Effect of Initial Acceleration History on Transition to Turbulence in Pipe Flow

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Abstract
Turbulent flows in pipes are usually accelerated from rest using blowers and pumps in practical applications. The history of cross-sectional mean velocity consists of the initial stage of flow acceleration and the subsequent stage of constant cross-sectional mean velocity. The effect of the initial acceleration on the transition to turbulence in such flows is not fully understood yet. In the present study experimental investigation is carried out to reveal this effect in a circular pipe using air as the working fluid.

Key words
Acceleration, Transition to Turbulence, Unsteady Flow, Circular Pipe, Flow Control

1. Introduction
In our previous studies [1, 2], transition to turbulence in a pipe flow was found to be significantly delayed by imposing constant-acceleration on the flow until the cross-sectional mean velocity reached a constant value. Such a flow with a constant-acceleration followed by a constant cross-sectional mean velocity was generated using a newly developed unsteady flow generation system. A butterfly valve was used and it was driven by a stepper motor following rotation speeds programmed to generate an accelerating flow in a brass pipe. The valve was not rotated but swung around its supporting rod. The axial velocity component of air flow was measured with a hot-wire anemometer.

Previous experimental results [1]-[15] collectively suggest that the transition to turbulence and re-transition to laminar flow can be controlled by applying acceleration and deceleration periodically with predetermined intervals. These intervals can readily be changed by controlling the opening area between the butterfly valve and the pipe wall. The degree of acceleration and deceleration can also be arbitrarily changed. Accordingly, the unsteady flows mentioned here are basically different from usual pulsating flows having constant pulsation period and constant amplitude [16]. If the control of the above-mentioned two transitions is possible, the control of frictional loss, heat transfer, and mass transfer is practically realized.

In this study three types of cross-sectional mean velocity histories were chosen on the initial stage of flow acceleration to investigate the acceleration history on the transition to turbulence. Two of the histories were not linear with respect to time unlike the previous study [1, 2]. The delay time for the transition to turbulence, i.e., the time lag, \( T_{\text{lag}} \), from the initiation of the constant velocity condition to the onset of turbulence was measured. The data on \( T_{\text{lag}} \) were found to be dependent on the history of acceleration. In addition, the radial distribution of the axial velocity component in the acceleration condition was compared with the analytical solution [17].

2. Wave Forms of Flows with Different Three Types of Histories of Cross-Sectional Mean Velocity
Fig.1 shows a schematic diagram of the histories of the cross-sectional mean velocity in composite acceleration/constant-velocity pipe flows. Symbols used in this study also are described in this figure.

Fig.2 provides details of the predetermined histories of three types of flows, where \( t_1 \) [s] is the time duration of the initial acceleration, \( T_{\text{lag}} \) is the delay time for the transition to turbulence, \( t_\text{tr} \) is the transition time measured from the start of flow, \( u_{\text{m,st}} \) [m/s] is the final cross-sectional mean velocity. The time duration, \( t_1 \), can be predetermined in this study, and \( T_{\text{lag}} \) and \( t_\text{tr} \) are measured with a hot-wire anemometer.

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Fig.1 Schematic diagram of the histories of the cross-sectional mean velocity

Fig.2 Predetermined waveforms of three types of acceleration flows followed by constant-velocity flow
3. Experimental Apparatus and Procedure

Fig. 3 shows a schematic diagram of the experimental apparatus. The circular test pipe made of brass had an inner diameter of \( D = 78 \text{mm} \). The length and wall thickness of the pipe were 5000mm and 5mm, respectively. The volumetric flow rate, \( Q \), was obtained by integrating the radial distribution of the axial velocity component measured with a hot-wire anemometer. Subsequently, the cross-sectional mean velocity, \( u_m \), was calculated by dividing the volumetric flow rate, \( Q \), by the cross-sectional area of the pipe, \( A = \pi D^2 / 4 \).

The arbitrary-acceleration/constant-velocity flow generating system consists of a butterfly valve and a stepper motor. The valve is driven following a programmed rotation speed to generate an arbitrary-acceleration flow followed by a constant-velocity condition. Target input histories shown in Fig. 2 were employed and denoted as Run 1, Run 2, and Run 3, respectively. The axial velocity component of air flow was measured using a hot-wire anemometer. Transition to turbulence was ascertained by monitoring the root-mean-square (RMS) level of the axial velocity signal.

The output signal of the hot-wire anemometer was digitized and then processed on a personal computer. The start time of measurement (\( t = 0 \)) was defined by using a trigger function for the A/D converter that was coupled to the initial motion of the butterfly valve.

The velocity was measured in the cross-sections at \( x = 2000 \text{mm} \) (25.6\( D \)), 3000mm (38.5\( D \)) and 4000mm (51.3\( D \)), as shown in Fig. 3. An I-probe was traversed in the vertical direction with measurements being made at twenty radial positions: \( r/R = 0.00, 0.08, 0.15, 0.23, 0.31, 0.38, 0.46, 0.54, 0.62, 0.67, 0.72, 0.77, 0.82, 0.87, 0.90, 0.92, 0.95, 0.96, 0.97 \) and 0.99, where \( r \) was measured from the centerline towards the bottom wall. The position of \( r/R = 0.00 \) is located on the centerline of the pipe.

The root-mean-square (RMS) value of the axial turbulence component can be calculated from

\[
\sqrt{\frac{1}{N-1} \sum_{j=1}^{N} (u_j - u_{\text{ta},j})^2}
\]

where \( u_{\text{ta},j} \) is the short-time averaged value of the axial velocity component (\( u_{\text{ta},j} = 5^{1/2} \text{ms} \)), \( u \), and \( N \) is the number of measurement points with respect to time. The long-time averaging is used in investigations of steady turbulent flows. In this case the period of the averaging is very long. Meanwhile, the short-time averaging is commonly used for investigating unsteady turbulent flows. The period should be much longer than the characteristic time such as the burst period relating to the turbulence generation but much shorter than the characteristic time describing the unsteady nature of the flow such as the pulsation period. The details of the calculation method are the same as those given in Refs. [1, 2].

The initiation time for turbulence was defined as the time from the start of the flow until the instant at which turbulent fluctuations appeared on the axial velocity component. The RMS fluctuation level of the axial turbulence component was used to ascertain the transition to turbulence in the same manner to the previous investigations.

4. Results and Discussion

4.1 Axial velocity component and transition to turbulence

Table 1 summarizes the measured values of the initiation time of turbulence, \( t_{\text{tr}} \), acceleration imposition period, \( t_1 \), time lag, \( T_{\text{lag}} \), cross-sectional mean velocity of the final steady pipe flow, \( u_{m,\text{st}} \), and the final Reynolds number, \( \text{Re}_{\text{st}} \).

Figures 4, 5 and 6 show the axial velocity component in Run 1 through Run 3 at \( r/R = 0.00 \) (centerline), 0.72 and 0.92 (near the bottom wall) for \( x = 4000 \text{mm} \) (\( x/D = 51.3 \)), respectively. The velocity signal before the transition to turbulence does not approach a constant value in each run.

\[
\Delta u_j = \sqrt{\frac{1}{N-1} \sum_{j=1}^{N} (u_j - u_{\text{ta},j})^2}
\]

\[
u_{\text{ta},j} = \frac{1}{N} \sum_{j=1}^{N} u_j
\]
The amplification of the initial velocity fluctuations would be most significantly suppressed in Run 3. This is inferred from the fact that the cross-sectional mean velocity in Run 3 is much lower than those in Run 1 and Run 2 in the initial acceleration stage \((t<1.05s)\). It is evident that the history of the initial acceleration causes different delay of transition to turbulence. As expected, the longest time delay was observed in Run 3.

Figure 7 shows the measured values of the root-mean-square (RMS) values of the axial velocity component, \(u\). The RMS value is denoted by \(\Delta u\). The \(\Delta u\) values for \(t<1.05s\) in Run 3 are kept at the lowest level among Run 1 through Run 3.

Table 1  Measured values for three cases

<table>
<thead>
<tr>
<th>(x=2000mm)</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_0) [s]</td>
<td>1.52</td>
<td>1.55</td>
<td>1.60</td>
</tr>
<tr>
<td>(t_1) [s]</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>(T_{lag}) [s]</td>
<td>0.47</td>
<td>0.50</td>
<td>0.55</td>
</tr>
<tr>
<td>(u_{max}) [m/s]</td>
<td>3.71</td>
<td>3.74</td>
<td>3.73</td>
</tr>
<tr>
<td>(Re_{st}) [-]</td>
<td>19300</td>
<td>19400</td>
<td>19400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(x=3000mm)</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_0) [s]</td>
<td>1.77</td>
<td>1.80</td>
<td>1.84</td>
</tr>
<tr>
<td>(t_1) [s]</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>(T_{lag}) [s]</td>
<td>0.71</td>
<td>0.75</td>
<td>0.79</td>
</tr>
<tr>
<td>(u_{max}) [m/s]</td>
<td>3.68</td>
<td>3.67</td>
<td>3.64</td>
</tr>
<tr>
<td>(Re_{st}) [-]</td>
<td>19100</td>
<td>19100</td>
<td>18900</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(x=4000mm)</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
</tr>
</thead>
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<tr>
<td>(t_0) [s]</td>
<td>1.93</td>
<td>2.00</td>
<td>2.04</td>
</tr>
<tr>
<td>(t_1) [s]</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>(T_{lag}) [s]</td>
<td>0.88</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td>(u_{max}) [m/s]</td>
<td>3.69</td>
<td>3.64</td>
<td>3.68</td>
</tr>
<tr>
<td>(Re_{st}) [-]</td>
<td>19200</td>
<td>18900</td>
<td>19100</td>
</tr>
</tbody>
</table>

4.2 Propagation of turbulence

The measured values of time lag, \(T_{lag}\), are shown against the axial distance, \(x\), in Fig.8. The three kinds of lines indicate the time lags calculated from the data on the propagation velocity of the leading edge of a turbulent slug in a steady pipe flow [16]. The measured and estimated values of \(T_{lag}\) are in good agreement with each other. This fact suggests that the propagation velocity of turbulence measured in this study is equal to that in a steady pipe flow.
It is interesting to note that the period $T_{lag}$ (in which the axial velocity exhibits unsteadiness) is almost equivalent to the duration of the initial acceleration, i.e., the flow before the transition to turbulence in this experiment is not in a steady state and still behaves in an unsteady manner.

The measured value of the non-dimensional cross-sectional mean velocity, $u_n'$, for Run 1, 2 and 3 are plotted in Fig.9 against the non-dimensional time, $t'$. Non-dimensional parameters used in this paper are defined as follows

$$u' = \frac{Ru}{\nu}, \quad u_n' = \frac{u_nR}{\nu}, \quad r' = \frac{r}{R}, \quad t' = \frac{t\nu}{R^2}$$  \hspace{1cm} (3)$$

Comparison of Fig.2 with Fig.9 indicates that the predetermined histories can be satisfactorily realized using the previously developed flow generation system. The solid lines are the best-fit lines for the measured values in each run. The best-fit line was drawn so as to pass through the mean of the measured values as close as possible. These lines were used for calculating the laminar analytical solution of the axial velocity component.

We deal with an arbitrary acceleration flow whose cross-sectional mean velocity is given by the third-order polynomial [17], i.e.,

$$u_n'(t') = \begin{cases} a_1 t'^3 + a_2 t'^2 + a_3 t', & (0 \leq t' \leq t_1) \\ a_1 t_1'^3 + a_2 t_1'^2 + a_3 t_1', & (t' > t_1') \end{cases}$$  \hspace{1cm} (4)$$

where $a_1$, $a_2$ and $a_3$ are constant values. The dimensionless cross-sectional mean velocities for Run1 through Run3 are given as follows:

- **Run1**
  $$u_n'(t') = 6302385797 \ t'^3 - 142212942 \ t'^2 + 1741065 \ t'$$  \hspace{1cm} (5)$$

- **Run2**
  $$u_n'(t') = 913952 \ t'$$  \hspace{1cm} (6)$$

- **Run3**
  $$u_n'(t') = 8010953445 \ t'^3 - 39556131 \ t'^2 + 481793 \ t'$$  \hspace{1cm} (7)$$

The exact solution to the axisymmetric Navier-Stokes equations is solved with the no-slip boundary condition on the pipe wall and the centerline velocity being finite, using the Laplace transform. The dimensionless axial velocity component, $u'_n$, is then given by (see [17])

$$u'(r'; t') = a_1 W_1(r'; t') + 2 a_2 W_2(r'; t') + 6 a_3 W_3(r'; t')$$

$$- H(t'' - t') \left[ \left( 3 a_1 t''^3 + 2 a_2 t''^2 + a_3 \right) Z_1(r' - t'') \\
+ 6 a_1 t''^3 + a_3 Z_2(r' - t'') + 6 a_3 Z_3(r' - t'') \right]$$

$$W_1(r'; t') = 2(1 - r'^2) \ t''^3 - \frac{1}{24} (1 - r'^2) (1 - 3r'^2)$$  \hspace{1cm} (8)$$

$$W_2(r'; t') = (1 - r'^2) \left( t''^3 - \frac{1}{6} t''^2 + \frac{5}{576} \right) + \frac{1}{8} (1 - r'^2) \left( t'' - \frac{1}{12} \right)$$

$$+ \frac{1}{288} (1 - r'^6)$$  \hspace{1cm} (9)$$

$$W_3(r'; t') = \frac{1}{3} (1 - r'^2) \left( t''^3 - \frac{1}{4} t''^2 + \frac{5}{192} t'' - \frac{351}{311040} \right)$$

$$+ \frac{1}{16} (1 - r'^2) \left( t''^2 - \frac{1}{6} t'' + \frac{5}{576} \right)$$

$$+ \frac{1}{288} (1 - r'^6) \left( t'' - \frac{1}{12} \right) + \frac{1}{18432} (1 - r'^8)$$  \hspace{1cm} (10)$$

$$Z_n(r'; t') = \left[ W_1(r'; t') + \sum_{n=1}^{\infty} G_n(r'; t') \right]$$  \hspace{1cm} (11)$$

$$G_n(r'; t') = (-1)^n 4 \exp(-y_n^2 r') \frac{y_n^2}{y_n^4 J_n(y_n)} (J_n(y_n) - J_n(y_n r'))$$  \hspace{1cm} (12)$$

where $y_n$ is the $n$-th zero of the Bessel function, $J_n(y)$.

In Figs.10, 11 and 12 the measured and analytical values of the axial velocity component in the acceleration periods are compared with each other. Satisfactory agreement between the measured and analytical velocity distributions can be seen in each run. Fig.13 shows the same degree of agreement for the velocity distributions in the constant-velocity period. The solid line denotes the analytical solutions for Run2. The analytical solutions for Run1 and Run 3 are not shown in order to avoid crowding in the figure.
where Re (= $2u_{in}/\nu$) is the Reynolds number. The measured velocity distribution after the transition to turbulence in Run 2 was approximated by the $1/n$-th power law, as shown in Fig.14. The same degree of approximation was observed for Run 1 and Run 3 as well although the $1/n$-th power law distributions were not drawn in Fig.14 in order to avoid crowding in the figure.
5. Conclusions
Experimental investigation was carried out using air as the working fluid to understand the effect of the initial acceleration history on the transition to turbulence in a composite acceleration/constant-velocity pipe flow. The main findings obtained in this study can be summarized as follows:

(1) The predetermined history of a composite acceleration/constant-velocity pipe flow could be satisfactorily realized using a previously developed flow generation system.

(2) The initial acceleration history affected the transition to turbulence significantly. This fact is closely related to the suppression of the amplification of the initial velocity fluctuations.

(3) The propagation velocity of turbulence measured in this study agreed with that reported for a steady pipe flow.

(4) The measured values of the axial velocity component were satisfactorily approximated by the laminar analytical solution in the period in which the flow was laminar. After the transition to turbulence the measured axial velocity component was in good agreement with the $1/n$-th power law.

Nomenclature

\begin{itemize}
\item $a$ acceleration, m/s$^2$
\item $D$ pipe diameter, m
\item $r$ radial distance measured from pipe centerline, m
\item $R$ pipe radius, m
\item $Re$ Reynolds number, [-] $u_m D/\nu = 2u_m^*\nu$
\item $Re_d$ final Reynolds number, [-] $u_m D/\nu$
\item $T_{lag}$ time lag, s
\item $t$ time, s
\item $t_i$ initiation time of turbulence, s
\item $u$ axial velocity component, m/s
\item $u_m$ cross-sectional mean velocity, m/s
\item $x$ axial distance measured from the pipe inlet, m
\item $\nu$ kinematic viscosity, m$^2$/s
\end{itemize}

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References


