Orientation Dependence of In-Plane Tensile Properties of Paperboard and Cardboard: Experiments and Theories

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Abstract
The in-plane tensile properties of commercial paperboard and cardboard for packaging containers or cartons are characterized using both a compact testing machine equipped with a non-contacting optical extensometer and a high-speed digital image sensor. Uncoated white paperboard and two kinds of coated water-resistant cardboard are tested in a controlled standard atmosphere. Thin strip tensile specimens are cut on a paper cutter at five different orientation angles between machine direction (MD) and cross-machine direction (CD). The in-plane Young's moduli, Poisson's ratios and tensile strengths as a function of orientation angle are measured and compared with theoretical predictions from both the orthotropic elasticity theory and the Tsai-Hill failure theory in composite materials. It is shown that the orientation dependence of the in-plane tensile properties can be explained using composite theories.

Key words
Cardboard, Elastic Constants, Orientation Dependence, Orthotropic Elasticity Theory, Paperboard, Strip Tensile Specimen, Tensile Strength, Tsai-Hill Failure Theory

1. Introduction
Paperboard and cardboard have widely been used as paper-based materials in packaging industries. They generally exhibit highly anisotropic and non-linear mechanical behavior, strongly affected by moisture and temperature. A sufficient understanding of their mechanical behavior is of much importance in most converting operations such as folding, cutting, creasing and corrugating, as well as in end-use applications. Anisotropic elastic constants for machine-made paper have been determined by many researchers (for example, see Ref. [1]). Several constitutive models were proposed by Xia et al. [2], and Mäkelä and Östlund [3] to describe the anisotropic elastic-plastic behavior of paper and paperboard. Furthermore, the stress-strain properties of paperboard in the thickness direction were measured by Stenberg et al. [4]. An elastic-plastic model was subsequently suggested by Stenberg et al. [5] to predict the out-of-plane plastic behavior of paperboard. Nevertheless, except for highly oriented paper sheets [6] and selected commercial paper [7], their orientation dependence of the in-plane tensile properties has not been fully understood.

The primary objective of the present study is to examine the orientation dependence of in-plane elastic and tensile properties of commercial paperboard and cardboard. Tension tests were conducted on constant-width strip specimens in a controlled standard atmosphere of 23 ± 2 °C and 50 ± 2% relative humidity. The in-plane Young's moduli, Poisson's ratios and tensile strengths at five different orientation angles were measured and compared with theoretical predictions from both the orthotropic elasticity theory [8] and the Tsai-Hill failure criterion [9]. It is expected that this work will offer useful guidelines for the proper selection of packaging boards.

2. Paperboard Structures and Specimen Preparation
Commercially available multi-layered paperboard and cardboard were chosen for tension tests. Uncoated white paperboard (Dainichi Paper Corp.) is constructed from 7 layers of 100 % recycled mechanical pulps (see Fig. 1). Two types of coated water-resistant cardboard (Hokuetsu Paper Mills Ltd.) are composed of 6 layers. Two outer layers sandwiching the core are constructed from chemical pulps, whereas the four-layer core is constructed from mechanical pulps. During the manufacturing process, the axes of pulp fibers usually tend to be aligned parallel to paperboard flow through a paperboard machine. This phenomenon leads to highly anisotropy in the mechanical properties of paperboard and cardboard. Basically, there are three principal material axes for them, and MD corresponds to the direction of a moving wire of the paperboard machine. Figure 2 gives the geometry of a constant-width strip specimen specified in the ASTM D828-97[10], which was cut at five different orientation angles from MD (see Fig. 3). The strip specimen's ends were over-wrapped with chipboard doublers to prevent localized end failure during tensile loading. Their basic properties in both MD and CD under the controlled standard atmosphere are listed in Table 1.

Fig. 1 Typical macroscopic structure of multi-layered paperboard and three principal material coordinates. MD is the machine direction, CD the cross-machine direction and ZD the thickness direction. The notations 1, 2 and 3 are also used for three principal material axes.
The thicknesses of each paperboard and cardboard was carefully measured with a digital micrometer (Mitutoyo: GMA-25 DM) having a flat ground circular movable face with a 20 mm diameter under a constant pressure of 31 kPa with an accuracy of $\pm 4 \mu m$. The surface roughness $R_a$ (center-lined average roughness) was contactlessly measured with a confocal laser scanning microscope.

### Table 1 Basic properties of paperboard and two types of cardboard tested at 23°C and 50% RH

<table>
<thead>
<tr>
<th>Type of board</th>
<th>Basis weight $g$ (g/m²)</th>
<th>Thickness $t$ (µm)</th>
<th>Apparent density $\rho$ (kg/m³)</th>
<th>Roughness $R_a$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncoated white paperboard (7-layer)</td>
<td>473(430) *¹</td>
<td>700 (690)</td>
<td>614 (623)</td>
<td>5.3 MD, 5.5 CD</td>
</tr>
<tr>
<td>NEW DV310 ® (6-layer)</td>
<td>353(310) *²</td>
<td>401(N/A)</td>
<td>773(N/A)</td>
<td>0.73 (T) $^*²$ 0.66 (T)</td>
</tr>
<tr>
<td>NEW DV400 ® (6-layer)</td>
<td>466(400) *²</td>
<td>511(N/A)</td>
<td>783(N/A)</td>
<td>0.74 (T) 0.63 (T)</td>
</tr>
</tbody>
</table>

*¹( ) given by paper manufacturers; *² T (top side); *³ B (bottom side) N/A = not available

**Fig. 2** Geometry of strip tensile specimen for paperboard and cardboard (ASTM D828-97)

**Fig. 3** Five different orientation angles at which strip tensile specimens (paperboard) were cut.

**Fig. 4** SEM micrographs of top sides and longitudinal cross-sections of uncoated white paperboard (a1) (a2) and coated water-resistant cardboard (NEW DV-310) (b1) (b2)
The measured surface roughness $R_a$ of cardboard is much smaller than that of paperboard, since the former is coated on both sides with a slurry of kaolin containing other pigments.

Figure 4 shows SEM micrographs of top sides and longitudinal cross-sections of paperboard and cardboard (NEW DV-310). Pulp fibers preferentially aligned to MD are definitely observed on the top side of uncoated paperboard (Fig. 4(a1)), while no pulp fibers can be found on the top side of cardboard (Fig. 4(b1)). This is because cardboard is coated on both sides, where the respective coated layers are nearly 20 $\mu$m in thickness. A lot of pores with different sizes are visible on both longitudinal cross sections (Figs. 4(a2) and 4(b2)).

3. Test Apparatus and Procedure
It is impossible to mechanically measure the deformation of the strip specimen with a conventional strain-gage extensometer. Hence, we used a compact testing machine (JT Tohsi: Little Senstar LSC-1/30) equipped with a non-contacting optical extensometer using two CCD cameras as shown in Fig. 5. Tension tests were carried out on the specimen at a crosshead velocity of 3 mm/min. Before testing, a small pretension load of 10 N was applied to remove slack from the specimen. The applied load was measured with a load cell of 1kN capacity (load resolution: 0.5N). The longitudinal elongation over a 50-mm gage length stamped in black in the middle of the specimen was recorded with the optical extensometer (measurement accuracy: ±3 $\mu$m) at a sampling rate of 5 Hz. The resulting tensile load-elongation relation was then converted to the nominal tensile stress-strain curve.

For the measurements of Poisson's ratios, the average transverse strain during tensile loading was simultaneously determined by recording the width contraction of the gage section over a distance of 40 mm along the longitudinal direction using a high-speed digital image sensor (also referred to as a multi-camera universal machine vision system) at a sampling rate of 5 Hz (see Fig. 6). The displacement measurement accuracy of the image sensor is less than ±2 $\mu$m.

4. Results and Discussion
4.1 In-plane tensile stress-strain behavior
Figures 7 to 9 show, respectively, typical in-plane tensile stress-strain curves for paperboard and the two types of cardboard tested at five different orientation angles. These non-linear stress-strain curves clearly depict the anisotropic elastic, initial yielding and strain hardening behavior. Fracture always occurs at the maximum tensile stress (defined as the tensile strength), and the strain corresponding to the maximum tensile stress is defined as the strain-to-fracture (or the elongation). Both Young's modulus and the tensile strength decrease, whereas both the strain-to-fracture and the absorbed energy (or the area under the stress-strain curve up to fracture) increase with increasing orientation angle from MD. This is because machine-made boards have more fibers aligned in MD.
Comparison among Figs. 7 to 9 reveals that the deformation stress level of two types of cardboard is higher than that of paperboard at any orientation angle. This is probably due to the differences in the strength between the fiber core layers or those in the coupling strength between the pulp fibers used.

Figure 10 displays typical macroscopic fracture appearance of the paperboard specimens, indicating that each fracture path takes places mostly along the pulp fibers preferentially aligned. Similar macroscopic fractures occurred in the cardboard specimens tested at the five different orientation angles. It appears that the presence of the coated layers has little effect on the macroscopic fracture modes in the cardboard specimens. Meanwhile, it is impossible to identify at which edge of the strip specimen the crack was initiated, from post-test observations of the fracture surface. In contrast, microscopic fracture mode obviously varies with increasing orientation angle, from mainly pulp fiber pull-out fracture in MD to transverse tensile fracture in CD.

4.2 Orientation dependence of elastic constants

Figure 11 provides typical two relations between longitudinal and transverse strains in the paperboard specimen under 1-direction (MD) and 2-direction (CD) loadings. Small variations in the two relations arise from the non-homogeneous deformation within the gage length.

Two in-plane Poisson’s ratios $\nu_{12}$ and $\nu_{21}$ are defined as

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} \quad \text{(MD loading)}$$

$$\nu_{21} = -\frac{\varepsilon_1}{\varepsilon_2} \quad \text{(CD loading)}$$

and are determined from the initial slopes of the two relations depicted in Fig. 11. Note that the major Poisson’s ratio $\nu_{12}$ is three times as large as the minor Poisson’s ratio $\nu_{21}$ due to anisotropy, and the values of both Poisson’s ratios are quite similar to those for a five-layer paperboard reported in Ref.[2].
Following the orthotropic elasticity theory [8], Young's modulus and Poisson's ratio in the MD-CD plane can be expressed in terms of an orientation angle \( \theta \) from MD (see Fig. 3) as

\[
\frac{1}{E_x} = \frac{m^2}{E_1} \left( m^2 - n^2 v_{12} \right) + \frac{n^2}{E_2} \left( n^2 - m^2 v_{21} \right) + \frac{m^2 n^2}{G_{12}} \tag{1}
\]

\[
\frac{v_{xy}}{E_x} = \frac{m^2}{E_1} \left( m^2 v_{12} - n^2 \right) + \frac{n^2}{E_2} \left( n^2 v_{21} - m^2 \right) + \frac{m^2 n^2}{G_{12}} \tag{2}
\]

where \( m = \cos \theta \); \( n = \sin \theta \); the subscripts 1 and 2 denote MD and CD, respectively. When \( \theta = 45^\circ \) is substituted into the right-hand side of Eq. (1), we obtain the following relation:

\[
\left( \frac{1}{E_x} \right)_{\theta=45^\circ} = \frac{1 - v_{12}}{4E_1} + \frac{1 - v_{21}}{4E_2} + \frac{1}{4G_{12}} \tag{3}
\]

Since the left-hand side of Eq. (3) is found from Young's modulus measured from a 45° off-axis specimen, the shear modulus \( G_{12} \) can be then determined based on the measured four in-plane elastic constants \( E_1, E_2, v_{12} \) and \( v_{21} \). Figure 12 depicts Young's modulus as a function of orientation angle for paperboard and cardboard. Error bars on each data plot indicate the standard deviation from the mean of ten specimens at various orientation angles. Young's modulus decreases monotonically from its maximum value \( E_1 \) at \( \theta = 0^\circ \) to its minimum value \( E_2 \) at \( \theta = 90^\circ \). The measured Young's moduli are in fairly good agreement with the theoretical ones from Eq. (1). Similarly, Fig. 13 shows Poisson's ratio against orientation angle for the respective boards. Poisson's ratio also decreases monotonically from its maximum value \( v_{12} \) at \( \theta = 0^\circ \) to its minimum value \( v_{21} \) at \( \theta = 90^\circ \). The measured Poisson's ratios also agree fairly well with the theoretical ones from Eq. (2). Some possible reasons for the discrepancies between the predictions and the experimental results may be that all of the pulp fibers in the respective boards are not necessarily aligned to MD, and both sides of cardboard are coated.

4.3 Orientation dependence of tensile properties

The tensile strength for a unidirectional lamina as a function of fiber orientation \( \theta \) is given from the well-known Tsai-Hill failure criterion [9] as

\[
\frac{1}{\sigma^2} = \frac{m^4}{\sigma_1^4} \left( 1 - \frac{1}{S} \right) + \frac{n^4}{\sigma_2^4} \left( 1 - \frac{1}{S} \right) + \frac{m^2 n^2}{\sigma_1^2 \sigma_2^2} \tag{4}
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the tensile strengths in MD (\( \theta = 0^\circ \)) and CD (\( \theta = 90^\circ \)); \( S \) is the in-plane pure shear strength. Figure 14 indicates the orientation dependence of the tensile strength for paperboard and cardboard. The measured tensile strengths are in close agreement with the theoretical ones obtained from the Tsai-Hill failure criterion. This implies that the Tsai-Hill failure criterion is valid for predicting the tensile strength of the respective boards at an arbitrary orientation angle \( \theta \). Figures 15 and 16, respectively, show the orientation dependence of the strain-to-fracture and the energy absorption. Unlike the tensile strength, no theories are available for predicting the strain-to-fracture and the energy absorption at an arbitrary orientation angle \( \theta \).
Figure 17 presents the relation between tensile strength and Young’s modulus measured at three specified orientation angles (or MD, 45˚ and CD) for the respective boards, indicating that there is a strong correlation between tensile strength and Young’s modulus.

5. Conclusions

The in-plane tensile stress-strain characteristics of multi-layered paperboard and water-resistant cardboard have been determined at five different orientation angles to MD in the controlled standard atmosphere. The orientation dependence of the in-plane elastic and tensile properties was evaluated using composite theories. From the current work, we conclude the following:

1. The orientation dependence of Young’s moduli and Poisson’s ratios can be well predicted from the orthotropic elasticity theory.
2. The orientation dependence of tensile strengths can be analyzed using the Tsai-Hill failure criterion for composite laminates.
3. There is a strong correlation between tensile strength and Young’s modulus.

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References