A K -SAMPLE RANK TEST BASED ON MODIFIED BAUMGARTNER STATISTIC AND ITS POWER COMPARISON

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ABSTRACT

The purpose of this paper is to develop a nonparametric k -sample test based on a modified Baumgartner statistic. We define a new modified Baumgartner statistic B^* and give some critical values. Then we compare the power of the B^* statistic with the t -test, the Wilcoxon test, the Kolmogorov-Smirnov test, the Cramér-von Mises test, the Anderson-Darling test and the original Baumgartner statistic. The B^* statistic is more suitable than the Baumgartner statistic for the location parameter when the sample sizes are not equal. Also, the B^* statistic has almost the same power as the Wilcoxon test for location parameter. For scale parameter, the power of the B^* statistic is more efficient than the Cramér-von Mises test and the Anderson-Darling test when the sizes are equal. The power of the B^* statistic is higher than the Kolmogorov-Smirnov test for location and scale parameters. Then the B^* statistic is generalized from two-sample to k -sample problems. The B_k^* statistic denotes a k -sample statistic based on the B^* statistic. We compare the power of the B_k^* statistic with the Kruskal-Wallis test, the k -sample Kolmogorov-Smirnov test, the k -sample Cramér-von Mises test, the k -sample Anderson-Darling test and the k -sample Baumgartner statistic. Finally, we investigate the behavior of power about the B_k^* statistics by simulation studies. As a result, we obtain that the B_k^* statistic is more suitable than the other statistics.

1. Introduction

Let $\mathbf{X} = (X_1, \dots, X_n)$ and $\mathbf{Y} = (Y_1, \dots, Y_m)$ be two random samples of size n and m independent observations from different populations with continuous distribution $F(x)$ and $G(y)$, respectively. The well known test of Wilcoxon (Hollander and Wolfe, 1999) is a test for location as $F(x) = G(y - \theta)$. Baumgartner, Weiß and Schindler (1998) presented a new nonparametric two-sample test which has almost same power as the Wilcoxon test for the location parameter. They assert that the Baumgartner statistic can be applied for the scale parameter as $F(x) = G(\frac{y}{\sigma})$, where $\sigma \neq 0$. Let $R_1 < \dots < R_n$ and $H_1 < \dots < H_m$ denote the combined-samples ranks of the X -values and Y -values in increasing order of magnitude, respectively. The test statistic B proposed by Baumgartner *et al.* (1998) is

$$B = \frac{1}{2}(B_X + B_Y),$$

where

$$B_X = \frac{1}{n} \sum_{i=1}^n \frac{(R_i - \frac{n+m}{n}i)^2}{\frac{i}{n+1} (1 - \frac{i}{n+1}) \frac{m(n+m)}{n}}$$

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and

$$B_Y = \frac{1}{m} \sum_{j=1}^m \frac{(H_j - \frac{m+n}{m}j)^2}{\frac{j}{m+1}(1 - \frac{j}{m+1}) \frac{n(m+n)}{m}}.$$

Recently, Neuhäuser (2003) researched ties of the Baumgartner statistic. Also, Neuhäuser (2000) suggested an exact two-sample test based on the Baumgartner statistic. In this paper, first, we propose a new nonparametric test by modifying the Baumgartner statistic. Second, we extend the new statistic to the k -sample problem. Finally, to compare the new method with previous methods, we will investigate its power by simulation study.

In Section 2, we consider the modified Baumgartner statistic B^* and compare the power of the B^* statistic with the t -test, the Wilcoxon test W , the Kolmogorov-Smirnov test KS (Hollander and Wolfe, 1999), the Cramér-von Mises test CV (Anderson, 1962), the Anderson-Darling test AD (Pettitt, 1976) and the B statistic. Some critical values of the B^* statistic are listed in Table 1. We know that the B^* statistic is more efficient than the B statistic when $n \neq m$. In Section 3, since the k -sample problems are important in statistical practice, we propose the statistic B_k^* for the k -sample test and investigate the behavior of the B_k^* statistic. Then, we compare the power of the B_k^* statistic with the Kruskal-Wallis test W_k (Hájek, Šidák and Sen, 1999), the k -sample Kolmogorov-Smirnov test KS_k (Hájek *et al.*, 1999), the k -sample Cramér-von Mises test CV_k (Kiefer, 1959), the k -sample Anderson-Darling test AD_k (Scholz and Stephens, 1987) and the k -sample Baumgartner statistic B_k . For comparison of the statistics, we perform simulation studies using the normal, double exponential, logistic and exponential distributions as different populations. We perform the simulation studies for $k = 3$. For all simulation studies, we have 1,000,000 times repetition.

2. Modified Baumgartner statistic

In this section, we propose the modified Baumgartner statistic B^* , and compare the power with the t -test, the W statistic, the KS statistic, the CV statistic, the AD statistic and the B statistic by simulation studies. It is well known that the expected value and variance of R_i are

$$E[R_i] = \frac{n+m+1}{n+1}i$$

and

$$\text{Var}[R_i] = \frac{i}{n+1} \left(1 - \frac{i}{n+1}\right) \frac{m(n+m+1)}{n+2},$$

respectively (Arnold, Balakrishnan and Nagaraja, 1992). Similarly, the expected value and variance of H_j are

$$E[H_j] = \frac{m+n+1}{m+1}j$$

and

$$\text{Var}[H_j] = \frac{j}{m+1} \left(1 - \frac{j}{m+1}\right) \frac{n(m+n+1)}{m+2}.$$

Here, the modified Baumgartner statistic B^* is defined by

$$B^* = \frac{1}{2}(B_X^* + B_Y^*),$$

where

$$B_X^* = \frac{1}{n} \sum_{i=1}^n \frac{(R_i - E[R_i])^2}{\text{Var}[R_i]}$$

and

$$B_Y^* = \frac{1}{m} \sum_{j=1}^m \frac{(H_j - E[H_j])^2}{\text{Var}[H_j]}.$$

Now, we compare the power of the B^* statistic with some other statistics before we elaborate. Suppose that $F(x)$ and $G(y)$ are distributed as $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, $DE(\mu_1, \sigma_1)$ and $DE(\mu_2, \sigma_2)$, $L(\mu_1, \sigma_1)$ and $L(\mu_2, \sigma_2)$, and $\eta(\lambda_1)$ and $\eta(\lambda_2)$, respectively. $N(\mu, \sigma^2)$ denotes the normal distribution with mean μ and variance σ^2 . $DE(\mu, \sigma)$ and $L(\mu, \sigma)$ denote the double exponential and logistic distributions with mean μ and variance σ , respectively. Also, $\eta(\lambda)$ denotes the exponential distribution with the mean parameter λ . Neuhäuser (2000) calculated the critical values of the B statistic ($\Pr(B \geq b)$) for equal sample sizes only. The critical values of B statistic and B^* statistic are listed in Table 1 including different sample sizes for simulation studies. Baumgartner *et al.* (1998) derived the asymptotic distribution, and its critical values were $\Pr(B \geq 3.880) = 0.010$ and $\Pr(B \geq 2.493) = 0.050$. The asymptotic distribution of the B^* statistic is same as the B statistic. From the Table 1, we can easily see that b^* converges to the point of the asymptotic distribution.

Table 1-(a): Exact critical value

n	m	$\Pr(B \geq b)$	b	$\Pr(B \geq b)$	b	$\Pr(B^* \geq b^*)$	b^*	$\Pr(B^* \geq b^*)$	b^*
5	5	0.0079	4.894	0.0476	2.533	0.0079	5.536	0.0476	2.870
10	5	0.0100	4.033	0.0496	2.552	0.0100	4.326	0.0500	2.703
10	10	0.0100	4.120	0.0500	2.583	0.0100	4.359	0.0500	2.700

Table 1-(b): Critical value by simulation

n	m	$\Pr(B \geq b)$	b	$\Pr(B \geq b)$	b	$\Pr(B^* \geq b^*)$	b^*	$\Pr(B^* \geq b^*)$	b^*
10	10	0.0100	4.119	0.0500	2.583	0.0100	4.359	0.0500	2.700
20	20	0.0100	4.129	0.0500	2.596	0.0100	4.219	0.0500	2.633
30	30	0.0100	4.087	0.0500	2.584	0.0100	4.144	0.0500	2.604
40	40	0.0100	4.063	0.0500	2.576	0.0100	4.097	0.0500	2.578
50	50	0.0100	4.021	0.0500	2.565	0.0100	4.081	0.0500	2.572
100	100	0.0100	3.962	0.0500	2.535	0.0100	3.991	0.0500	2.539
200	200	0.0100	3.929	0.0500	2.522	0.0100	3.937	0.0500	2.523
300	300	0.0100	3.910	0.0500	2.512	0.0100	3.905	0.0500	2.504

The values in Tables 2 to 5 are results for the case of $\mu_1 \neq \mu_2$ and $\sigma_1 = \sigma_2$ or $\mu_1 = \mu_2$ and $\sigma_1 \neq \sigma_2$ when $n = m$ and $n \neq m$, respectively, and the significant level is 5 %.

Table 2-(a): The case of $n = m = 10$ for $N(0, 1)$ and $N(\mu_2, 1)$

	μ_2						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
t	0.05	0.19	0.56	0.89	0.99	1.00	1.00
W	0.05	0.18	0.54	0.86	0.98	1.00	1.00
KS	0.05	0.16	0.47	0.80	0.96	1.00	1.00
CM	0.05	0.17	0.52	0.85	0.98	1.00	1.00
AD	0.05	0.17	0.52	0.86	0.98	1.00	1.00
B	0.05	0.17	0.52	0.85	0.98	1.00	1.00
B^*	0.05	0.17	0.52	0.85	0.98	1.00	1.00

Table 2-(b): The case of $n = m = 10$ for $N(0, 1)$ and $N(0, \sigma_2^2)$

	σ_2						
	1.0	5.0	10.0	15.0	20.0	25.0	30.0
W	0.05	0.08	0.09	0.10	0.10	0.10	0.10
KS	0.05	0.32	0.49	0.57	0.61	0.64	0.66
CM	0.05	0.23	0.39	0.48	0.53	0.57	0.60
AD	0.05	0.30	0.54	0.66	0.74	0.78	0.82
B	0.05	0.34	0.58	0.70	0.77	0.81	0.84
B^*	0.05	0.33	0.58	0.70	0.77	0.81	0.84

Table 2-(c): The case of $n = 10, m = 5$ for $N(0, 1)$ and $N(\mu_2, 1)$

	μ_2						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
t	0.05	0.14	0.39	0.72	0.92	0.99	1.00
W	0.05	0.13	0.38	0.70	0.90	0.98	0.99
KS	0.06	0.13	0.36	0.64	0.86	0.96	0.99
CM	0.05	0.13	0.36	0.67	0.89	0.98	1.00
AD	0.05	0.13	0.36	0.67	0.89	0.98	1.00
B	0.05	0.10	0.31	0.62	0.86	0.97	1.00
B^*	0.05	0.13	0.36	0.67	0.89	0.98	1.00

Table 2-(d): The case of $n = 10, m = 5$ for $N(0, 1)$ and $N(0, \sigma_2^2)$

	σ_2						
	1.0	5.0	10.0	15.0	20.0	25.0	30.0
W	0.05	0.10	0.08	0.08	0.07	0.07	0.07
KS	0.06	0.22	0.29	0.31	0.33	0.34	0.34
CM	0.05	0.21	0.28	0.31	0.33	0.34	0.34
AD	0.05	0.23	0.30	0.32	0.34	0.34	0.35
B	0.05	0.22	0.29	0.31	0.33	0.34	0.34
B^*	0.05	0.22	0.29	0.31	0.33	0.34	0.34

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Table 3-(a): The case of $n = m = 10$ for $DE(0, 1)$ and $DE(\mu_2, 1)$

	μ_2						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
W	0.05	0.15	0.42	0.71	0.88	0.96	0.99
KS	0.05	0.15	0.43	0.71	0.89	0.96	0.99
CM	0.05	0.15	0.42	0.72	0.89	0.97	0.99
AD	0.05	0.15	0.42	0.71	0.89	0.97	0.99
B	0.05	0.15	0.42	0.71	0.89	0.97	0.99
B^*	0.05	0.15	0.42	0.71	0.89	0.97	0.99

Table 3-(b): The case of $n = m = 10$ for $DE(0, 1)$ and $DE(0, \sigma_2)$

	σ_2						
	1.0	3.0	5.0	7.0	9.0	11.0	13.0
W	0.05	0.06	0.07	0.08	0.09	0.09	0.10
KS	0.05	0.13	0.23	0.31	0.36	0.41	0.45
CM	0.05	0.10	0.17	0.22	0.27	0.31	0.35
AD	0.05	0.11	0.20	0.29	0.37	0.43	0.49
B	0.05	0.12	0.23	0.32	0.40	0.47	0.52
B^*	0.05	0.12	0.22	0.32	0.40	0.46	0.52

Table 3-(c): The case of $n = 10, m = 5$ for $DE(0, 1)$ and $DE(\mu_2, 1)$

	μ_2						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
W	0.05	0.12	0.30	0.52	0.71	0.84	0.92
KS	0.06	0.13	0.32	0.55	0.74	0.87	0.93
CM	0.05	0.11	0.29	0.52	0.72	0.86	0.93
AD	0.05	0.11	0.29	0.52	0.72	0.86	0.93
B	0.05	0.09	0.25	0.47	0.68	0.83	0.92
B^*	0.05	0.12	0.30	0.52	0.72	0.86	0.93

Table 3-(d): The case of $n = 10, m = 5$ for $DE(0, 1)$ and $DE(0, \sigma_2)$

	σ_2						
	1.0	3.0	5.0	7.0	9.0	11.0	13.0
W	0.05	0.09	0.10	0.10	0.10	0.09	0.09
KS	0.06	0.13	0.18	0.22	0.24	0.26	0.28
CM	0.05	0.12	0.17	0.21	0.24	0.26	0.27
AD	0.05	0.13	0.19	0.23	0.26	0.28	0.29
B	0.05	0.12	0.18	0.21	0.24	0.26	0.27
B^*	0.05	0.12	0.18	0.21	0.24	0.26	0.27

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Table 4-(a): The case of $n = m = 10$ for $L(0, 1)$ and $L(\mu_2, 1)$

	μ_2						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
W	0.05	0.09	0.23	0.44	0.67	0.84	0.93
KS	0.05	0.09	0.21	0.41	0.62	0.80	0.91
CM	0.05	0.09	0.22	0.43	0.66	0.83	0.93
AD	0.05	0.09	0.22	0.43	0.66	0.83	0.93
B	0.05	0.09	0.22	0.43	0.65	0.83	0.93
B^*	0.05	0.09	0.22	0.43	0.66	0.83	0.93

Table 4-(b): The case of $n = m = 10$ for $L(0, 1)$ and $L(0, \sigma_2)$

	σ_2						
	1.0	3.0	5.0	7.0	9.0	11.0	13.0
W	0.05	0.06	0.08	0.08	0.09	0.09	0.09
KS	0.05	0.16	0.29	0.38	0.44	0.49	0.52
CM	0.05	0.12	0.21	0.28	0.34	0.39	0.42
AD	0.05	0.14	0.27	0.39	0.47	0.53	0.59
B	0.05	0.16	0.31	0.42	0.51	0.58	0.63
B^*	0.05	0.16	0.30	0.42	0.51	0.57	0.63

Table 4-(c): The case of $n = 10, m = 5$ for $L(0, 1)$ and $L(\mu_2, 1)$

	μ_2						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
W	0.05	0.08	0.17	0.31	0.49	0.66	0.79
KS	0.06	0.08	0.17	0.30	0.47	0.64	0.78
CM	0.05	0.08	0.16	0.30	0.47	0.64	0.79
AD	0.05	0.08	0.16	0.30	0.47	0.65	0.79
B	0.05	0.06	0.13	0.26	0.41	0.59	0.74
B^*	0.05	0.08	0.16	0.30	0.47	0.65	0.79

Table 4-(d): The case of $n = 10, m = 5$ for $L(0, 1)$ and $L(0, \sigma_2)$

	σ_2						
	1.0	3.0	5.0	7.0	9.0	11.0	13.0
W	0.05	0.09	0.10	0.09	0.09	0.09	0.08
KS	0.06	0.15	0.21	0.25	0.27	0.29	0.30
CM	0.05	0.14	0.20	0.24	0.27	0.29	0.30
AD	0.05	0.15	0.22	0.26	0.29	0.30	0.31
B	0.05	0.14	0.20	0.24	0.27	0.29	0.30
B^*	0.05	0.14	0.20	0.24	0.27	0.29	0.30

Table 5-(a): The case of $n = m = 10$ for $\eta(1)$ and $\eta(\lambda_2)$

	λ_2						
	1.0	2.0	3.0	4.0	5.0	6.0	7.0
W	0.05	0.24	0.51	0.69	0.80	0.87	0.91
KS	0.05	0.22	0.46	0.64	0.77	0.84	0.90
CM	0.05	0.23	0.49	0.68	0.80	0.87	0.91
AD	0.05	0.24	0.50	0.69	0.81	0.88	0.92
B	0.05	0.24	0.51	0.70	0.81	0.89	0.92
B^*	0.05	0.24	0.50	0.70	0.81	0.88	0.92

Table 5-(b): The case of $n = 10, m = 5$ for $\eta(1)$ and $\eta(\lambda_2)$

	λ_2						
	1.0	2.0	3.0	4.0	5.0	6.0	7.0
W	0.05	0.19	0.37	0.51	0.61	0.69	0.74
KS	0.06	0.18	0.34	0.47	0.58	0.66	0.72
CM	0.05	0.18	0.35	0.50	0.60	0.69	0.75
AD	0.05	0.18	0.37	0.52	0.63	0.70	0.76
B	0.05	0.15	0.32	0.46	0.57	0.66	0.72
B^*	0.05	0.18	0.36	0.51	0.61	0.69	0.75

From the Tables, we can see that the power of the B and B^* statistics are almost equal for the location test when $n = m$. However, the power of the B^* statistic is greater than that of the B statistic when $n \neq m$ for the location test. For the case of $\mu_1 = \mu_2, \sigma_1 \neq \sigma_2$, both the B and B^* statistics have almost equal power when $n = m$ or $n \neq m$. For a location parameter, the B^* statistic is almost the same as the Wilcoxon test, and the B^* statistic is more efficient than the Kolmogorov-Smirnov test for location and scale parameters. Also, the B^* statistic is more suitable for a scale parameter than the Cramér-von Mises test and the Anderson-Darling test when $n = m$.

3. Generalization of the modified Baumgartner statistic B^*

In Section 2, we have obtained the result that the B^* statistic is superior to the B statistic. Now, we generalize the modified Baumgartner statistic B^* from the two-sample to k -sample test. Recently, Neuhäuser (2001) considered a trend test based on the Baumgartner statistic for pairwise comparison.

Let $\{X_{ij} | i = 1, \dots, k, j = 1, \dots, n_i\}$ be k independent samples of size $n_1, \dots, n_k$ of independent observations. The observation X_{ij} is assumed to be obtained from a continuous distribution function $F_i(x)$. Let R_{ij} be the increasing-order rank of X_{ij} in the combined $N = n_1 + \dots + n_k$ samples. One of the problems is to test the hypothesis $H_0 : F_1 = F_2 = \dots = F_k$ against $H_1 : \text{not } H_0$. Let B_k^* be the k -sample modified Baumgartner statistic defined by

$$B_k^* = \frac{1}{k} \sum_{i=1}^k \left\{ \frac{1}{n_i} \sum_{j=1}^{n_i} \frac{(R_{ij} - E[R_{ij}])^2}{\text{Var}[R_{ij}]} \right\},$$

where

$$E[R_{ij}] = \frac{N+1}{n_i+1}j$$

and

$$\text{Var}[R_{ij}] = \frac{j}{n_i+1} \left(1 - \frac{j}{n_i+1}\right) \frac{(N-n_i)(N+1)}{n_i+2}.$$

Now, we investigate the behavior of the k -sample modified Baumgartner statistic B_k^* . We compare the power of the B_k^* statistic with the Kruskal-Wallis test W_k , the k -sample Kolmogorov-Smirnov test KS_k , the k -sample Cramér-von Mises test CV_k , the k -sample Anderson-Darling test AD_k and the k -sample Baumgartner statistic B_k . The B_k statistic is defined by

$$B_k = \frac{1}{k} \sum_{i=1}^k \left\{ \frac{1}{n_i} \sum_{j=1}^{n_i} \frac{(R_{ij} - \alpha_{ij})^2}{\beta_{ij}} \right\},$$

where

$$\alpha_{ij} = \frac{N}{n_i}j$$

and

$$\beta_{ij} = \frac{j}{n_i+1} \left(1 - \frac{j}{n_i+1}\right) \frac{(N-n_i)N}{n_i}.$$

Suppose that $F_i(x)$ is distributed as $N(\mu_i, \sigma_i^2)$, $DE(\mu_i, \sigma_i)$, $L(\mu_i, \sigma_i)$ and $\eta(\lambda_i)$, respectively. We examine the powers when the location and scale parameters are different, and the selected critical values are listed in Table 6 as estimated by simulation study.

Table 6: Critical values

n_1	n_2	n_3	$\Pr(B_k \geq b_k)$	b_k	$\Pr(B_k \geq b_k)$	b_k	$\Pr(B_k^* \geq b_k^*)$	b_k^*	$\Pr(B_k^* \geq b_k^*)$	b_k^*
10	7	5	0.010	2.989	0.050	2.120	0.010	3.170	0.050	2.213
10	10	10	0.010	3.047	0.050	2.138	0.010	3.171	0.050	2.198

The critical values were estimated by simulation study.

Table 7 to Table 10 are the results of the normal distribution, the double exponential distribution, the logistic distribution and the exponential distribution when $n_1 = n_2 = n_3$ or $n_1 \neq n_2 \neq n_3$. The significant level of this simulation is 5 %.

From Table 7 to Table 10, we can see that the power of the B_k^* statistic is almost the same as the B_k statistic when sample sizes are equal. However, the B_k^* statistic is more efficient than the B_k statistic for different sample sizes. For the location parameter, the B_k^* statistic is almost the same as the W_k statistic. For the scale parameter, the power of the B_k^* statistic is higher than those of the CV_k statistic and AD_k statistic. Also, the power of the B_k^* statistic is more suitable than that of the KS_k statistic for location and scale parameters.

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Table 7-(a): The case of $n_1 = n_2 = n_3 = 10$ for $N(\mu_i, 1)$

	μ_1 μ_2 μ_3				
	0, 0, 0	0, 0, 0.5	0, 0, 1	0, 0.5, 1	0, 1, 2
W_k	0.05	0.17	0.55	0.43	0.96
KS_k	0.05	0.14	0.45	0.34	0.91
CV_k	0.05	0.16	0.53	0.42	0.96
AD_k	0.05	0.17	0.54	0.42	0.96
B_k	0.05	0.16	0.53	0.41	0.95
B_k^*	0.05	0.16	0.53	0.41	0.95

Table 7-(b): The case of $n_1 = n_2 = n_3 = 10$ for $N(0, \sigma_i^2)$

	σ_1 σ_2 σ_3				
	1, 1, 1	1, 5, 5	1, 5, 10	1, 10, 10	1, 10, 20
W_k	0.05	0.07	0.08	0.08	0.08
KS_k	0.05	0.23	0.32	0.39	0.46
CV_k	0.05	0.22	0.34	0.35	0.47
AD_k	0.05	0.25	0.45	0.39	0.60
B_k	0.05	0.32	0.52	0.59	0.76
B_k^*	0.05	0.32	0.50	0.58	0.75

Table 7-(c): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $N(\mu_i, 1)$

	μ_1 μ_2 μ_3				
	0, 0, 0	0, 0, 0.5	0, 0, 1	0, 0.5, 1	0, 0.5, 1
W_k	0.05	0.11	0.32	0.30	0.85
KS_k	0.05	0.10	0.28	0.26	0.76
CV_k	0.05	0.11	0.31	0.29	0.83
AD_k	0.05	0.11	0.32	0.29	0.84
B_k	0.05	0.08	0.22	0.23	0.78
B_k^*	0.05	0.10	0.28	0.29	0.83

Table 7-(d): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $N(0, \sigma_i^2)$

	σ_1 σ_2 σ_3				
	1, 1, 1	1, 5, 5	1, 5, 10	1, 10, 10	1, 10, 20
W_k	0.05	0.11	0.12	0.12	0.12
KS_k	0.05	0.23	0.30	0.35	0.40
CV_k	0.05	0.22	0.29	0.32	0.38
AD_k	0.05	0.28	0.40	0.40	0.51
B_k	0.05	0.31	0.40	0.49	0.56
B_k^*	0.05	0.33	0.43	0.51	0.60

Table 8-(a): The case of $n_1 = n_2 = n_3 = 10$ for $DE(\mu_i, 1)$

	$\mu_1 \quad \mu_2 \quad \mu_3$				
	0, 0, 0	0, 0, 0.5	0, 0, 1	0, 0.5, 1	0, 1, 2
W_k	0.05	0.14	0.42	0.33	0.84
KS_k	0.05	0.14	0.41	0.32	0.81
CV_k	0.05	0.15	0.43	0.35	0.86
AD_k	0.05	0.14	0.42	0.34	0.84
B_k	0.05	0.14	0.42	0.33	0.84
B_k^*	0.05	0.14	0.42	0.34	0.84

Table 8-(b): The case of $n_1 = n_2 = n_3 = 10$ for $DE(0, \sigma_i)$

	$\sigma_1 \quad \sigma_2 \quad \sigma_3$				
	1, 1, 1	1, 5, 5	1, 5, 10	1, 10, 10	1, 10, 20
W_k	0.05	0.07	0.07	0.07	0.08
KS_k	0.05	0.16	0.25	0.30	0.38
CV_k	0.05	0.17	0.25	0.28	0.37
AD_k	0.05	0.19	0.31	0.31	0.46
B_k	0.05	0.21	0.34	0.43	0.59
B_k^*	0.05	0.21	0.34	0.43	0.58

Table 8-(c): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $DE(\mu_i, 1)$

	$\mu_1 \quad \mu_2 \quad \mu_3$				
	0, 0, 0	0, 0, 0.5	0, 0, 1	0, 0.5, 1	0, 1, 2
W_k	0.05	0.10	0.24	0.23	0.65
KS_k	0.05	0.10	0.24	0.24	0.65
CV_k	0.05	0.10	0.25	0.24	0.68
AD_k	0.05	0.10	0.25	0.24	0.68
B_k	0.05	0.07	0.18	0.19	0.61
B_k^*	0.05	0.10	0.24	0.24	0.68

Table 8-(d): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $DE(0, \sigma_i)$

	$\sigma_1 \quad \sigma_2 \quad \sigma_3$				
	1, 1, 1	1, 5, 5	1, 5, 10	1, 10, 10	1, 10, 20
W_k	0.05	0.10	0.11	0.11	0.12
KS_k	0.05	0.18	0.24	0.28	0.34
CV_k	0.05	0.18	0.23	0.27	0.32
AD_k	0.05	0.22	0.31	0.34	0.43
B_k	0.05	0.23	0.29	0.39	0.46
B_k^*	0.05	0.24	0.32	0.41	0.49

A K -sample Baumgartner Statistic

Table 9-(a): The case of $n_1 = n_2 = n_3 = 10$ for $L(\mu_i, 1)$

	$\mu_1 \quad \mu_2 \quad \mu_3$				
	0, 0, 0	0, 0, 0.5	0, 0, 1	0, 0.5, 1	0, 1, 2
W_k	0.05	0.09	0.22	0.18	0.57
KS_k	0.05	0.08	0.19	0.15	0.48
CV_k	0.05	0.09	0.22	0.17	0.56
AD_k	0.05	0.09	0.22	0.17	0.55
B_k	0.05	0.09	0.22	0.17	0.55
B_k^*	0.05	0.09	0.22	0.17	0.55

Table 9-(b): The case of $n_1 = n_2 = n_3 = 10$ for $L(0, \sigma_i)$

	$\sigma_1 \quad \sigma_2 \quad \sigma_3$				
	1, 1, 1	1, 5, 5	1, 5, 10	1, 10, 10	1, 10, 20
W_k	0.05	0.07	0.08	0.08	0.08
KS_k	0.05	0.21	0.31	0.37	0.45
CV_k	0.05	0.21	0.31	0.33	0.45
AD_k	0.05	0.24	0.41	0.38	0.56
B_k	0.05	0.29	0.47	0.55	0.72
B_k^*	0.05	0.29	0.46	0.54	0.71

Table 9-(c): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $L(\mu_i, 1)$

	$\mu_1 \quad \mu_2 \quad \mu_3$				
	0, 0, 0	0, 0, 0.5	0, 0, 1	0, 0.5, 1	0, 1, 2
W_k	0.05	0.07	0.13	0.13	0.39
KS_k	0.05	0.07	0.13	0.12	0.36
CV_k	0.05	0.07	0.14	0.13	0.39
AD_k	0.05	0.07	0.14	0.13	0.39
B_k	0.05	0.06	0.10	0.10	0.32
B_k^*	0.05	0.07	0.13	0.13	0.39

Table 9-(d): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $L(0, \sigma_i)$

	$\sigma_1 \quad \sigma_2 \quad \sigma_3$				
	1, 1, 1	1, 5, 5	1, 5, 10	1, 10, 10	1, 10, 20
W_k	0.05	0.11	0.11	0.12	0.12
KS_k	0.05	0.22	0.29	0.33	0.39
CV_k	0.05	0.21	0.28	0.30	0.36
AD_k	0.05	0.27	0.37	0.39	0.49
B_k	0.05	0.29	0.37	0.46	0.54
B_k^*	0.05	0.30	0.40	0.49	0.58

Table 10-(a): The case of $n_1 = n_2 = n_3 = 10$ for $\eta(\lambda_i)$

	λ_1		λ_2	λ_3	
	1, 1, 1	1, 2, 2	1, 2, 3	1, 3, 3	1, 3, 5
W_k	0.05	0.23	0.42	0.52	0.75
KS_k	0.05	0.18	0.34	0.43	0.67
CV_k	0.05	0.23	0.40	0.51	0.74
AD_k	0.05	0.23	0.41	0.51	0.75
B_k	0.05	0.24	0.42	0.54	0.76
B_k^*	0.05	0.23	0.41	0.53	0.77

Table 10-(b): The case of $n_1 = 10, n_2 = 7, n_3 = 5$ for $\eta(\lambda_i)$

	λ_1		λ_2	λ_3	
	1, 1, 1	1, 2, 2	1, 2, 3	1, 3, 3	1, 3, 5
W_k	0.05	0.20	0.31	0.43	0.60
KS_k	0.05	0.18	0.28	0.38	0.55
CV_k	0.05	0.20	0.31	0.43	0.59
AD_k	0.05	0.20	0.32	0.43	0.61
B_k	0.05	0.19	0.28	0.43	0.58
B_k^*	0.05	0.22	0.33	0.47	0.63

4. Conclusion and discussion

In this paper, we propose a modified Baumgartner statistic B^* and a k -sample Baumgartner statistic B_k^* . By simulation studies, we find that the B^* statistic is superior to the B statistic. Then, the B^* statistic is generalized from a two-sample to a k -sample statistic. By a power comparison, the B_k^* statistic is more efficient than the W_k statistic, the KS_k statistic, the CV_k statistic, the AD_k statistic and the k -sample Baumgartner statistic B_k . Therefore, we can say that our new statistic is more competent than the W_k statistic, the KS_k statistic, the CV_k statistic, the AD_k statistic and the B_k statistic. In future work, one of the problems is deriving the asymptotic distribution of k -sample Baumgartner statistic.

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