \). The first empirical expression of the total reaction cross section, \( \sigma_R \), for nucleus–nucleus collisions was proposed by Bradt and Peters where \( A_p \) and \( A_t \) are the mass numbers of the projectile and the target, respectively, and \( b_0 \) is the overlap, or transparency parameter, and \( r_0 \) being the constant of proportionality in the expression for the geometrical nuclear radius \( r = r_0 A_i^{1/3} \). Both \( b_0 \) and \( r_0 \) are energy–independent parameters. Although adequate for high energies \( \geq 1.5 \) GeV/nucleon, where both the total cross sections and the total reaction cross sections are expected to be energy–independent, there are significant differences between the calculated values and the experimental data at energies below 1.5 GeV/nucleon. Therefore, Sihver et al. have developed more accurate semiempirical formulas for calculation of both total proton–nucleus and nucleus–nucleus reaction cross sections. The agreement between these calculated cross sections is illustrated in Figure 5a, together with experimental data for the reactions of protons with Be, B, C and Al.

\[
\sigma_{\text{reac}} = \pi r_0^2 \left( A_p^{1/3} + A_t^{1/3} - b_0 \right)^2,
\]

where \( A_p \) and \( A_t \) are the mass numbers of the projectile and the target, respectively, and \( b_0 \) is the overlap, or transparency parameter, and \( r_0 \) being the constant of proportionality in the expression for the geometrical nuclear radius \( r = r_0 A_i^{1/3} \). Both \( b_0 \) and \( r_0 \) are energy–independent parameters. Although adequate for high energies \( \geq 1.5 \) GeV/nucleon, where both the total cross sections and the total reaction cross sections are expected to be energy–independent, there are significant differences between the calculated values and the experimental data at energies below 1.5 GeV/nucleon. Therefore, Sihver et al. have developed more accurate semiempirical formulas for calculation of both total proton–nucleus and nucleus–nucleus reaction cross sections. The agreement between these calculated cross sections is illustrated in Figure 5a, together with experimental data for the reactions of protons with Be, B, C and Al.
sections and the experimental data is much better than all earlier published results. In Figure 5a, I show the calculated \( \sigma_R \), together with experimental data \(^{28}\), for the reactions of protons with Be, B, C and Al.

If Figure 5b, I show the calculated ratios \( \sigma_{\text{reac}}(\text{calc})/\sigma_{\text{reac}}(\text{reac}) \) as function of \((A_p^{1/3}+A_t^{1/3})\) for nucleus–nucleus reactions above 100 MeV/nucleon. The experimental data are taken from ref. 26 and 29–34.

Figure 5b. The calculated ratios \( \sigma(\text{calc})/\sigma(\text{reac}) \) as function of \((A_p^{1/3}+A_t^{1/3})\) for nucleus–nucleus reactions above 100 MeV/nucleon. The experimental data are taken from ref. 26 and 29–34.

IV. TARGET AND PROJECTILE FRAGMENTATION

When a heavy–ion beam impinges upon a target, the collision can result in nuclear fragmentation. Depending on impact parameters, these events are characterized as projectile and/or target fragmentation. On the one hand, the target fragments are typically large, high Z fragments, which carry little momentum. On the other hand, the projectile fragments lose very little momentum and travel nearly in the beam direction with relatively minor deflection. Since the projectile fragments have approximately the same velocity as the incident beam, they open a possibility of producing a beam of unstable nuclei ("radioactive beam") for the study of nuclear properties. After that the high–energy heavy–ion beams became available, the projectile fragmentation process has been extensively studied. Target fragmentation is complementary to that of projectile fragmentation, although it is not studied as extensively due to the experimental difficulties of detecting these low energy (0.02
\( \leq E_{\text{fragment}} \leq 5 \text{ MeV/nucleon} \) high Z fragments. One way of detecting these large, relatively slow moving fragments is to perform single particle inclusive measurements of these fragments using radioanalytical techniques without chemical separations. One big advantage with this method as compared to using detectors such as time–of–flight spectrometers, is the lack of velocity cutoffs below which fragments are not detected. Counter experiments may miss a significant fraction of all events from the heavy target fragments due to these low energy cutoffs.

V. PARTIAL NUCLEUS–NUCLEUS CROSS SECTIONS

The partial nucleus–nucleus cross sections of high–energy reactions are of considerable astrophysical interest. The ability to calculate with precision nuclear fragmentation cross sections is of special importance for modeling of cosmic rays composition and propagation as most cosmic ray nuclei with nuclear charge \( Z \geq 2 \) suffer nuclear collisions in the interstellar medium. These collisions alter the elemental and isotopic composition of the source. The partial nucleus–nucleus cross sections are also of great importance in therapeutic and diagnostic medicine, since the projectile fragments lower the average stopping power relative to that of the incident beam. They will therefore make up a "tail", which extends beyond the stopping region of the primary beam (i.e. beyond the Bragg Peak). The partial inelastic cross sections have systematic regularities that permit the formulation of semiempirical equations. Rudstam noted these systematic regularities and developed a semiempirical cross section formula which is particularly useful for targets heavier than calcium. Silberberg and Tsao have constructed a semiempirical equation resembling Rudstam's with additional parameters, and have defined regions of target and product mass intervals where these parameters apply. They have also developed equations for calculating the cross sections for the breakup of nuclides \((Z_p, A_p)\) colliding with \((Z_t, A_t)\) by scaling their semiempirical systematics. However, these equations are based on a first iteration and the agreement with experimental data needs improvement. J.R. Cummings et al. have developed a procedure to calculate the fragmentation cross sections for both hydrogen targets and "heavy" targets, by fitting experimental data. However, there has not been any attempt to make these fits for "light" \((Z \leq 26)\) projectile–target combinations yet. We have therefore constructed a procedure for calculating these cross sections, by scaling semiempirical proton–nucleus partial cross section systematics. The scaling is done by a scaling parameter, which is based on a Bradt–Peters–type law and also takes advantage of the weak–factorization property of projectile fragments. Olson et al. have found that projectile fragments appear to obey the so–called weak–factorization property expressed as

\[
\sigma_f = \gamma_p \gamma_{p,t},
\]
Figure 6. The calculated partial nucleus–nucleus cross sections for the reaction of 2.1 GeV/nucleon $^{16}$O with $^{12}$C, together with the experimental data from ref 48. The filled circles are the calculated values, and the open circles are the experimental data. When only a filled circle can be seen, the calculated value is "in coincidence with the experimental data point".

where $\sigma_f$ is the partial cross section for the production of the projectile fragment $f$, $\gamma_p^{f}$ is a factor that depends only on the species of the projectile and the fragment, and $\gamma_{p,t}$ is a factor that depends only on the species of the projectile and target. The following relation was found$^{48}$ to describe $\gamma_{p,t}$:

$$\gamma_{p,t} = g \left( A_p^{1/3} + A_t^{1/3} - \delta \right),$$

where $g$ and $\delta$ are free parameters. Scaling to proton–nucleus cross sections in our procedure essentially means

$$S_C \rightarrow \gamma_{p,t}.$$  

Semiempirical expressions for $g$ and $\delta$ can be written as

$$g = \left( 1 + A_t^{1/3} - b_0^{(PM)} \right)^{-1},$$
\[ \delta = B_{o}^{(NN)}, \]  

where \( b_{o}^{(pN)} \) and \( b_{o}^{(NN)} \) are overlap parameters for the proton–nucleus and for nucleus–nucleus reactions, respectively. \( \delta \) in eq. (4) can easily be identified with the geometric overlap parameter \( b_{o} \), and the parameter \( g \) and can be identified as a scaling–specific parameter.

Figure 7. The calculated partial nucleus–nucleus cross sections for the reaction of 600 MeV/nucleon \(^{16}\text{O}\) with \(^{12}\text{C}\), together with the experimental data from ref 30.

There are, however, deviations from complete proportionality between proton–nucleus and nucleus–nucleus reactions. The deviations have been explained by Lindstrom et al.\(^{45}\) in terms of (1) nuclear transparency (the energy deposition is less in p–nucleus interactions; so, as \( \Delta A \) increases, the relative yields of proton–nucleus reactions diminish progressively), and (2) giant dipole resonance, as a result of which single–nucleon stripping is enhanced in collisions with heavy nuclei. (3)

132
The lightest products (Li, Be and B) are also enhanced by a factor of 2–3 beyond the scaling factor for heavier targets \cite{46,47}. We have therefore added enhancement factors for single–nucleon stripping, large \( \Delta A \) reactions and the lightest products (Li, Be and B) are also added. Our equation for calculating the cross sections for the breakup of nuclides \((Z_i, A_i)\) colliding with \((Z_j, A_j)\) is

\[
\sigma_{\text{reac}}(Z_i, A_i, Z_j, A_j, E_i) = S_c \varepsilon_L \varepsilon_\Delta \varepsilon_1 \sigma_{\text{reac}}(Z_i, A_i, p, E_p),
\]

where \( S_c \), as described in eq. (5), \( \sigma_{\text{reac}}(Z_i, A_i, Z_j, A_j, E_i) \) and \( \sigma_{\text{reac}}(Z_i, A_i, p, E_p) \) are the total reaction cross sections for nucleus–nucleus and proton–nucleus cross sections, respectively. \( \varepsilon_1 \), \( \varepsilon_\Delta \) and \( \varepsilon \) are the enhancements factors for the lightest products, for reactions with a large values of \( \Delta A \), and for single–nucleon stripping, respectively.

All products from the \( Z \) of the projectile down to \( Z=2 \) (He) can be calculated with our formulas. In Figure 6 and 7, I show the calculated partial nucleus–nucleus cross sections for the reactions of 2.1 GeV/nucleon \(^{16}\)O with \(^{12}\)C and 600 MeV/nucleon \(^{16}\)O with \(^{12}\)C, respectively, together with the experimental data from ref. 30 and 48.

We have also developed an algorithm to scale nucleus–nucleus collisions projectile–fragment cross sections from proton–nucleus ones over the whole energy–range 0.1–2.1 GeV/nucleon with no restriction on the sizes of target (apart from the special case of \(^4\)He) \cite{49}. This algorithm also takes advantage of the weak factorization property of projectile fragments. It uses the participant–spectator model and Glauber scattering theory and approximates the collision's sum rules.

VI. ACKNOWLEDGEMENTS

The author gratefully acknowledge the support of the STA Fellow–ship program. I also wish to thank all my colleagues and friends at NIRS for all their help and support during my time in Japan.
REFERENCES


134