Behavior of a Droplet on an Inclined Plate under Various Wettability Conditions

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(Received 18 December 2009; received in revised form 30 April 2010; accepted 12 June 2010)

Abstract
The aim of this study is to numerically and experimentally investigate dynamic behavior of a droplet impacting and bouncing on an inclined plate under various wettability conditions. The visualization techniques employed include Computational Fluid Dynamics (CFD) and experimental observation with a high-speed camera. The present computational model is verified for a droplet impacting on a horizontal plate. The dynamic behavior of a droplet bouncing on an inclined plate is then observed under various high-hydrophobic conditions. This paper presents some results for snapshots of bouncing motion, trajectory, the coefficient of restitution and the deformation rate at the impact on the plate.

Keywords
Droplet, Wettability, Contact Angle, Surface Tension, Experimental Visualization, CFD

1. Introduction
Dynamic behavior of a droplet on a solid wall is an intriguing subject in some practical engineering applications, e.g., ink-jet printer, soldering and spray cooling [1-3]. In the application, wettability plays a key role to promote a chemical reaction or heat and mass transfer (the wettability is usually poor, i.e., hydrophobic). Also, in the steelmaking processes, the refractory with poor wettability is usually used to inhibit reactions between the refractory and molten steel.

Recent review on the wettability and surface tension physics is found in the textbook of de Gennes, Brochard-Wyart & Quéré [4]. The ability of drops to stick to non-horizontal surface of solid was analyzed on the basis of the method of asymptotic expansions [5]. The critical inclination angle of plate for sliding down of a liquid drop has been investigated in a series of articles of Kato et al. [6-8] which presented delicate experimental results and established the mathematical model based on the usual axisymmetric Laplace-Young equation. Sonoyama & Iguchi [9] also calculated the advancing and receding contact angles of a bubble or droplet by solving the system that the net energy of the bubble or droplet including the influence of the gravity and surface tension becomes minimum in an equilibrium state. Later, following Sonoyama & Iguchi [9], Kagawa & Iguchi [10] investigated a detachment shape of a single silicone oil droplet from a downward single-hole nozzle in a vessel filled with water. In addition, Yokoi et al. [11] computed the influence of the dynamic contact angle on a droplet impacting on a dry surface with the use of a hybrid model of a level-set method and a volume-of-fluid (VOF) method, and compared the results against the experimental ones with a high-speed camera.

Although only one research group [12] in France dealt with dynamic behavior of a droplet on a super-hydrophobic inclined plate (the contact angle being approximately 170 deg), the detailed mechanism on the target issue is still open as far as the authors know.

This study investigates the dynamic behavior of a water droplet bouncing and moving down on an inclined plate (see Fig.1) whose wettability is experimentally adopted at the two values of 70 (deg) and 147 (deg) and numerically varied between 70 (deg) and 178 (deg).

2. Experimental Procedure
Figure 2 shows a schematic of the experimental apparatus and physical setting. The flat plate made of transparent acrylic resin was 342 (mm) long and 75 (mm) wide. The angle \( \theta_p \) of the inclined plate was set at 15 (deg). The acrylic plate, in general, exhibits an equilibrated contact angle of 70 (deg) between the clean water and the air. The present experiment used a water repellent to vary the contact angle \( \theta_c \) to 147 (deg). A single water droplet, whose volume \( V_D \) was either 10.0 or 100 (mm\(^3\)), was smoothly released from a micropipette at the height of 4.97 (mm) from the plate (see Fig.2). The behavior of the

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Fig. 1 Photograph of a water droplet of \( V_D = 2.0 \) (mm\(^3\)) impacting and bouncing on a hydrophobic (\( \theta_c = 147 \)deg) inclined flat plate (\( \theta_p = 4.0 \) deg). The photo was taken in a long-time exposure with continuous point lighting.

Fig. 2 Experimental apparatus and physical setting
3. Numerical Procedure

In all numerical predictions, FLUENT™ ver.6.2.16 was employed on 2.0 (GHz) Intel Core 2 Duo processor with 2.0 (GB) RAM. GAMBIT was used for the construction of the three-dimensional computational grids.

The computational grid was made of cubic 404,250 elements (−2.61D<x<21.95D and 0<y<2.61D and 0<z<2.61D, D: diameter of an initial droplet), as shown in Fig. 3. To save a CPU time the droplet and flow field were assumed to be symmetric with respect to the symmetry face shown in Fig.3. The contact angle of the plate was varied between 70 (deg) and 178 (deg). The spherical water droplet, having \( V_D = 10.0 \) (mm\(^3\)), was set at the height of 4.97 (mm) from the inclined plate, which was identical with the above-mentioned experimental condition.

In the FLUENT code a segregated implicit solver and first-order upwind interpolation scheme were employed. A small time-step of \( \Delta t = 1.0 \times 10^{-4} \) (s) was adapted to achieve a convergence in every iteration. A free surface behavior was tracked by the VOF model. The contribution of a volume force from a wall adhesion was added to the momentum equation as the source term, \( F_{vol} = \sigma_i (\rho \kappa_i \cdot \text{grad} \alpha_i) / [(1/2) (\rho_1 + \rho_2)] \) with the curvature \( \kappa_i = \text{div} \ n \) and the surface normal \( n = \text{grad} \ alpha \). Here, \( \rho \) is the density of the liquid \( i \), \( \rho_1 \) is the density of the gas \( j \), \( \rho_2 \) is the volume-averaged density, \( \alpha_i \) is the volume fraction of the liquid \( i \), and \( \sigma_i \) is the surface tension. The wettability (meniscus) was taken into account using the geometrical condition \( n = n_x \cos \alpha + t_x \sin \alpha \) on the contact line where \( n_x \) and \( t_x \) are the normal and tangential vectors on a wall. In this computation, we adopted the sufficiently small time-step \( \Delta t \) during which the droplet would deform a little. This allows us employ the following Laplace-Young equation: \( \Delta p = \sigma_{12} (1/R_1 + 1/R_2) \), where \( \Delta p \) is the pressure difference between the two fluids and, \( 1/R_1 \) and \( 1/R_2 \) are the curvature, respectively (see FLUENT 6.2 User’s Guide [13] for more details).

4. Results and Discussion

4.1 A water droplet impacting on a horizontal plate

The dynamic behavior of a water droplet impacting on a horizontal plate (\( \theta_i = 0.0 \) deg) makes it possible to verify the present numerical model against the experimental one obtained with the high-speed camera.

Figure 4 shows a comparison of computed temporal surface behavior of the droplet on two values of the contact angles of 70 (deg) (hydrophilic) and 147 (deg) (hydrophobic). At \( t = 0.0 \) (s) a spherical water droplet, having a diameter \( D = 5.76 \) (mm) and made of 1,290 elements, is set to contact on the plate (see the left of Fig.4). The droplet is observed to impact on the plate due to the gravity and then reaches a steady-regime as oscillating. As seen in Fig.5, the computed steady-regime of the droplet for \( \theta_i = 147 \) (deg) is in good agreement with the experimental photograph.

4.2 Behavior of a bouncing droplet on an inclined plate

This subsection considers the behavior of a spherical droplet of \( V_D = 10.0 \) (mm\(^3\)) bouncing on an inclined plate (\( \theta_i = 15\)deg) with various contact angles. The single water droplet is set at the height of 4.97 (mm) from the inclined flat plate. Then, the droplet impacts on the plate due to the gravity and bounces down on the plate under a hydrophobic condition in which the contact angle is larger than a certain value (see Fig.1).

Figure 6 shows the selected computational results (\( \theta_i = 70, 147 \) and 170 deg) of instantaneous snapshots around the first impact of the droplet on the plate together with two sets of experimental photographs (\( \theta_i = 70 \) and 147 deg) shown in Fig.7. In the computational results the vector plots are also drawn to understand an instantaneous situation of the impact motion of the droplet. It seems that the present computation is in good agreement with the
Fig. 6 Computational snapshots of a droplet impacting on an inclined plate ($\theta_p = 15$ deg) during the first impact for (top): $\theta_c = 70$ (deg); (middle): $\theta_c = 147$ (deg); (bottom): $\theta_c = 170$ (deg)

Fig. 7 Experimental photographs of a droplet impacting on an inclined plate ($\theta_p = 15$ deg) during the first impact for (top): $\theta_c = 70$ (deg); (bottom): $\theta_c = 147$ (deg)
experimental one. As shown in Figs. 6 and 7, there is no bouncing of the droplet for the contact angle being smaller than approximately 120 (deg) while the droplet is apparently observed to bounce on the hydrophilic plate ($\theta_c \geq 130$ deg). The droplet is observed to remarkably deform like a pancake at the impact and then loses the energy due to the surface tension rather than the gravity.

The deformation rate of the droplet is defined as $e_d = D_e / D$, where $D_e$ denotes the maximum width of the droplet during the impact motion. As shown in Fig. 8, the value of $e_d$ becomes smaller with the increase of $\theta_c$ while it becomes slightly larger for $\theta_c = 170$ and 178 (deg) (this is still open and further investigation is required as a future work). Figure 9 shows the coefficient of restitution (COR) $e$ which is defined as $e = (h_{n-1} / h_n)^{1/2}$. The values of $e$ are estimated on the experimental results for $\theta_c = 15.0$ (deg) and is approximately obtained as $e = 0.65$ (first impact), 0.70 (second impact), 0.84 (third impact). The COR is observed to increase in a gradient of $\theta_c^{1/2}$ for the experimental results of the first impact (notwithstanding the COR exhibiting almost constant value for the second impact) and to approach a constant value of 0.75 at around $\theta_c = 170$ (deg).

**Fig. 8** Deformation rate $e_d$ of the droplet at the impact on the plate.

**Fig. 9** The coefficient of restitution of the droplet at the impact on the plate. The CFD results of the first impact is fitted as $e = 0.074 \times \theta_c^{1/2} - 0.213$

Figure 10 shows the trajectory of the bouncing droplet on the inclined plate of 15 (deg) with the contact angles being larger than 130 (deg). Trajectory of a point object of Eqs. (7)–(9) derived from Newton's second law is also depicted in red line. The values of $e$ were 0.65 (first impact), 0.70 (second impact), 0.84 (third impact). (Blue): experimental result for $\theta_c = 147$ (deg), (Circle): CFD for $\theta_c = 178$ (deg), (Triangle): CFD for $\theta_c = 147$ (deg) and (Square): CFD for $\theta_c = 130$ (deg).

**Fig. 10** Trajectory of a bouncing droplet on the inclined plate of 15 (deg) with the contact angles being larger than 130 (deg). Trajectory of a point object of Eqs. (7)–(9) derived from Newton’s second law is also depicted in red line. The values of $e$ were 0.65 (first impact), 0.70 (second impact), 0.84 (third impact). (Blue): experimental result for $\theta_c = 147$ (deg), (Circle): CFD for $\theta_c = 178$ (deg), (Triangle): CFD for $\theta_c = 147$ (deg) and (Square): CFD for $\theta_c = 130$ (deg).

5. Conclusions

This study has investigated the dynamic behavior of a water droplet impacting and bouncing on an inclined plate with various contact angles. The visualization techniques employed were the high-speed camera and the computation with the aid of the FLUENT numerical software. Main findings are summarized as follows:

1. The computed shape of an equilibrium droplet on a hydrophobic plate was in good agreement with the experimental result. The present numerical model is thus thought to be valid for a simulation of a bouncing movement of a droplet.

2. On a bouncing movement of a droplet, the dynamic behavior was observed from the viewpoint of numerical simulation and experiment with the high-speed camera.

3. At the impact of the droplet on the plate, the droplet deforms and then loses the energy due to the surface tension.

4. This study measured the coefficient of restitution at the impact, and estimated the deformation rate of the droplet.
Appendix – Trajectory of a point object –

The movement of a point object can be easily obtained on the basis of Newton’s second law. The point object is set at the height of \( h \) (see Fig. 11) and it is freely released. Then, the object falls due to the gravity and collides on the wall whose slope is \( \theta_p \).

The velocity of the object at the collision on the slope is obtained as \( v_0 = (2gh)^{1/2} \) using the well-known conservation of energy between the kinetic energy and the potential energy. Here, \( g \) is the acceleration due to gravity.

Now, we transform the coordinates \((X, Y)\) to the one \((x, y)\) along the slope surface, as shown in Fig. 12. This transform simplifies the calculation of the trajectory of the object. Then, the \( x \)- and \( y \)-components of the velocities, \( v_{x1} \) and \( v_{y1} \), after the first collision have the relation of

\[
v_{x1} = v_{x0} \quad \text{and} \quad v_{y1} = e_1 v_{y0}
\]

where the coefficient of restitution at the \( n \)th impact on the slope is \( e_n \). The bouncing object after the first collision collides again on the slope in the time of \( t_1 = 2v_{y1} / (g \cos \theta_p) \).

Similarly, the velocities, \( v_{x2} \) and \( v_{y2} \), after the second collision are described as

\[
v_{x2} = v_{x0} + 2e_1 v_{y0} \tan \theta_p \quad \text{(2)}
\]
\[
v_{y2} = e_2 v_{y1} = e_1 e_2 v_{y0} \quad \text{(3)}
\]

Therefore, we have the general description on the velocity after the \( n \)th collision:

\[
v_{x1} = v_{x0} \quad \text{for } n = 1 \quad \text{(4)}
\]
\[
v_{xn} = v_{x0} + 2v_{y0} (\tan \theta_p) \sum_{j=1}^{n-1} (\prod_{i=1}^{j} e_i) \quad \text{for } n \geq 2 \quad \text{(5)}
\]
\[
v_{yn} = (\prod_{j=1}^{n} e_j) v_{y0} \quad \text{for } n \geq 1 \quad \text{(6)}
\]

The moving distance in the \( x \)-direction (together with \( y_n \)) during the \( n \)th bounce (Fig. 13) is easily obtained from the above velocities.

\[
x_i = 0 \quad \text{for } n = 1 \quad \text{(7)}
\]
\[
x_i = x_{i-1} + v_{x_{i-1}} t_{i-1} + \frac{1}{2} g t_{i-1}^2 \sin \theta_p \quad \text{for } n \geq 2 \quad \text{(8)}
\]
\[
y_n = v_{y0} \left( \frac{1}{2} t_0 \right) - \frac{1}{2} g \left( \frac{1}{2} t_n \right)^2 \cos \theta_p \quad \text{for } n \geq 1 \quad \text{(9)}
\]

with \( t_n = 2v_{y0} / (g \cos \theta_p) \).

References


