New Equations to Calculate Temperature Correction Factors for P_{O_2} in Human Blood

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Abstract Effects of hemoglobin concentration (Hb), pH, and body temperature (T) on the relationships between $\Delta \log P_{O_2}/\Delta T$ and P_{O_2} were studied by means of a mathematical model using a Newton-Raphson iteration method. The functions between $\Delta \log P_{O_2}/\Delta T$ and P_{O_2} were affected by the above three factors. New equations considering the effects of Hb, pH, and T were proposed by modifying the equation reported by Severinghaus:

$$\Delta \log P_{O_2}/\Delta T = (L + (U - L)/(A(vP_{O_2}^{37})^B + 1))(10^{-2})$$

where

$$U = 3.15 - 0.45(7.4 - \text{pH}^{37})$$

$$L = 0.68 - 0.09(7.4 - \text{pH}^{37})$$

$$A = 5.86(\exp_{10}(0.074(T) - 0.294(7.4 - \text{pH}^{37}) - 11))((\text{Hb})^{0.913})$$

$$B = 6.33(\exp_{10}(-0.0051(T)))((\text{Hb})^{-0.113}) + 0.24(7.4 - \text{pH}^{37})$$

and $vP_{O_2}^{37}$ is virtual $P_{O_2}^{37}$ which may exist when $P_{O_2}^{37}$ is corrected to standard conditions (pH = 7.4, BE = 0) by the following equations:

$$vP_{O_2}^{37} = P_{O_2}^{37}(\exp_{10}(f_B(7.4 - \text{pH}^{37}) - 0.0013(\text{BE})))$$

$$f_B = (P_{O_2}^{37}/26.6)^{0.08} - 1.52$$

where f_B is the Bohr factor.

The above equations provided values of $\Delta \log P_{O_2}/\Delta T$ which fit closely to those obtained by the complex iteration method with maximum differences of less than 1.3×10^{-3} at $T = 27$, indicating that maximum % errors for P_{O_2} at T ($P_{O_2}^T$) are less than 3.0% at $T = 27$ and that our equations can be applied over a wide range of Hb, pH³⁷ and T .

Key words: anaerobic temperature correction, partial pressure of oxygen, hemoglobin concentration, pH, human blood.

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SYMBOLS AND DEFINITIONS

Symbols and units of measurement used in this text are defined as follows.

T : body temperature ($^{\circ}\text{C}$)

$P_{\text{O}_2}^{37}$, $P_{\text{O}_2}^T$: partial pressures of oxygen at 37°C and T (mmHg)

$vP_{\text{O}_2}^{37}$, $vP_{\text{O}_2}^T$: $P_{\text{O}_2}^{37}$, $P_{\text{O}_2}^T$ corrected to standard conditions ($\text{pH}=7.4$, $\text{BE}=0$, $T=37$ and $P_{50}=26.7$), which are virtual values needed for the calculation of $S_{\text{O}_2}^{37}$ and $S_{\text{O}_2}^T$

$S_{\text{O}_2}^{37}$, $S_{\text{O}_2}^T$: fractional oxygen saturations of hemoglobin at 37°C and T

α_{B}^{37} , α_{B}^T : Bunsen solubility coefficients for P_{O_2} in blood ($\text{ml}\cdot\text{ml}^{-1}\cdot\text{atm}^{-1}$) reported by CHRISTOFORIDES and HEDLEY-WHITE (1969)

Hb: hemoglobin concentration in blood (g/dl blood)

$C_{\text{O}_2}^{37}$, $C_{\text{O}_2}^T$: content of oxygen in blood at 37°C and T (ml STPD/dl blood)

pH^{37} , pH^T : pH at 37°C and T

$P_{\text{CO}_2}^{37}$, $P_{\text{CO}_2}^T$: partial pressure of carbon dioxide at 37°C and T

HCO_3^- : plasma bicarbonate concentration (mEq/l)

BE: base excess (mEq/l)

P_{50} : partial pressure of oxygen at half saturation under the conditions $\text{pH}=7.4$, $\text{BE}=0$, and $T=37$, Normal P_{50} was defined as 26.7 (SEVERINGHAUS, 1979).

$\exp_{10} X$: 10^X

$\exp_e X$: e^X

In modern blood gas analyzers currently available, the maintenance of accuracy depends on keeping the temperature around their electrode at 37°C . Thus, when body temperatures of patients deviate from 37°C , anaerobic temperature correction is indispensable. Temperature correction for P_{O_2} , as in P_{CO_2} , is known to follow an exponential relationship:

$$P_{\text{O}_2}^T = P_{\text{O}_2}^{37} (\exp_{10}(f(T-37)))$$

It has been well known that f or $\Delta \log P_{\text{O}_2}/\Delta T$ is not constant and varies with $P_{\text{O}_2}^{37}$ or S_{O_2} (HEDLEY-WHITE and LAVER, 1964; NUNN *et al.*, 1965; SEVERINGHAUS, 1966). Several algorithms for the calculation of f from $P_{\text{O}_2}^{37}$ or $S_{\text{O}_2}^T$ have been reported, which are derived from the data of NUNN *et al.* (1965) and values calculated by SEVERINGHAUS (1966). However there are wide differences between the data of NUNN *et al.* and values calculated by Severinghaus, especially in P_{O_2} values around 120 mmHg (GABEL, 1980). We obtained f by a Newton-Raphson iteration method based on the constancy of C_{O_2} during anaerobic temperature change for various pH^{37} , P_{50} , Hb, and T values and studied their effects on the functions between f and $vP_{\text{O}_2}^{37}$.

CALCULATIONS AND ITERATIONS

Since C_{O_2} remains constant when blood is cooled or warmed anaerobically as expected in blood gas analyzers, the following equation can be obtained:

$$C_{O_2}^{37} = C_{O_2}^T \quad (1)$$

therefore

$$(\alpha_B^{37}/7.6)(P_{O_2}^{37}) + 1.34(Hb)(S_{O_2}^{37}) = (\alpha_B^T/7.6)(P_{O_2}^T) + 1.34(Hb)(S_{O_2}^T). \quad (2)$$

S_{O_2} can be calculated from vP_{O_2} by several algorithms reported previously (GABEL, 1980). We used the algorithm reported by SASAI *et al.* (1984) which, to the best of our knowledge, provides exact expression for the "standard" oxygen hemoglobin equilibrium curve reported by SEVERINGHAUS (1979). The algorithms of SASAI *et al.* are shown in Table 1. By combining Eq. 2 with the following equations for calculating $P_{O_2}^{37}$ from $vP_{O_2}^{37}$ and $vP_{O_2}^T$ from $P_{O_2}^T$ (GABEL, 1980; SEVERINGHAUS, 1979),

Table 1. Algorithms of Sasai *et al.*

P	S	$A3$	$A2$	$A1$	$A0$
[0, 4)	[0.60, 2.56)	0.11/12	-0.13/4	1.87/3	0.00
[4, 10)	[2.56, 9.58)	0.03/16	0.38/8	1.90/4	0.08
[10, 16)	[9.58, 21.40)	0.005/8	0.37/8	0.89/2	-0.12
[16, 22)	[21.40, 37.60)	-0.11/8	0.8075	-12.97	78.52
[22, 28)	[37.60, 53.16)	0.085/24	-0.30625	33.7/3	-99.02
[28, 34)	[53.16, 65.16)	-0.009375	0.815	-21.4175	219.69
[34, 40)	[65.16, 74.69)	-0.1/12	0.880	-29.2317	369.29
[40, 46)	[74.69, 81.71)	0.1/20	-0.6725	31.2250	-418.31
[46, 52)	[81.71, 86.59)	0.1/24	-0.63125	32.6258	-488.92
[52, 58)	[86.59, 89.95)	-0.11/16	1.12875	-61.15	1180.93
[58, 70)	[89.95, 94.06)	-3571E-8	-4036E-6	1.29921	35.14
[70, 85)	[94.06, 96.42)	56E-5/3	-0.048	12.67/8	-30.40
[85, 100)	[96.42, 97.49)	-5333E-8	0.0126	-2.663/3	113.59
[100, 130)	[97.49, 98.44)	25E-6/3	-0.00335	1.409/3	75.69
[130, 175)	[98.44, 99.03)	4.127E-7	-97E-5/3	0.331/4	92.24
[175, 250)	[99.03, 99.41)	64E-8/3	-168E-6	0.1418/3	94.76
[250, 500)	[99.41, 99.72)	22E-9	-289E-7	0.01329	97.55
[500, 1000)	[99.72, 100.00)	3.238E-9	-639E-8	0.004472	98.68

$S = A3 \times P^3 + A2 \times P^2 + A1 \times P + A0$, where $S = S_{O_2}$, $P = vP_{O_2}$, $[a, b)$ means $a < (P \text{ or } S) < b$ and $E-n$ means $\times 10^{-n}$. It is assumed in these equations that $S_{O_2} = 100.00$ when $P_{O_2} = 1000$. Since S_{O_2} is expressed as a cubic function of vP_{O_2} , it is relatively easy to calculate vP_{O_2} from S_{O_2} . The theoretical limitation of error for S_{O_2} is 0.02% S_{O_2} for the range of 0 to 500 in P_{O_2} , but the differences are practically less than 0.004% S_{O_2} , when the values of S_{O_2} calculated by these equations are compared with the 51 data pairs for the "standard" oxygen hemoglobin equilibrium curve reported by SEVERINGHAUS (1979).

Fig. 1. Suggested flow-chart to calculate $P_{O_2}^T$. Maximum errors for $P_{O_2}^T$ are calculated from the following: $(dP_{O_2}^T/dC_{O_2}^T) \times 10^{-6}$. $dP_{O_2}^T/dC_{O_2}^T$, which is almost proportional to $dP_{O_2}^T/dS_{O_2}^T$, becomes larger as $P_{O_2}^T$ increases and as Hb decreases. However, the error for $P_{O_2}^T$ is as much as 6.0×10^{-5} when $P_{O_2}^T = 500$ and Hb = 5, indicating that values of $\Delta \log P_{O_2}/\Delta T$ obtained from $P_{O_2}^T$ are accurate unless values of ΔT are very small.

$$P_{O_2}^{37} = vP_{O_2}^{37}(\exp_{10}(f_B(\text{pH}^{37} - 7.4) + 0.0013(\text{BE}))(P_{50}/26.7)) \quad (3)$$

$$vP_{O_2}^T = P_{O_2}^T(\exp_{10}(f_B(7.4 - \text{pH}^T) - 0.0013(\text{BE}) + 0.024(37 - T)))(26.7/P_{50}) \quad (4)$$

where f_B is the Bohr factor, which may be calculated from the following equation derived from the data of HLASTALA and WOODSON (1975),

$$f_B = (vP_{O_2}^{37}/26.6)^{0.08} - 1.52 \quad (5)$$

and

$$\text{pH}^T = \text{pH}^{37} - (T - 37)(0.0147 + 0.0065(\text{pH}^{37} - 7.4)) \quad (6)$$

$$\text{BE} = (1 - 0.0143(\text{Hb}))((\text{HCO}_3^-) - (9.5 + 1.63(\text{Hb}))(7.4 - \text{pH}^{37}) - 24) \quad (7)$$

$$\text{HCO}_3^- = 0.0306(P_{\text{CO}_2}^{37})(\exp_{10}(1.04214(\text{pH}^{37}) - 6.4104)) \quad (8)$$

$C_{O_2}^T$ can be expressed as a function of $P_{O_2}^T$, when Hb, pH^{37} , $P_{\text{CO}_2}^{37}$, P_{50} , and T are given. We obtained $P_{O_2}^T$ corresponding to $P_{O_2}^{37}$ and $vP_{O_2}^{37}$ where differences between $C_{O_2}^{37}$ and $C_{O_2}^T$ are less than 10^{-6} by a Newton-Raphson iteration method following the flow-chart which is shown in Fig. 1.

RESULTS

Figure 2 shows the effects of Hb on the relationship between $\Delta \log P_{O_2}/\Delta T$ or f and $vP_{O_2}^{37}$ ($= P_{O_2}^{37}$) for normal pH^{37} and P_{50} . A decrease in Hb caused a leftward

Fig. 2. Effects of Hb on the relationship between $\Delta \log P_{O_2}/\Delta T$ and $vP_{O_2}^{37}$ ($=P_{O_2}^{37}$) at pH=7.4, $P_{CO_2}^{37}=40$ and $T=27$, — our values (Hb=20, 16, 12, 8, 4, right to left); ---- the values calculated by the equation of Kelman-Nunn (Eq. 9); ···· the values calculated by the equation of Severinghaus (Eq. 11).

shift of the functional curve between f and vP_{O_2} . In this figure the curve obtained from the equations of KELMAN and NUNN (1966, Eq. 9) and SEVERINGHAUS (1979, Eq. 10) are also shown.

$$\Delta \log P_{O_2}/\Delta T = 0.0052 + 0.0268(1 - \exp_e(-0.3(100 - S_{O_2}^{37}))) \quad (9)$$

$$\Delta \ln P_{O_2}/\Delta T = 0.013 + 0.058(0.243(P_{O_2}^{37}/100)^{3.88} + 1)^{-1} \quad (10)$$

Equation 10 may be transformed to the following equation:

$$\begin{aligned} \Delta \log P_{O_2}/\Delta T &= (\Delta \ln P_{O_2}/\Delta T)/\ln 10 = 0.0056 \\ &+ 0.0252(0.243(P_{O_2}^{37}/100)^{3.88} + 1)^{-1}. \end{aligned} \quad (11)$$

The curve obtained for Hb=20 was in good agreement with the curve derived from the equation of SEVERINGHAUS (1979).

As was the case with decreasing Hb, the curve illustrating the relationship between f and $vP_{O_2}^{37}$ was displaced downward and to the left by a decrease in pH³⁷ (Fig. 3), while the curve between f and $P_{O_2}^{37}$ shifted downward and to the right (Fig. 4) under the condition of Hb=15. $P_{CO_2}^{37}=40$ mmHg, and $P_{50}=26.7$. Values of f

Fig. 3. Effects of pH^{37} on the relationship between $\Delta \log P_{\text{O}_2} / \Delta T$ and $vP_{\text{O}_2}^{37}$ at $\text{Hb} = 15$, $P_{\text{CO}_2}^{37} = 40$, $T = 27$, for $\text{pH}^{37} = 7.6, 7.4, 7.2, 7.0$ (— right to left; ---- Kelman-Nunn).

were also calculated when the changes in pH^{37} were introduced by the changes in $P_{\text{CO}_2}^{37}$ with constant BE. Comparison of these f to those indicated in Figs. 3 and 4 showed negligible differences (less than 0.4×10^{-3}). The effects of changes in P_{50} were also studied. It was verified that increasing P_{50} had same effects with decreasing pH^{37} .

The effects of T on the values of f are shown in Fig. 5. The magnitude of f determined by varying $vP_{\text{O}_2}^{37}$ (ranging from 40 to 400 mmHg) was almost constant in spite of large changes in T . However, it showed slight temperature dependency when observed with the condition of moderate hyperoxia ($vP_{\text{O}_2}^{37} = 100$ and 200 mmHg).

Therefore, $\Delta \log P_{\text{O}_2} / \Delta T$ or f should be determined not only by $vP_{\text{O}_2}^{37}$ or $P_{\text{O}_2}^{37}$ but also by Hb , pH^{37} , P_{50} and T .

DESIGN OF NEW EQUATIONS AND NOMOGRAMS

We modified Eq. 11 reported by Severinghaus for the design of new equations in which the effects of Hb , pH^{37} , and T are taken into account.

Equation 11 was rewritten as follows.

Fig. 4. Effects of pH^{37} on the relationship between $\Delta \log P_{\text{O}_2} / \Delta T$ and $P_{\text{O}_2}^{37}$ at $\text{Hb} = 15$, $P_{\text{CO}_2}^{37} = 40$, and $T = 27$, for $\text{pH}^{37} = 7.6, 7.4, 7.2, 7.0$ (— left to right; ···· Severinghaus).

$$\Delta \log P_{\text{O}_2} / \Delta T = (0.56 + (3.08 - 0.56)(0.422(10^{-7})(P_{\text{O}_2}^{37})^{3.88} + 1)^{-1})(10^{-2}) \quad (12)$$

Then, constants 3.08, 0.56, 0.422×10^{-7} , and 3.88 were defined as U , L , A , and B , respectively, and $P_{\text{O}_2}^{37}$ was replaced by $vP_{\text{O}_2}^{37}$.

$$\Delta \log P_{\text{O}_2} / \Delta T = (L + (U - L)/(A(vP_{\text{O}_2}^{37})^B + 1))(10^{-2}) \quad (13)$$

Equation 13 may be mathematically transformed to the following linear form.

$$\log((U/100 - \Delta \log P_{\text{O}_2} / \Delta T) / (\Delta \log P_{\text{O}_2} / \Delta T - L/100)) = B(\log vP_{\text{O}_2}^{37}) + \log A \quad (14)$$

We analyzed values of $\Delta \log P_{\text{O}_2} / \Delta T$ which were obtained by the iteration method for $\text{Hb} = 5-10$, $\text{pH}^{37} = 7.0-7.6$, $T = 27-40$, and $vP_{\text{O}_2} = 20-500$, and expressed each of constants as a function of Hb , pH^{37} , and T .

Since U and L , which provide the upper and lower limitations of $\Delta \log P_{\text{O}_2} / \Delta T$, were affected mainly by pH^{37} , they were determined by the following equations.

$$U = 3.15 - 0.45(7.4 - \text{pH}^{37}) \quad (15)$$

$$L = 0.68 - 0.09(7.4 - \text{pH}^{37}) \quad (16)$$

Fig. 5. Relationships between $\Delta \log P_{O_2} / \Delta T$ and T at $Hb = 20$, $pH^{37} = 7.4$ and $P_{CO_2}^{37} = 40$, for $vP_{O_2}^{37} = 40, 100, 200, 400$ (top to bottom). f values for $T = 36-38$ are not shown, as errors by the iteration method are large in these T ranges.

A changed exponentially depending upon pH^{37} and T . B changed exponentially depending on T and linearly on pH^{37} . However both A and B can be fundamentally expressed as a power function of Hb .

$$A = 5.86(\exp_{10}(0.074(T) - 0.294(7.4 - pH^{37}) - 11))((Hb)^{0.913}) \quad (17)$$

$$B = 6.33(\exp_{10}(-0.0051(T)))((Hb)^{-0.113}) + 0.24(7.4 - pH^{37}) \quad (18)$$

The differences between the values of f calculated from the above equations and those obtained by the iteration method were very small (less than 1.3×10^{-3} at $T = 27$). These values of difference are extremely small as compared with those obtained by the equations of Kelman-Nunn and Severinghaus, indicating that these equations can be applied in a wide range of Hb , pH^{37} , and T , provided that about 3% of maximum error for $P_{O_2}^T$ is permitted (Table 2).

Nomograms for the calculation of $P_{O_2}^T$ based on the above observation are shown in Fig. 6. In these nomograms, the effects of T on f were not included.

DISCUSSION

Judging from the above results, it is surprising that the equations which did not

Fig. 6. Nomograms for the calculation of $P_{O_2}^T$. First, read f corresponding to $vP_{O_2}^{37}$ in the lower right graph. When pH^{37} differs from 7.4, read $\Delta f / \Delta \text{pH}$ in the upper graph and add Δf to f . Plot this corrected f on vertical line x and this plot is horizontally moved on line y . Draw a line between the plot on line y and 37 in the left graph and read $P_{O_2}^T / P_{O_2}^{37}$ corresponding to T on this line.

take the effect of Hb, pH^{37} , and T into consideration have been used clinically and in computer programs for blood gas analyzers. It can be speculated easily that the values of f calculated by equations 9 and 11 may differ widely from those obtained by the iteration method during deep hypothermia under ether anesthesia and slight hemodilution which may cause a decrease in pH^{37} and Hb. Figure 7 shows % differences between the values of $P_{O_2}^T$ calculated by the equations of Severinghaus, Kelman-Nunn, or our equations and those obtained from the iteration method at $\text{Hb} = 5$, $\text{pH}^{37} = 7.2$, and $T = 32$. The % difference in $P_{O_2}^T$ caused by the application of the equation of Severinghaus amounted to as much as 10% in

Table 2. The ranges of difference between $\Delta \log P_{O_2}/\Delta T$ ($\times 10^{-2}$) obtained by the iteration method and those calculated from the equation of Kelman-Nunn (K-N), Severinghaus (S), or our equations (A) under various conditions at $T=27$.

		Hb = 20	Hb = 15	Hb = 10	Hb = 5
pH = 7.4	K-N	-0.66-0.03 (-0.7-15.1)	-0.51-0.05 (-1.1-12.5)	-0.33-0.14 (-3.2-7.9)	0.01-0.33 (-7.3-0.2)
	S	0.01-0.17 (-3.8-0.3)	-0.10-0.14 (-3.2-2.3)	-0.08-0.42 (-9.2-1.9)	-0.05-0.90 (-18.7-1.2)
	A	-0.08-0.02 (-0.5-1.9)	-0.09-0.02 (-0.5-2.0)	-0.10-0.03 (-0.7-2.3)	-0.13-0.05 (-1.1-3.0)
pH = 7.2	K-N	-0.50-0.20 (-4.5-12.2)	-0.40-0.21 (-4.7-9.6)	-0.22-0.23 (-5.2-5.2)	0.14-0.37 (-8.2-3.2)
	S	-0.34-0.01 (-0.2-8.1)	-0.18-0.02 (-0.5-4.2)	-0.06-0.13 (-2.9-1.4)	-0.03-0.59 (-12.7-0.7)
	A	-0.07-0.07 (-1.6-1.6)	-0.08-0.02 (-0.5-1.9)	-0.08-0.07 (-1.5-1.9)	-0.11-0.07 (-1.6-2.6)
pH = 7.0	K-N	-0.38-0.27 (-6.0-9.1)	-0.27-0.28 (-6.2-6.4)	-0.07-0.23 (-5.2-1.6)	0.15-0.46 (-10.1-3.3)
	S	-0.57-0.06 (-1.4-14.0)	-0.41-0.06 (-1.4-9.9)	-0.20-0.07 (-1.6-4.7)	-0.02-0.27 (-6.1-0.5)
	A	-0.07-0.08 (-1.8-1.6)	-0.03-0.05 (-1.1-0.7)	-0.08-0.06 (-1.4-1.9)	-0.08-0.05 (-1.1-1.9)
pH = 7.6	K-N	-0.68-0.02 (-0.5-16.9)	-0.63-0.10 (-2.3-15.6)	-0.48-0.09 (-2.1-11.7)	-0.11-0.25 (-5.6-2.6)
	S	-0.13-0.21 (-4.7-3.0)	-0.06-0.36 (-8.0-1.4)	-0.11-0.62 (-13.3-2.6)	-0.10-1.07 (-21.8-2.3)
	A	-0.08-0.10 (-2.3-1.9)	-0.07-0.09 (-2.1-1.6)	-0.11-0.07 (-1.6-2.6)	-0.13-0.09 (-2.1-3.0)

% differences in $P_{O_2}^T$ are shown in parentheses. These may be calculated by the following equation in which differences in $\Delta \log P_{O_2}/\Delta T$ are defined as Δf : $100(\exp_{10}(\Delta f(T-37))-1)$.

$P_{O_2}^{37}$ values around 110.

Our results are in good agreement with the work of THOMAS (1972), and with the recent report of SIGGARD-ANDERSON *et al.* (1984) except for the results demonstrating the effects of P_{50} on the relationship between f and $S_{O_2}^{37}$, which showed a leftward shift of the curve upon decreasing P_{50} . We did not show the effects of P_{50} on the relationship between f and $P_{O_2}^{37}$ or $vP_{O_2}^{37}$ in Figs. 3 and 4, but it was verified that a decrease in P_{50} has the same effects with increasing pH³⁷, so the curve between f and $S_{O_2}^{37}$ should be displaced upward and to the right from our results and those of Thomas. This discrepancy may be partly caused by the difference of algorithms applied for the expression of the hemoglobin-oxygen dissociation curve. Their algorithms gave higher values of S_{O_2} for high vP_{O_2} values

Fig. 7. % difference between $P_{O_2}^T$ calculated by the equations of Kelman-Nunn, Severinghaus, or our equations and that obtained by the iteration method for Hb = 5, $pH^{37} = 7.0$, and $T = 30$ (— our equations; ---- Kelman-Nunn; Severinghaus).

than those derived from the data of SEVERINGHAUS (1979) and calculated from the algorithm of SASAI *et al.* which we used, indicating that they may have overestimated the slope of the hemoglobin-oxygen dissociation curve and that may have affected the calculations of f values. It is possible that the factor which expresses the effects of abnormal P_{50} values on the relationship between f and $vP_{O_2}^{37}$ is added to our equations. The following equations showing the relationship between pH^{37} and P_{50} can be applied.

$$P'_{50}/P_{50} = \exp_{10}(0.54 (7.4 - pH^{37})) \quad (19)$$

where P'_{50} is $P_{O_2}^{37}$ at pH^{37} and $S_{O_2}^{37} = 0.5$, and 0.54 is the Bohr factor at $S_{O_2} = 0.5$. Thus,

$$7.4 - pH^{37} = (\log P'_{50} - \log P_{50})/0.54 \quad (20)$$

It is difficult to measure P_{50} directly in usual laboratory work. However P_{50} can be estimated by measuring $S_{O_2}^{37}$ directly from venous blood of the same patient with $S_{O_2}^{37}$ values of from 0.6 to 0.8. For this range of $S_{O_2}^{37}$, the magnitude of the Bohr factor can be considered to be almost constant (0.5, HLASTLA and WOODSON, 1975). Therefore Eq. 18 may be rewritten as follows:

$$7.4 - pH^{37} = (\log vP_{O_2}^{37'} - \log vP_{O_2}^{37})/0.5 = C \quad (21)$$

where $vP_{O_2}^{37'}$ is $vP_{O_2}^{37}$ calculated by algorithms of SASAI *et al.* from directly measured $S_{O_2}^{37}$, and $vP_{O_2}^{37}$ is a value calculated from Eq. 3 to 8. By adding Eq. 19 to those from 13 to 16, useful equations are obtained.

$$U = 3.15 - 0.45 (7.4 - pH^{37} + C) \quad (15')$$

$$L = 0.68 - 0.09 (7.4 - pH^{37} + C) \quad (16')$$

$$A = 5.86 (\exp_{10}(0.074(T) - 0.294 (7.4 - \text{pH}^{37} + C) - 11))((\text{Hb})^{0.913}) \quad (17')$$

$$B = 6.33 (\exp_{10}(-0.0051(T)))((\text{Hb})^{-0.113}) + 0.24 (7.4 - \text{pH}^{37} + C) \quad (18')$$

When the above equations and Eq. 12 are applied, $vP_{\text{O}_2}^{37}$ should be calculated by the following equation derived from Eq. 3.

$$vP_{\text{O}_2}^{37} = P_{\text{O}_2}^{37} (\exp_{10}(f_B(7.4 - \text{pH}^{37}) - 0.0013(\text{BE}))) (1/D) \quad (3')$$

where D is $vP_{\text{O}_2}^{37}/vP_{\text{O}_2}^{37}$, obtained from the analysis of venous blood mentioned above.

We could not find any reports on previous experimental data about anaerobic temperature correction for P_{O_2} , in which the effects of Hb, pH, P_{50} , and T are fully considered. For example, NUNN *et al.* (1965) gave a few values of f , but they did not show the pH value. Therefore, it is impossible to compare the values of f obtained by the iteration method with those obtained from the measured values of $P_{\text{O}_2}^T$.

Each value of f determined by us is based on the data concerning factors and variables for blood gases and acid-base reported previously. Assuming that the data from which the equations applied by us were derived are valid, the accuracy for f therefore depends mainly on the fidelity with which the equations represent the data. Among those equations, equations for the expression of the oxygen hemoglobin equilibrium curve are most important, because the shape or the slope of curve has strong effects on the value of f . The equations of SASAI *et al.* retain excellent accuracy with practical error of less than 0.004% S_{O_2} . This means that the errors for f derived from these equations are so small that we cannot show them, because SEVERINGHAUS (1979) gave the values of S_{O_2} only down to two decimal places in his table for the "standard" oxygen hemoglobin equilibrium curve. We therefore believe that our values of f or $P_{\text{O}_2}^T$ fit closely to those obtained by direct measurements, provided that the oxygen hemoglobin equilibrium curve of blood is "standard" and in good agreement with that reported by SEVERINGHAUS (1979). However, our equations may not give exact values of $P_{\text{O}_2}^T$ for blood containing CO hemoglobin which causes changes in the shape of the oxygen hemoglobin equilibrium curve. The only way, therefore, to obtain exact values of $P_{\text{O}_2}^T$ for such blood is for those values to be measured directly with blood gas analyzers in which the temperature around their electrodes is kept equal to T . However this procedure requires time-consuming elaboration.

We therefore believe that a time-saving and reliable method of temperature correction for P_{O_2} of "normal" blood is for P_{O_2} to be measured at 37°C and then corrected to the value at T by our equations or algorithms reported recently by SIGGARD-ANDERSON *et al.* (1983). However, as mentioned above, our equations may give different values of f from those derived from the algorithms of SIGGARD-ANDERSON *et al.* Further studies are necessary to elucidate this discrepancy between these two observations.

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